Introduction to Logic

What comes next?

a) $9^2 = \_\_\_\_, \hspace{1cm} b) \ 12345 \cdot 9 = \_\_\_\_\_, \hspace{1cm} c) \ 2, 4, \_\_, \_\_, \_\_\_

$$99^2 = \_\_\_\_, \hspace{1cm} 12345 \cdot 18 = \_\_\_\_\_,$$
$$999^2 = \_\_\_\_, \hspace{1cm} 12345 \cdot 27 = \_\_\_\_\_,$$
$$9,999^2 = \_\_\_\_, \hspace{1cm} \_\_\_\_\_\_ = \_\_\_\_\_\_$$

- How did you know what came next in the above examples?

- You used **inductive reasoning**: you looked for a pattern, and applied it as a rule.

Examples of **conjectures** using inductive reasoning:

- All ice I have ever observed in cold, therefore all ice is cold.
- The sun has risen every day of my life, therefore it will rise tomorrow.
- I have always gotten an A in math class, therefore I will get an A in this math class.
- All members of a sample got well from a medication, therefore the entire population will get well from this medication.

What are some problems with inductive reasoning? __________________________________________

What is useful about inductive reasoning?

- Use inductive reasoning to **disprove** a conjecture by finding a **counterexample**

**Example:** **All odd numbers are prime**.

Prove this conjecture **false** by finding a **counterexample**, an odd number that is not prime.

- A counterexample to this conjecture is the number ______.

- An example of a conjecture that uses inductive reasoning that can be disproved by a counterexample is (give the counterexample, too): __________________________________________

**Vocabulary Review** Fill in the descriptions for each term...

<table>
<thead>
<tr>
<th>Notation/Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>conjecture</td>
<td></td>
</tr>
<tr>
<td>inductive reasoning</td>
<td></td>
</tr>
<tr>
<td>counterexample</td>
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</table>
Conditional Statements

Conditional Statements (If-Then):

- Examples:
  - If the weather is nice, then I will wash the car.
  - If 2 divides evenly into x, then x is a positive number.
  - Your turn: ___________________________________________________________________

- Sometimes have to put into if-then form...
  - All birds have feathers
  - Two angles are supplementary if they are a linear pair.

Forms of Conditional Statements

Notation: Let $p$ represent the hypothesis of a conditional, and $q$ represent the conclusion

- **If $p$ then $q$** also written as $p \rightarrow q$; stated as “$p$ implies $q$”
- Conditionals have converse, inverse, and contrapositive statements

**Example 1**: All birds have feathers
- **Conditional**: If an animal is a bird, then it has feathers
- **Converse**: $q \rightarrow p$; exchange hypothesis and conclusion
- **Inverse**: $\sim(p \rightarrow q)$ or $\sim p \rightarrow \sim q$; negate hypothesis and conclusion
- **Contrapositive**: $\sim q \rightarrow \sim p$; converse of the inverse

**Example 2**: Two angles are supplementary if they are a linear pair.
- **Conditional**: __________________________________________________________________
- **Converse**: ____________________________________________________________________
- **Inverse** ______________________________________________________________________
- **Contrapositive** __________________________________________________________________
Write the conditional if-then form, converse, inverse, and contrapositive forms of the following statements. Assuming the original statement is true; decide whether the other forms are true or false.

<table>
<thead>
<tr>
<th>Statement 1</th>
<th>Statement 2</th>
<th>Statement 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>All cats are mammals.</td>
<td>Baseball players are athletes.</td>
<td>All 180° ∠s are straight ∠s.</td>
</tr>
<tr>
<td>if-then:</td>
<td>if-then:</td>
<td>if-then:</td>
</tr>
<tr>
<td>converse:</td>
<td>converse:</td>
<td>converse:</td>
</tr>
<tr>
<td>inverse:</td>
<td>inverse:</td>
<td>inverse:</td>
</tr>
<tr>
<td>contrapositive:</td>
<td>contrapositive:</td>
<td>contrapositive:</td>
</tr>
</tbody>
</table>

You Try...

1) Guitar players are musicians.  
   if-then:  
   converse:  
   inverse:  
   contrapositive:  

2) All Great Danes are large.  
   if-then:  
   converse:  
   inverse:  
   contrapositive:  

3) A polygon is regular if it is equilateral.  
   if-then:  
   converse:  
   inverse:  
   contrapositive:  

- What can we inductively conclude about the converse and inverse of a statement?  
- What can we inductively conclude about a conditional statement and its contrapositive?  
- They are ___________________________ statements (have the same truth value).

**Biconditional Statements**

- Statements where the original statement and converse are BOTH true  
- Use the words “if and only if” (IFF)  
- Notation:  \( p \leftrightarrow q \)  
- Example: An animal meows IFF it is a cat. Other examples?  
- Which of the previous examples are biconditional?
Compound Logic Statements

- **conjunction**: A compound logic statement formed using the word **and**
- **disjunction**: A compound logic statement formed using the word **or**

  - **Example**:  
    - \( p \): Joes eats fries  
    - \( q \): Maria drinks soda  
    - \( p \land q \): Joe eats fries and Maria drinks soda  
    - \( p \lor q \): Joe eats fries or Maria drinks soda  
    - A conjunction is true IFF only both parts are true  
    - A disjunction is false IFF only both parts are false

  - **You try**: Write the statement in symbolic form, or translate the symbols to English...

    a: We go to school on a holiday  
    b: Arbor Day is a holiday  
    c: We work on Arbor Day  

    1) We work on Arbor Day or Arbor Day is a holiday.  
      ____________________________________________

    2) Arbor Day is a holiday and we do not work on Arbor Day.  
      ____________________________________________

    3) If we go to school on a holiday and Arbor Day is a holiday then we work on Arbor Day.  
      ____________________________________________

    4) \( a \land c \)  
      ____________________________________________

    5) \( b \lor c \land \sim a \)  
      ____________________________________________

    6) \(( \sim a \land b ) \rightarrow c \)  
      ____________________________________________

Vocabulary Review

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<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>conditional statement</td>
<td>A logical statement that has a hypothesis and conclusion; can be put in the form “if-then.”</td>
<td>( p \rightarrow q )</td>
</tr>
<tr>
<td>hypothesis</td>
<td>The “if” part of a conditional statement.</td>
<td>( p )</td>
</tr>
<tr>
<td>conclusion</td>
<td>The “then” part of a conditional statement.</td>
<td>( q )</td>
</tr>
<tr>
<td>negation</td>
<td>The opposite of the original statement or clause.</td>
<td>( \sim p )</td>
</tr>
<tr>
<td>converse</td>
<td>The statement formed if the hypothesis and conclusion are switched.</td>
<td>( q \rightarrow p )</td>
</tr>
<tr>
<td>inverse</td>
<td>The statement formed by negating both the hypotheses and conclusion.</td>
<td>( \sim p \rightarrow \sim q )</td>
</tr>
<tr>
<td>contrapositive</td>
<td>The statement formed by writing the converse of the inverse.</td>
<td>( \sim q \rightarrow \sim p )</td>
</tr>
<tr>
<td>biconditional statement</td>
<td>A statement whose converse is equivalent to the original form of the statement; contains “if and only if” (IFF).</td>
<td>( p \leftrightarrow q )</td>
</tr>
<tr>
<td>equivalent statements</td>
<td>Statements that have the same truth value (true or false).</td>
<td>N/A</td>
</tr>
<tr>
<td>conjunction</td>
<td>Compound logic statement using <strong>and</strong>.</td>
<td>( p \land q )</td>
</tr>
<tr>
<td>disjunction</td>
<td>Compound logic statement using <strong>or</strong>.</td>
<td>( p \lor q )</td>
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