Geometry SOL Review

Study Guide of Important Information

G.1 Logic

<table>
<thead>
<tr>
<th>Conditional</th>
<th>Converse</th>
<th>Inverse</th>
<th>Contrapositive</th>
<th>Biconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>p → q</td>
<td>q → p</td>
<td>~p → ~q</td>
<td>~q → ~p</td>
<td>p ↔ q</td>
</tr>
</tbody>
</table>

- Contrapositive is true when the conditional is true.
- Converse and inverse have the same truth value
- Additional symbols: “and” “or”

Law of Detachment

*one conditional statement
*second statement sounds like first statement hypothesis
*conclusion sounds like first conclusion

Law of Syllogism

*two conditional statements
*first conclusion repeats as second hypothesis
*conclusion is: If (1st hypothesis then 2nd conclusion)

G.2 Coordinate Formulas and Transformations

Formulas:
- Midpoint: \( \left( \frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right) \)
- Distance: \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
- Slope: \( \frac{y_2 - y_1}{x_2 - x_1} \)

Transformations:
- Translation (slide)
- Reflection (flip/fold)
- Rotation (spin/turn)

Symmetry:
- Line: fold line; folds figure exactly in half, one half onto the other
- Rotational: spin figure by a degree value and figure matches onto itself
- Point: has rotational symmetry of 180°

Caution!
- Parallelogram – point symmetry only!!!
- Rhombus – line symmetry & point symmetry!!!

Venn Diagram

* all small in large
* some large in small
* some of each overlap in the other
* none when no overlap

* slopes of parallel lines are equal
* slopes of perpendicular lines are negative reciprocals; product is -1

Vertical Lines:
- Slope is undefined
- Equation is \( x = # \)

Horizontal Lines:
- Slope is 0
- Equation is \( y = # \)
**G.3 – Angle Relationships**

**Congruent Angles**

*(If lines are parallel)*
- Vertical \( \angle 2 \cong \angle 3 \)
- Alternate Interior \( \angle 4 \cong \angle 5 \)
- Alternate Exterior \( \angle 1 \cong \angle 8 \)
- Corresponding \( \angle 3 \cong \angle 7 \)

**Triangles**

- Sum of interior angles is 180
  \[
  m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ
  \]

- Measure of an exterior angle is equal to the sum of its two remote interior angles.
  \[
  m\angle 4 = m\angle 1 + m\angle 2
  \]

**G.4 Ways to prove lines are parallel**

- Alternate interior angles are congruent
- Corresponding angles are congruent
- Consecutive interior angles are supplementary
- The two lines are perpendicular to the same line

**G.5 Congruent and Similar Triangles**

**Congruent Triangles**

- Corresponding angles are congruent
- Corresponding sides are congruent
- Ways to prove triangles are congruent
  - SSS, SAS, ASA, AAS, HL (for right triangles)

**Similar Triangles**

- Corresponding angles are congruent
- Corresponding sides are proportional
- Ways to prove triangles are similar
  - AA~, SSS~, SAS~

**G.6 Triangle Inequalities**

To form a triangle, sum of smaller two lengths must be greater than the largest

- \( L > S + M \) triangle
- \( L = S + M \) flat
- \( L < S + M \) gap

Largest angle is opposite largest side, smallest angle is opposite smallest side

Base angles of an isosceles triangle are congruent

Sides opposite congruent angles are congruent

To find the possible lengths for the third side given the other sides:
subtract given #’s < \( x \) < add given #’s

**G.7 Right Triangles**

- Pythagorean Theorem \( c^2 = a^2 + b^2 \)
  - *Used when two sides of a right triangle are given*

Converse of the Pythagorean Theorem

- *Identify a triangle as right, obtuse, or acute*
  - \( c^2 = a^2 + b^2 \) Right
  - \( c^2 > a^2 + b^2 \) Obtuse
  - \( c^2 < a^2 + b^2 \) Acute

  *Check to see if triangle is possible*

Right Triangle Trigonometry – SOH CAH TOA

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

**Special Right Triangles:** only when given angle

- 45 – 45 – 90
- 30 – 60 – 90
G.8 Quadrilaterals

Parallelogram
- Opposite sides are parallel
- Opposite sides are congruent
- Opposite angles are congruent
- Consecutive angles are supplementary
- Diagonals bisect each other

Kite
- 2 pair of adjacent sides are congruent
- No opposite sides are congruent

Rectangle
- Parallelogram with:
  - All right angles
  - Diagonals are congruent

Rhombus
- Parallelogram with:
  - Four congruent sides
  - Diagonals are perpendicular
  - Diagonals bisect opposite angles

Square
- Parallelogram
- Rectangle
- Rhombus
- *all 10 properties listed above

G.9 Polygons

Formulas:
- \( (n - 2)180 \) Sum of interior angles
- \( \frac{(n-2)180}{n} \) Each interior angle (regular)
- \( \frac{360}{n} \) Sum of exterior angles
- \( \frac{360}{n} \) Each exterior angle (regular)

*Exterior angle + interior angles = 180
*Exterior angle and its interior angle are supplementary

Tessellation Information
*Each vertex must have a sum of 360 degrees

Regular polygons that tessellate:
- Triangle – each angle measures 60°
- Square – each angle measures 90°
- Hexagon – each angle measures 120°

Combinations of regular polygons that tessellate:
- square and octagon
- square and triangle
- triangle and hexagon

Other common regular polygon measurements (do not tessellate):
- Pentagon – each angle measures 108°
- Octagon – each angle measures 135°

Isosceles Trapezoid
- Legs are congruent
- Pairs of base angles are congruent
- Diagonals are congruent

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**Non-regular figures can tessellate. Make sure that the sum of the angles at any vertex add to 360**
### G.10 Circles

#### Angles & Arcs

**Central Angle**
\[ m\angle 1 = m\overarc{AB} \]

**Inscribed Angle**
\[ m\angle 2 = \frac{1}{2} m\overarc{AB} \]

**Vertex inside circle**
\[ m\angle 3 = \frac{1}{2} (m\overarc{DE} + m\overarc{FG}) \]

**Vertex outside circle**
\[ m\angle 4 = \frac{1}{2} (m\overarc{HJ} - m\overarc{LK}) \]

#### Segments

**Two Chords**
(product of segments from one chord = product of segments from the other)

\[ ab = cd \]

**Two Secants**
(outer secant segment₁ x whole secant₁ = outer secant segment₂ x whole secant₂)

\[ \overline{AB} \cong \overline{CD} \iff \overline{arcAB} \cong \overline{arcCD} \]

A diameter perpendicular to a chord bisects the chord and its arc.

**Tangent and Secant**
Tangent² = outer secant segment x whole secant

\[ c^2 = a^2 + b^2 \]

### Miscellaneous Topics

Congruent chords have congruent arcs

A diameter perpendicular to a chord bisects the chord and its arc.

Chords equidistant from the center are congruent.

### Miscellaneous Topics

Tangents from the same exterior point are congruent.

Tangent is perpendicular to the radius drawn to the point of tangency.

#### Arc Length

\[ \text{degree} \cdot \frac{2\pi r}{360} \]

#### Area of a sector

\[ \text{degree} \cdot \frac{\pi r}{360} \]

### G.13 Lateral Area, Surface Area & Volume of 3-D Figures

**Lateral Area** – does not include base areas (ex: toilet paper roll, b-day party hat)

**Surface Area** – does include base areas (ex: soda can, closed box)

**Volume** – amount filled inside 3-D figure (ex: soda in a can, helium in a balloon)

### G.14 Proportions in similar figures

**Scale Factor**
\[ a:b \]

**Perimeter ratio**
\[ a:b \]

**Any Area ratio**
\[ a^2:b^2 \]

**Volume ratio**
\[ a^3:b^3 \]

Think about the measurement units for perimeter, area, and volume to help you remember the power of the ratio.