AP Stats Ch 16 and 17 Review Worksheet

Chapter 16:

1. Patients receiving artificial knees often experience pain after surgery. The pain is measured on a subjective scale with possible values of 1 to 5. Assume that $X$ is a random variable representing the pain score for a randomly selected patient. The table gives part of the probability distribution of $X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

a. Find $P(X = 5)$. 

b. Find the probability that the pain score is less than 3.

\[ P(X < 3) = 0.3 \]

c. Find the mean and standard deviation for this distribution.

\[ \mu_X = 3.1 \]
\[ \sigma_X = 1.1358 \]

d. Suppose the pain scores for two randomly selected patients are recorded. Let $Y$ be the random variable representing the sum of the two scores. Find the mean and standard deviation of $Y$.

\[ \mu_Y = \mu_X + \mu_X = 3.1 + 3.1 = 6.2 \]
\[ \sigma_Y = \sqrt{\sigma_X^2 + \sigma_X^2} = \sqrt{1.1358^2 + 1.1358^2} = 1.6063 \]

2. Suppose that a discrete random variable has the following probability distribution.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>

a. Find the mean and standard deviation of $X$.

\[ \mu_X = 3.5 \]
\[ \sigma_X = 1.0583 \]

b. Define the new random variable $Y = 3X + 1$. Find the mean and standard deviation of $Y$.

\[ \mu_Y = 3(3.5) + 1 = 11.5 \]
\[ \sigma_Y = 3(1.0583) = 4.9749 \]
3. A carnival game offers a $100 cash prize for anyone who can break a balloon by throwing a dart at it. It costs $5 to play, and you're willing to spend up to $20 trying to win. You estimate that you have about a 10% chance of hitting the balloon on any throw.

   a. Find the expected number of darts you'll throw.

   \[
   \begin{array}{c|cccc}
   X=\#\text{ darts} & 1 & 2 & 3 & 4 \\
   p(x) & .1 & .09 & .081 & .0729 \\
   \end{array}
   \]

   \[E(X) = 2.3087\]

   b. Find your expected winnings.

   \[
   \begin{array}{c|cccc}
   Y & 95 & 90 & 85 & 80 \\
   p(y) & .1 & .09 & .081 & .0729 \\
   \end{array}
   \]

   \[E(Y) = 86.1541\]

4. A time and motion study measures the time required for an assembly line worker to perform a repetitive task. The data shows that the time required to bring a part from a bin to its position on an automobile chassis varies from car to car with mean 11 seconds and standard deviation 2 seconds. The time required to attach the part to the chassis varies with mean 20 seconds and standard deviation 4 seconds.

   \[\mu_p = 11 \quad \sigma_p = 2 \quad \mu_A = 20 \quad \sigma_A = 4\]

   a. What is the mean time required for the entire operation of positioning and attaching the part?

   \[\mu_{p+A} = 11 + 20 = 31\]

   \[\sigma_{p+A} = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.4721\]

   b. Find the standard deviation of the time required for the two-step assembly operation.

   \[\sqrt{2^2 + 4^2} = \sqrt{20} = 4.4721\]

5. The academic motivation and study habits of female students as a group are better than those of males. The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures these factors. The distribution of SSHA scores among the women at a college has mean 120 and standard deviation 28, and the distribution of scores among men students has mean 105 and standard deviation 35. You select one male student and one female student at random and give them the SSHA test.

   a. Explain why it is reasonable to assume that the scores of the two students are independent.

   Test score one student does not effect another student

   b. What are the mean and standard deviation of the difference between their scores?

   \[\mu_{w-m} = 120 - 105 = 15\]

   \[\sigma_{w-m} = \sqrt{28^2 + 35^2} = \sqrt{2009} = 44.819\]
6. The amount of cereal a manufacturer puts in a cereal box is a random variable with mean 16.2 ounces and standard deviation 0.1 ounces. If the weight of the cereal can be described by the normal model, what is the probability that a randomly selected box contains

- less than 16.0 ounces.

\[ P(X < 16) = P(Z < -2) = \text{normalcdf}(-10, -2, 1) = .0228 \]

b. between 16.0 and 16.35 ounces.

\[ P(16 < X < 16.35) = P(-2 < Z < 1.5) = \text{normalcdf}(-2, 1.5, 0, 1) = .9104 \]

7. In the 4 x 100 medley relay event, four swimmers swim 100 yards, each using a different stroke. A college team preparing for the conference championship looks at the times their swimmers have posted and find the means and standard deviations of the times in seconds (shown in the table). Assume that the swimmer’s performances are independent.

<table>
<thead>
<tr>
<th>Swimmer</th>
<th>Mean</th>
<th>St Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50.72</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>55.51</td>
<td>0.22</td>
</tr>
<tr>
<td>3</td>
<td>49.43</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>44.91</td>
<td>0.21</td>
</tr>
</tbody>
</table>

a. What are the mean and standard deviation for the relay team’s total time in this event?

\[ \mu = 50.72 + 55.51 + 49.43 + 44.91 = 200.57 \]

\[ \sigma = \sqrt{.24^2 + .22^2 + .25^2 + .21^2} = .46 \]

b. The team’s best time so far this season was 3:19.48 (199.48 seconds). Do you think the team is likely to swim faster than this at the conference championship? Explain.

\[ P(X < 199.48) = \text{normalsdf} (-10, -2.36, 0, 1) = .009 \]

Chapter 17

8. Jamie is taking a multiple choice test that she has not studied for. There are 20 questions and each question has five choices. She plans to guess on each question. She needs to get 14 questions right in order to pass.

\[ x = \text{correct} = \frac{14}{20} = .2 \]

\[ n = 20 \text{ Binomial} \]

a. What is the probability that she gets exactly 14 questions right?

\[ P(X = 14) = \text{binompdf}(20, .2, 14) = .000001 \]

b. What is the probability that she gets at least 14 questions right?

\[ P(X \geq 14) = 1 - P(X \leq 13) = 1 - \text{binomcdf}(20, .2, 13) = .0000018 \]
9. A scientist concerned with the deadly disease mycobacteriosis (found in rockfish in the Chesapeake Bay waters) knows that 19% of rockfish have the disease. Suppose a charter boat takes a group of amateur anglers out for a tour of the Chesapeake Bay. They catch 24 from different parts of the bay.

a. What is the probability that none of the fish have the disease?
\[ P(X=0) = \text{binomcdf}(24, 0.19, 0) = 0.0064 \]
\[ n = 24 \quad p = 0.19 \quad \text{(have disease)} \]

b. What is the probability that at least two have the disease?
\[ P(X \geq 2) = 1 - P(X \leq 1) = 1 - \text{binomcdf}(24, 0.19, 1) = 0.9578 \]

c. What is the probability that no more than 3 have the disease?
\[ P(X \leq 3) = \text{binomcdf}(24, 0.19, 3) = 0.3050 \]

d. What is the probability that less than 3 have the disease?
\[ P(X < 3) = P(X \leq 2) = \text{binomcdf}(24, 0.19, 2) = 0.1388 \]

e. What is the probability that more than 5 have the disease?
\[ P(X > 5) = 1 - P(X \leq 5) = 1 - \text{binomcdf}(24, 0.19, 5) = 0.2982 \]

f. What is the probability that between (not including) 10 and 15 fish have the disease?
\[ P(10 < X < 15) = P(X \leq 14) - P(X \leq 10) = \text{binomcdf}(24, 0.19, 14) - \text{binomcdf}(24, 0.19, 10) = 0.0025 \]

g. What is the probability that between 13 and 18 (inclusive) fish have the disease?
\[ P(13 \leq X \leq 18) = P(X \leq 18) - P(X \leq 12) = \text{binomcdf}(24, 0.19, 18) - \text{binomcdf}(24, 0.19, 12) = 0.001 \]

h. How many fish should they expect to have the disease?
\[ \mu = np = 24(0.19) = 4.56 \]

i. What is the standard deviation of the number of fish that have the disease?
\[ \sigma = \sqrt{24(0.19)(0.81)} = 1.5458 \]
10. Suppose that Roberto, a well-known major league baseball player, finished last season with a .325 batting average. He wants to calculate the probability that he will get his first hit of this new season in his first at-bat. You define a success as getting a hit and define the random variable \( X \) = number of at-bats until Roberto gets his first hit. \( \text{Geometric } P = .325 \)

a. What is the probability that Roberto will get a hit on his first at-bat?

\[ P(X=1) = \text{geometric-pdf (.325, 1)} = \boxed{.325} \]

b. What is the probability that it will take him at most 3 at-bats to get his first hit?

\[ P(X \leq 3) = \text{geometric-cdf (.325, 3)} = \boxed{.6925} \]

c. What is the probability that it will take him more than 4 at-bats to get his first hit?

\[ P(X > 4) = 1 - P(X \leq 4) = 1 - \text{geometric-cdf (.325, 4)} = \boxed{.2076} \]

d. Roberto wants to know the expected number of at-bats until he gets a hit. What would you tell him?

\[ M = \frac{1}{.325} = \boxed{3.0769} \]

11. There is a probability of 0.08 that a vaccine will cause a certain side effect. Suppose that a number of patients are inoculated with the vaccine. We are interested in the number of patients vaccinated until the first side effect is observed.

\( \text{Geometric } P = .08 \)

a. Define the random variable of interest.

\[ X = \# \text{ patients for 1st side effect to occur} \]

b. Verify that this describes a geometric setting.

- \( n \) not fixed until 1st side effect observed
- independent (patients will not affect one another)
- success (side effect) and failure (no side effect)

\( P = .08 \) same

c. Find the probability that exactly 5 patients must be vaccinated in order to observe the first side effect.

\[ P(X=5) = \text{geometric-pdf (.08, 5)} = \boxed{.0573} \]

d. Construct a probability distribution table for \( X \) (up through \( X = 5 \)).

<table>
<thead>
<tr>
<th>( X )</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X) )</td>
<td>.08</td>
<td>.0736</td>
<td>.0677</td>
<td>.0623</td>
<td>.0573</td>
</tr>
</tbody>
</table>

e. How many patients would you expect to have to vaccinate in order to observe the first side effect?

\[ M = \frac{1}{P} = \frac{1}{.08} = \boxed{12.5} \]