Arithmetic and Geometric Sequences

Name_____
Date: _____ Block: _____
Mrs. Mistron

Warm up→ Radio Station Prizes

1) Every morning a radio show has a contest with a prize of $150. Each day the prize is not awarded, they amount is increased by $75.

A) Make a list of the prize amount from the week if no one is awarded the money.

150	225	300	375	450	525	600
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	2	3	4	5	6	7

B) How much will someone be awarded on the 10th day?

$150+9(75)=\$825$

C) How much will be awarded on the 30th day?

$150+29(75)=2,325$

D) How much will be awarded on the nth day? (in terms of n)

$150+(n-1)(75)$

2) A rival radio show has a contest with a prize on $10 which doubles each day.

A) Make a list of the prize amounts from the week if no one is awarded the money.

10	20	40	80	160	320	640
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

B) How much will be awarded on the 10th day?

$10 \cdot 2^9 = 5120$

C) How much will be awarded on the 30th day?

$10 \cdot 2^{29} = 5,368,709,120$

D) How much will be awarded on the nth day? (in terms of n)

$\$ 10 \cdot 2^{n-1}$

Arithmetic Sequence

A **sequence** in which the difference in each consecutive term is constant (in order words, you are adding the same number each time)

The **explicit rule** (or equation) for an arithmetic sequence is $a_n = a_1 + d(n-1)$

$a_n = a_0 + dn$

The **recursive rule** for an arithmetic sequence is $a_n = a_{n-1} + d$

Example 1→ Write the rule for an arithmetic sequence

Write the explicit **and** recursive rule for the n^{th} term of the sequence and find a_{15}

a. 4, 9, 14, 19...

Recursive: $a_n = a_{n-1} + 5$

Explicit:

$a_n = 4 + 5(n-1)$

$= 4 + 5n - 5$

$a_n = -1 + 5n$

b. 60, 52, 44, 36,...

Recursive: $a_n = a_{n-1} - 8$

Explicit:

$a_n = 60 - 8(n-1)$

$= 60 - 8n + 8$

$a_n = 68 - 8n$

Example 2→ Write the explicit rule given a term and the common difference

$a_{19} = 48,\ d = 3$

$48 = a_1 + 3(18)$

$48 = a_1 + 54$

$-6 = a_1$

$a_n = -9 + 3n$

$a_{12} = 10,\ d = -3$

$10 = a_1 + (-3)(11)$

$10 = a_1 - 33$

$43 = a_1$

$a_n = 46 - 3n$

Example 3→ Write the explicit rule given two terms

Using a system of equations:

$a_8 = 21$ and $a_{27} = 97$

$-[21 = a_1 + d(7)]$

$97 = a_1 + d(26)$

$76 = 19d$

$4 = d$

$21 = a_1 + 4(7)$

$21 = a_1 + 28$

$-7 = a_1$

$\boxed{a_n = -11 + 4n}$

Finding the line:

$a_8 = 21$ and $a_{27} = 97$

$(8, 21), (27, 97)$

$d = \frac{97-21}{27-8} = \frac{76}{19} = 4$

$y - 21 = 4(x - 8)$

$y = 4x - 32 + 21$

$y = 4x - 11$

$\boxed{a_n = -11 + 4n}$

You try: (either method)

$a_{12} = -8$ and $a_{19} = -43$

$-(-8 = a_1 + 11d)$

$-43 = a_1 + 18d$

$-35 = 7d$

$-5 = d$

$-8 = a_1 - 55$

$63 = a_1$

$\boxed{a_n = 68 - 5d}$

***Arithmetic Mean*→** the terms between any two non-consecutive terms of an arithmetic sequence. EX: In the sequence 5, 8, 11, 14, 17, 20, there are three arithmetic means between 8 and 20. The arithmetic means between 8 and 20 are 11, 14, and

Example 4→ Find three arithmetic means between -2 *and* 8

$-2 \quad \frac{1}{2} \quad 3 \quad \frac{11}{2} \quad 8$

$-2 + d + d + d + d = 8$

$-2 + 4d = 8$

$4d = 10$

$d = \frac{5}{2}$

Geometric Sequence

A **sequence** in which the ratio between consecutive terms are constant (in order words, you are multiplying the same number each time)

The **explicit rule** (or equation) for a geometric sequence is $a_n = a_1(r^{n-1})$

The **recursive rule** for a geometric sequence is $a_n = r \bullet a_{n-1}$

Example 5→ Write the explicit and recursive rule given the sequence

a. 4, 20, 100, 500, ...

Recursive: $a_n = 5a_{n-1}$

Explicit: $a_n = 4(5)^{n-1}$

b. 152, -76, 38, -19,...

Recursive: $a_n = -\frac{1}{2}a_{n-1}$

Explicit: $a_n = (152)(-\frac{1}{2})^{n-1}$

Example 6→ Write the explicit rule given the common ratio and a term

a. $a_4 = 12$, $r = 2$

$12 = a_1(2)^3$

$12 = a_1 \cdot 8$

$\frac{12}{8} = a_1$

$\frac{3}{2} = a_1$

$a_n = \frac{3}{2}(2)^{n-1}$

b. $a_5 = 54$, $r = 3$

$54 = a_1(3)^4$

$54 = a_1(81)$

$\frac{54}{81} = a_1 = \frac{2}{3}$

$a_n = \frac{2}{3}(3)^{n-1}$

Example 7→ Write the explicit rule given two terms

a. $a_3 = -48$, $a_6 = 3072$

$-48 = a_1 r^2$

$\frac{-48}{r^2} = a_1$

$\frac{-48}{16} = a_1$

$-3 = a_1$

$3072 = a_1 r^5$

$3072 = (\frac{-48}{r^2})r^5$

$3072 = -48r^3$

$-64 = r^3$

$-4 = r$

$a_n = -3(-4)^{n-1}$

b. $a_2 = -12$, $a_4 = -3$

$-12 = a_1 r$

$\frac{-12}{r} = a_1$

$\frac{-12}{-\frac{1}{2}}$ or $\frac{-12}{\frac{1}{2}}$

$24 = a_1$, $-24 = a_1$

$a_n = 24(-\frac{1}{2})^{n-1}$

or

$a_n = -24(\frac{1}{2})^{n-1}$

$-3 = a_1 r^3$

$-3 = -\frac{12}{r} \cdot r^3$

$-3 = -12r^2$

$\frac{1}{4} = r^2$

$\pm\frac{1}{2} = r$

Example 8→ Finding a Job

You have been offered a job at two different advertising agencies. Agency A offers you a starting salary of $50,000 and an annual raise of $600 a year. Agent B offers you a starting salary of $48,000 and will increase your salary by 3% each year. Which agency do you go with and why?

Agency A

$a_1 = 50{,}000$

$d = 600$

$a_n = 49{,}400 + 600n$

Agency B

$a_1 = 48{,}000$

$r = 1.03$

$a_n = 48000(1.03)^{n-1}$

Agency A starts higher for the first 1.98 years, but after 2 years I will make more at Agency B

Example 9→ Find the geometric mean

(A) Find one geometric mean between 2 and 18

2 6 18 or 2 −6 18

$2 \cdot r \cdot r = 18$ $\quad r = \pm 3$

$2r^2 = 18$

$r^2 = 9$

(B) Find two geometric means between 4 and 32

4 8 16 32

$4 \cdot r \cdot r \cdot r = 32$

$4r^3 = 32$

$r^3 = 8$

$r = 2$

Homework→ Arithmetic and Geometric Sequences Worksheet