How is math used in packaging candy?

When marketing a product such as candy, how the product is packaged can be as important as how it tastes. A marketer must decide what shape container is best, how much candy the container should hold, and how much material it will take to make the chosen container. To make these decisions, you must be able to identify three-dimensional objects, calculate their volumes, and calculate their surface areas.

You will solve problems about packaging in Lesson 7-5.
Vocabulary Review
Choose the correct term to complete each sentence.

1. A quadrilateral with exactly one pair of parallel opposite sides is called a (parallelogram, trapezoid). (Lesson 6-4)
2. Polygons that have the same size and shape are called (congruent, similar) polygons. (Lesson 6-5)

Prerequisite Skills
Multiply. (Lesson 6-4)

3. $\frac{1}{3} \cdot 8 \cdot 12$
4. $\frac{1}{3} \cdot 4 \cdot 9^2$

Find the value of each expression to the nearest tenth.

5. $8.3 \cdot 4.1$
6. $9 \cdot 5.2$
7. $7.36 \div 4$
8. $12 \div 0.06$

Use the $\pi$ key on a calculator to find the value of each expression. Round to the nearest tenth.

9. $\pi \cdot 15$
10. $2 \cdot \pi \cdot 3.2$
11. $\pi \cdot 7^2$
12. $\pi \cdot (19 \div 2)^2$

Classify each polygon according to its number of sides.

13. 
14. 
15. 
16.
Area of Parallelograms, Triangles, and Trapezoids

**Virginia SOL Standard 8.17** The student will create and solve problems, using proportions, formulas, and functions.

**NEW Vocabulary**
- **base**
- **altitude**

**What You’ll Learn**
- Find the areas of parallelograms, triangles, and trapezoids.

**Mini Lab**

Work with a partner.

1. **Draw a rectangle on grid paper.**
2. **Shift the top line 3 units right and draw a parallelogram.**
3. **Draw a line connecting two opposite vertices of the parallelogram and form two triangles.**

1. What dimensions are the same in each figure?
2. Compare the areas of the three figures. What do you notice?

The area of a parallelogram can be found by multiplying the measures of its base and its height.

![Diagram showing a parallelogram with labeled base and altitude.]

**Key Concept**

**Area of a Parallelogram**

**Words**
The area $A$ of a parallelogram is the product of any base $b$ and its height $h$.

**Symbols**
$A = bh$

**EXAMPLE**

Find the area of the parallelogram.

The base is 5 feet. The height is 7 feet.

$A = bh$

Area of a parallelogram

$A = 5 \times 7$

Replace $b$ with 5 and $h$ with 7.

$A = 35$

Multiply.

The area is 35 square feet.
Lesson 7-1
Area of Parallelograms, Triangles, and Trapezoids

A diagonal of a parallelogram separates the parallelogram into two congruent triangles.

![Diagram showing a diagonal dividing a parallelogram into two congruent triangles]

Using the formula for the area of a parallelogram, you can find the formula for the area of a triangle.

### Key Concept

**Area of a Triangle**

- **Words**
  The area $A$ of a triangle is half the product of any base $b$ and its height $h$.

- **Symbols**
  $A = \frac{1}{2}bh$

### Example

**Find the Area of a Triangle**

**Find the area of the triangle.**

The base is 12 meters. The height is 8 meters.

\[
A = \frac{1}{2}bh \\
A = \frac{1}{2}(12)(8) \quad \text{Replace } b \text{ with 12 and } h \text{ with 8.} \\
A = \frac{1}{2}(96) \quad \text{Multiply. } 12 \times 8 = 96 \\
A = 48 \quad \text{Multiply. } \frac{1}{2} \times 96 = 48.
\]

The area is 48 square meters.

In Chapter 6, you learned that a trapezoid is a quadrilateral with exactly one pair of parallel sides. These parallel sides are its bases. A trapezoid can be separated into two triangles. Consider trapezoid $EFGH$.

**Area of Trapezoid**

\[
\text{area of trapezoid } EFGH = \text{area of } \triangle FGH + \text{area of } \triangle EFH \\
= \frac{1}{2}b_1h + \frac{1}{2}b_2h \\
= \frac{1}{2}h(b_1 + b_2) \quad \text{Distributive Property}
\]
Area of a Trapezoid

**Words**
The area $A$ of a trapezoid is half the product of the height $h$ and the sum of the bases, $b_1$ and $b_2$.

**Symbols**
$A = \frac{1}{2}h(b_1 + b_2)$

---

**EXAMPLE**

**Find the Area of a Trapezoid**

Find the area of the trapezoid.

The height is 4 yards. The lengths of the bases are 7 yards and 3 yards.

$A = \frac{1}{2}h(b_1 + b_2)$

Replace $h$ with 4, $b_1$ with 7, and $b_2$ with 3.

$A = \frac{1}{2}(4)(10)$ or 20

Simplify.

The area of the trapezoid is 20 square yards.

---

**Your Turn**

Find the area of each figure.

---

**Use Area to Solve a Real-Life Problem**

**LANDSCAPING** You are buying grass seed for the lawn surrounding three sides of an office building. If one bag covers 2,000 square feet, how many bags should you buy?

To find the area to be seeded, subtract the area of the rectangle from the area of the trapezoid.

**Area of trapezoid**

$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(80)(100 + 140)$

$A = 9,600$

**Area of rectangle**

$A = lw$

$A = (50)(62)$

$A = 3,100$

The area to be seeded is 9,600 $- 3,100$ or 6,500 square feet. If one bag seeds 2,000 square feet, then you will need 6,500 $/ 2,000$ or 3.25 bags. Since you cannot buy a fraction of a bag, you should buy 4 bags.
1. **Compare** the formulas for the area of a rectangle and the area of a parallelogram.

2. **OPEN ENDED** Draw and label two different triangles that have the same area.

3. **FIND THE ERROR** Anthony and Malik are finding the area of the trapezoid at the right. Who is correct? Explain.

   - Anthony
     \[
     A = \frac{1}{2}(14.2)(8.5) \\
     A = 60.35 \text{ mm}^2 
     \]

   - Malik
     \[
     A = \frac{1}{2}(8.5)(14.2 + 11.8) \\
     A = 110.5 \text{ mm}^2 
     \]

4. Find the area of each figure.
   - 4.
   - 5.
   - 6.

5. Find the area of each figure.
   - 7.
   - 8.
   - 9.
   - 10.
   - 11.
   - 12.

6. 13. parallelogram: base, 4 \(\frac{2}{3}\) in.; height, 6 in.
    14. parallelogram: base, 3.8 m; height, 4.2 m
    15. triangle: base, 12 cm; height, 5.4 cm
    16. triangle: base, 15\(\frac{3}{4}\) ft; height, 5\(\frac{1}{2}\) ft
    17. trapezoid: height, 3.6 cm; bases, 2.2 cm and 5.8 cm
    18. trapezoid: height, 8 yd; bases, 10\(\frac{1}{2}\) yd and 15\(\frac{1}{3}\) yd

19. **ALGEBRA** Find the height of a triangle with a base of 6.4 centimeters and an area of 22.4 square centimeters.

20. **ALGEBRA** A trapezoid has an area of 108 square feet. If the lengths of the bases are 10 feet and 14 feet, find the height.
**GEOGRAPHY** For Exercises 21–24, estimate the area of each state using the scale given.

21. Tennessee
22. Arkansas
23. Virginia
24. North Dakota

**RESEARCH** Use the Internet or another reference to find the actual area of each state listed above. Compare to your estimate.

25. **MULTI STEP** A deck shown is constructed in the shape of a trapezoid, with a triangular area cut out for an existing oak tree. You want to waterproof the deck with a sealant. One can of sealant covers 400 square feet. Find the area of the deck. Then determine how many cans of sealant you should buy.

**CRITICAL THINKING** For Exercises 27 and 28, decide how the area of each figure is affected.

27. The height of a triangle is doubled, but the length of the base remains the same.
28. The length of each base of a trapezoid is doubled and its height is also doubled.

29. **MULTIPLE CHOICE** Which figure does not have an area of 120 square feet?

30. **MULTIPLE CHOICE** Which of the following is the best estimate of the area of the shaded region?

For Exercises 31–33, use the following information.
Triangle XYZ has vertices X(−4, 1), Y(−1, 4), and Z(−3, −3). Graph \( \triangle XYZ \). Then graph the image of \( \triangle XYZ \) after the indicated transformation and write the coordinates of its vertices. (Lessons 6-7, 6-8, and 6-9)

31. translated by \((3, -2)\)  
32. reflected over the \(x\)-axis  
33. rotated 180°

**BASIC SKILL** Use the \(\pi\) key on a calculator to find the value of each expression. Round to the nearest tenth.

34. \(\pi \cdot 27\)  
35. \(2 \cdot \pi \cdot 9.3\)  
36. \(\pi \cdot 5^2\)  
37. \(\pi \cdot (15 \div 2)^2\)
Find the circumference and area of circles.

**circle**
**center**
**radius**
**diameter**
**circumference**
**pi**

**NEW Vocabulary**

- circle
- center
- radius
- diameter
- circumference
- pi

**Math Symbols**

\[ \pi \approx \text{approximately equal to} \]

**What You’ll Learn**

- Find the circumference and area of circles.

**Hands-On Mini Lab**

**Work with a partner.**

**STEP 1** Measure and record the distance \( d \) across the circular part of the object, through its center.

**STEP 2** Place the object on a piece of paper. Mark the point where the object touches the paper on both the object and on the paper.

**STEP 3** Carefully roll the object so that it makes one complete rotation. Then mark the paper again.

**STEP 4** Finally, measure the distance \( C \) between the marks.

1. What distance does \( C \) represent?
2. Find the ratio \( \frac{C}{d} \) for this object.
3. Repeat the steps above for at least two other circular objects and compare the ratios of \( C \) to \( d \). What do you observe?
4. Plot the data you collected as ordered pairs, \((d, C)\). Then find the slope of a best-fit line through these points.

**Materials**

- several different cylindrical objects like a can or battery
- ruler
- marker

**Study Tip**

The numbers 3.14 and \( \frac{22}{7} \) are often used as approximations for \( \pi \).

**A circle** is a set of points in a plane that are the same distance from a given point in the plane, called the center. The distance from the center to any point on the circle is called the radius. The distance across the circle through the center is its diameter. The distance around the circle is called the circumference.

The relationship you discovered in the Mini Lab is true for all circles. The ratio of the circumference of a circle to its diameter is always 3.1415926 ... The Greek letter \( \pi \) (pi) represents this number.
Finding the area of a circle can be related to finding the area of a parallelogram. A circle can be separated into congruent wedge-like pieces. Then the pieces can be rearranged to form the figure below.

Since the circle has an area that is relatively close to the area of the parallelogram-shaped figure, you can use the formula for the area of a parallelogram to find the area of a circle.

\[ A = bh \quad \text{Area of a parallelogram} \]
\[ A = \left( \frac{1}{2} \cdot C \right) r \quad \text{The base of the parallelogram is one-half the circumference and the height is the radius.} \]
\[ A = \left( \frac{1}{2} \cdot 2\pi r \right) r \quad \text{Replace } C \text{ with } 2\pi r. \]
\[ A = \pi \cdot r \cdot r \quad \text{or } \pi r^2 \quad \text{Simplify.} \]
**Find the Areas of Circles**

Find the area of each circle.

\[ A = \pi r^2 \]  
Area of a circle

\[ A = \pi \cdot 8^2 \]  
Replace \( r \) with 8.

\[ A = \pi \cdot 64 \]  
Evaluate \( 8^2 \).

\[ A \approx 201.1 \]  
Use a calculator.

The area is about 201.1 square kilometers.

\[ A = \pi (7.5)^2 \]  
Replace \( r \) with half of 15 or 7.5.

\[ A = \pi \cdot 56.25 \]  
Evaluate \( 7.5^2 \).

\[ A \approx 176.7 \]  
Use a calculator.

The area is about 176.7 square feet.

**Your Turn**

Find the circumference and area of each circle. Round to the nearest tenth.

a.  
\[ r = 11\, \text{cm} \]

b.  
\[ r = 5\, \text{mi} \]

c.  
\[ r = \frac{5}{4}\, \text{in.} \]

**Estimation**

To estimate the area of a circle, square the radius and then multiply by 3.

**REAL-LIFE MATH**

**TREES**

Trees should be planted so that they have plenty of room to grow. The planting site should have an area of at least 2 to 3 times the diameter of the circle the spreading roots of the maturing tree are expected to occupy.

*Source:* www.forestry.uga.edu

**EXAMPLE**

**TREES**

During a construction project, barriers are placed around trees. For each inch of trunk diameter, the protected zone should have a radius of \( 1\frac{1}{2} \) feet.

Find the area of this zone for a tree with a trunk circumference of 63 inches.

First find the diameter of the tree.

\[ C = \pi d \]  
Circumference of a circle

\[ 63 = \pi \cdot d \]  
Replace \( C \) with 63.

\[ \frac{63}{\pi} = d \]  
Divide each side by \( \pi \).

\[ 20.1 \approx d \]  
Use a calculator.

The diameter \( d \) of the tree is about 20.1 inches. The radius \( r \) of the protected zone should be \( 1\frac{1}{2}d \) feet. That is, \( r = \frac{1}{2}(20.1) \) or 30.15 feet. Use this radius to find the area of the protected zone.

\[ A = \pi r^2 \]  
Area of a circle

\[ A = \pi(30.15)^2 \]  
Replace \( r \) with 30.15 and use a calculator.

The area of the protected zone is about 2,855.8 square feet.
1. OPEN ENDED Draw and label a circle that has a circumference between 10 and 20 centimeters.

2. NUMBER SENSE If the radius of a circle is doubled, how will this affect its circumference? its area? Explain your reasoning.

Find the circumference and area of each circle. Round to the nearest tenth.

3. The diameter is 12 yards.
4. The radius is 18 centimeters.
5. The radius is 21 feet.
6. The radius is 14.5 meters.
7. The diameter is 5.3 miles.
8. The radius is $4 \frac{3}{4}$ inches.

Find the circumference and area of each circle. Round to the nearest tenth.

9. The radius is 10 inches.
10. The radius is 24 millimeters.
11. The radius is 38 miles.
12. The diameter is 17 kilometers.
13. The diameter is 19.4 meters.
14. The radius is $7 \frac{1}{4}$ feet.
15. The radius is 3.5 centimeters.
16. The diameter is 8.6 kilometers.
17. The diameter is $10 \frac{3}{8}$ feet.
18. The radius is $6 \frac{2}{5}$ inches.

19. CARS If the tires on a car each have a diameter of 25 inches, how far will the car travel in 100 rotations of its tires?

20. SPORTS Three tennis balls are packaged one on top of the other in a can. Which measure is greater, the can’s height or circumference? Explain.

21. ANIMALS A California ground squirrel usually stays within 150 yards of its burrow. Find the area of a California ground squirrel’s world.

22. LAWN CARE The pattern of water distribution from a sprinkler is commonly a circle or part of a circle. A certain sprinkler is set to cover part of a circle measuring 270°. Find the area of the grass watered if the sprinkler reaches a distance of 15 feet.
**Lesson 7-2  Circumference and Area of Circles**

33. **MULTIPLE CHOICE** One lap around the outside of a circular track is 352 yards. If you jog from one side of the track to the other through the center, about how far do you travel?

   ![Image](msmath3.net/self_check_quiz/sol)

   - A 11 yd
   - B 56 yd
   - C 112 yd
   - D 176 yd

34. **SHORT RESPONSE** The circumference of a circle is 16.5 feet. What is its area to the nearest tenth of a square foot?

   Find the area of each figure described.  **(Lesson 7-1)**

   35. triangle: base, 4 cm
       height, 8.7 cm

   36. trapezoid: height, 4 in.
       bases, 2.5 in. and 5 in.

   37. Graph $\triangle WXY$ with vertices $W(1, -3)$, $X(3, -1)$, and $Y(4, -2)$. Then graph its image after a rotation of 90° counterclockwise about the origin and write the coordinates of its vertices.  **(Lesson 6-9)**

**GETTING READY FOR THE NEXT LESSON**

**BASIC SKILL** Add.

38. $450 + 210.5$

39. $16.4 + 8.7$

40. $25.9 + 134.8$

41. $213.25 + 86.9$
What You’ll Learn
Solve problems by solving a simpler problem.

Solve a Simpler Problem

Mr. Lewis wants to know the largest number of pieces of pizza that can be made by using 8 straight cuts. That’s a hard problem!

Well, maybe we can make it easier by solving a simpler problem, or maybe even a few simpler problems.

Explore
Mr. Lewis said that a “cut” does not have to be along a diameter, but it must be from edge to edge. Also, the pieces do not have to be the same size.

Plan
Let’s draw diagrams to find the largest number of pieces formed by 1, 2, 3, and 4 cuts and then look for a pattern.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>Pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>8</td>
<td>37</td>
</tr>
</tbody>
</table>

So the largest number of pieces formed by 8 cuts is 37.

Examine
Two cuts formed 2 • 2 or 4 pieces, and 4 cuts formed about 3 • 4 or 12 pieces. It is reasonable to assume that 6 cuts would form about 4 • 6 or 24 pieces and 8 cuts about 5 • 8 or 40 pieces. Our answer is reasonable.

1. Explain why it was helpful for Kimi and Paige to solve a simpler problem to answer Mr. Lewis’ question.
2. Explain how you could use the solve a simpler problem strategy to find the thickness of one page in this book.
3. Write about a situation in which you might need to solve a simpler problem in order to find the solution to a more complicated problem. Then solve the problem.
Apply the Strategy

Solve. Use the solve a simpler problem strategy.

4. **GEOMETRY** How many squares of any size are in the figure at the right?

5. **TABLES** A restaurant has 25 square tables that can be pushed together to form one long table for a banquet. Each square table can seat only one person on each side. How many people can be seated at the banquet table?

Mixed Problem Solving

Solve. Use any strategy.

6. **PARTY SUPPLIES** Paper cups come in packages of 40 or 75. Monica needs 350 paper cups for the school party. How many packages of each size should she buy?

7. **SOFT DRINKS** The graph below represents a survey of 400 students. Determine the difference in the number of students who preferred cola to lemon-lime soda.

8. **GIFT WRAPPING** During the holidays, Tyler and Abigail earn extra money by wrapping gifts at a department store. Tyler wraps 8 packages an hour while Abigail wraps 10 packages an hour. Working together, about how long will it take them to wrap 40 packages?

9. **READING** For Exercises 9 and 10, use the following information. Carter Middle School has 487 fiction books and 675 nonfiction books. Of the nonfiction books, 84 are biographies. Draw a Venn diagram of this situation.

10. How many books are not biographies?

11. **MONEY** Mario has $12 to spend at the movies. After he pays the $6.50 admission, he estimates that he can buy a tub of popcorn that costs $4.25 and a medium drink that is $2.50. Is this reasonable? Explain.

12. **HEALTH** A human heart beats an average of 72 times in one minute. Estimate the number of times a human heart beats in one year.

13. **TRAVEL** When Mrs. Lopez started her trip from Jacksonville, Florida, to Atlanta, Georgia, her odometer read 35,400 miles. When she reached Atlanta, her odometer read 35,742 miles. If the trip took $\frac{5}{2}$ hours, what was her average speed?

14. **NUMBER SENSE** Find the sum of all the whole numbers from 1 to 40, inclusive.

15. **STANDARDIZED TEST PRACTICE** Three different views of a cube are shown. If the fish is currently faceup, what figure is facedown?

You will use the solve a simpler problem strategy in the next lesson.
Area of Complex Figures

**Carpeting** When carpeting, you must calculate the amount of carpet needed for the floor space you wish to cover. Sometimes the space is made up of several shapes.

1. Identify some of the polygons that make up the family room, nook, and foyer area shown in this floor plan.

We have discussed the following area formulas.

- **Parallelogram**
  \[ A = bh \]

- **Triangle**
  \[ A = \frac{1}{2}bh \]

- **Trapezoid**
  \[ A = \frac{1}{2}h(b_1 + b_2) \]

- **Circle**
  \[ A = \pi r^2 \]

You can use these formulas to help you find the area of complex figures. A **complex figure** is made up of two or more shapes.

To find the area of a complex figure, separate the figure into shapes whose areas you know how to find. Then find the sum of these areas.

**Example**

Find the area of the complex figure.

The figure can be separated into a rectangle and a triangle.

**Area of rectangle**
\[ A = \ell w \]
\[ A = 15 \cdot 12 \]
\[ A = 180 \]

**Area of triangle**
\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2} \cdot 15 \cdot 4 \]
\[ A = 30 \]

The area of the figure is 180 + 30 or 210 square feet.
Find the Area of a Complex Figure

Find the area of the complex figure.

The figure can be separated into a semicircle and a triangle.

**Area of semicircle**

\[ A = \frac{1}{2}\pi r^2 \]
\[ A = \frac{1}{2} \cdot \pi \cdot 3^2 \]
\[ A \approx 14.1 \]

**Area of triangle**

\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2} \cdot 6 \cdot 11 \]
\[ A = 33 \]

The area of the figure is about 14.1 + 33 or 47.1 square meters.

Your Turn

Find the area of each figure. Round to the nearest tenth if necessary.

a. 12 cm
b. 7 m
c. 20 in.

Use the Area of a Complex Figure

**SHORT-RESPONSE TEST ITEM**

The plans for one hole of a miniature golf course are shown. How many square feet of turf will be needed to cover the putting green if one square represents 1.5 square feet?

**Read the Test Item**

You need to find the area of the putting green in square units and then multiply this result by 1.5 to find the area of the green in square feet.

**Solve the Test Item**

Find the area of the green by dividing it into smaller areas.

**Region A**

Trapezoid

\[ A = \frac{1}{2}h(b_1 + b_2) \]

\[ A = \frac{1}{2}(3)(2 + 3) \text{ or } 7.5 \]

**Region B**

Parallelogram

\[ A = bh \]

\[ A = 6 \cdot 3 \text{ or } 18 \]

**Region C**

Trapezoid

\[ A = \frac{1}{2}h(b_1 + b_2) \]

\[ A = \frac{1}{2}(2)(4 + 5) \text{ or } 9 \]

The total area is 7.5 + 18 + 9 or 34.5 square units. So, 1.5(34.5) or 51.75 square feet of turf is needed to cover the green.
1. **OPEN ENDED** Draw an example of a complex figure that can be separated into at least two different shapes whose area you know how to find. Then show how you would separate this figure to find its area.

2. **Explain** at least two different ways of finding the area of the figure at the right.

Find the area of each figure. Round to the nearest tenth if necessary.

3. 4 cm  
5 cm
4 cm

4. 3 yd  
8 yd
10 yd

5. 12 in.  
11 in.
17 in.

6. 12 cm  
4 cm
5 cm

7. 6 yd  
16 yd
8 yd

8. 8 in.  
5 in.
8 in.
6 in.

9. 7 m  
7 m

10. 6.4 ft  
7 ft
3.6 ft
9 ft

11. 5 cm  
3.6 cm

12. What is the area of a figure that is formed using a square with sides 15 yards and a triangle with a base of 8 yards and a height of 12 yards?

13. What is the area of a figure that is formed using a trapezoid with one base of 9 meters, one base of 15 meters, and a height of 6 meters and a semicircle with a diameter of 9 meters?

**FLAGS** For Exercises 14–16, use the diagram of Ohio’s state flag at the right.

14. Find the area of the flag. Describe your method.

15. Find the area of the triangular region of the flag.

16. What percent of the total area of the flag is the triangular region?
HOME IMPROVEMENT  For Exercises 17 and 18, use the diagram of one side of a house and the following information.
Suppose you are painting one side of your house. One gallon of paint covers 350 square feet and costs $21.95.

17. If you are only planning to apply one coat of paint, how many cans should you buy? Explain your reasoning.

18. Find the total cost of the paint, not including tax.

19. **MULTI STEP** A school’s field, shown at the right, must be mowed before 10:00 A.M. on Monday. The maintenance crew says they can mow at a rate of 1,750 square feet of grass per minute. If the crew begins mowing at 9:30 that morning, will the field be mowed in time? Explain your reasoning.

20. **CRITICAL THINKING** In the diagram at the right, a 3-foot wide wooden walkway surrounds a garden. What is the area of the walkway?

21. **MULTIPLE CHOICE** What is the area of the figure below?
- A) 17.5 m²
- B) 25.5 m²
- C) 437.5 m²
- D) 637.5 m²

22. **MULTIPLE CHOICE** What is the best estimate for the area of the figure below?
- A) 36 units²
- B) 54 units²
- C) 56 units²

23. **MONUMENTS** Stonehenge is a circular array of giant stones in England. The diameter of Stonehenge is 30.5 meters. Find the approximate distance around Stonehenge. (Lesson 7-2)

Find the area of each figure. (Lesson 7-1)

24. triangle: base, 4 mm
   height, 3.5 mm
25. trapezoid: height, 11 ft
   bases, 17 ft and 23 ft

**BASIC SKILL** Classify each polygon according to its number of sides.

26. 27. 28. 29.

msmath3.net/self_check_quiz/sol
Building Three-Dimensional Figures

Different views of a stack of cubes are shown in the activity below. A point of view is called a **perspective**. You can build or draw three-dimensional figures using different perspectives.

**Work with a partner.**

The top, side, and front views of a three-dimensional figure are shown. Use cubes to build the figure. Then, draw your model on isometric dot paper.

Now draw your model on isometric dot paper as shown at the right. Label the front and the side of your figure.

**Your Turn**

The top, side, and front views of three-dimensional figures are shown. Use cubes to build each figure. Then draw your model on isometric dot paper, labeling its front and side.

**Writing Math**

1. **Determine** which view, top, side, or front, would show that a building has multiple heights.

2. **Build** your own figure using up to 20 cubes and draw it on isometric dot paper. Then **draw** the figure’s top, side, and front views. **Explain** your reasoning.
Identify and draw three-dimensional figures.

**NEW Vocabulary**
- plane
- solid
- polyhedron
- edge
- face
- vertex
- prism
- base
- pyramid

**CRYSTALS**
A two-dimensional figure has two dimensions, length and width. A three-dimensional figure, like the Amethyst crystal shown at the right, has three dimensions, length, width, and depth (or height).

1. Name the two-dimensional shapes that make up the sides of this crystal.

2. If you observed the crystal from directly above, what two-dimensional figure would you see?

3. How are two- and three-dimensional figures related?

A **plane** is a two-dimensional flat surface that extends in all directions. There are different ways that planes may be related in space.

<table>
<thead>
<tr>
<th>Intersect in a Line</th>
<th>Intersect at a Point</th>
<th>No Intersection</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Intersecting Planes" /></td>
<td><img src="image.png" alt="Intersection at a Point" /></td>
<td><img src="image.png" alt="Parallel Planes" /></td>
</tr>
</tbody>
</table>

Intersecting planes can also form three-dimensional figures or **solids**. A **polyhedron** is a solid with flat surfaces that are polygons.

A **prism** is a polyhedron with two parallel, congruent faces called **bases**.

A **pyramid** is a polyhedron with one base that is a polygon and faces that are triangles.

Prisms and pyramids are named by the shape of their bases.
Analyze Real-Life Drawings

ARCHITECTURE An artist’s drawing shows the plans for a new office building. Each unit on the drawing represents 50 feet. Draw and label the top, front, and side views.

You can see from the front and side views that the top floor is a rectangle that is 2 units wide by 4 units long. The actual dimensions are 4(50) feet by 2(50) feet or 200 feet by 100 feet.

\[ A = 200 \cdot 100 \quad A = \ell \cdot w \]
\[ A = 20,000 \quad \text{Simplify.} \]

The area of the top floor is 20,000 square feet.

Identify Prisms and Pyramids

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

1. The figure has two parallel congruent bases that are triangles, so it is a triangular prism. The other three faces are rectangles. It has a total of 5 faces, 9 edges, and 6 vertices.

2. The figure has one base that is a pentagon, so it is a pentagonal pyramid. The other faces are triangles. It has a total of 6 faces, 10 edges, and 6 vertices.

Your Turn Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

a. b. c.

Common Polyhedrons

rectangular pyramid triangular pyramid rectangular prism triangular prism
1. **Identify** the indicated parts of the polyhedron at the right.

2. **OPEN ENDED** Give a real-life example of three intersecting planes and describe their intersection.

**GUIDED PRACTICE**

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

3.  

4.  

5.  

6. **PETS** Your pet lizard lives in an aquarium with a hexagonal base and a height of 5 units. Draw the aquarium using isometric dot paper.

**Practice and Applications**

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

7.  

8.  

9.  

10.  

**ARCHITECTURE** For Exercises 11 and 12, complete parts a–c for each architectural drawing.

a. Draw and label the top, front, and side views.

b. Find the overall height of the solid in feet.

c. Find the area of the shaded region.

11. Sculpture Pedestal  
12. Porch Steps

1 unit = 6 in.  
1 unit = 8 in.

Determine whether each statement is **sometimes**, **always**, or **never** true. Explain your reasoning.

13. Three planes do not intersect in a point.

14. A prism has two congruent bases.

15. A pyramid has five vertices.
**CRYSTALS** For Exercises 16–18, complete parts a and b for each crystal.

a. Identify the solid or solids that form the crystal.

b. Draw and label the top and one side view of the crystal.

16. Emerald  
17. Fluorite  
18. Quartz

**19. CRITICAL THINKING** A pyramid with a triangular base has 6 edges and a pyramid with a rectangular base has 8 edges. Write a formula that gives the number of edges $E$ for a pyramid with an $n$-sided base.

**EXTENDING THE LESSON** Skew lines do not intersect, but are also not parallel. They lie in different planes. In the figure at the right, the lines containing $AD$ and $CG$ are skew. $BH$ is a diagonal of this prism because it joins two vertices that have no faces in common.

For Exercises 20–22, use the rectangular prism above.

20. Identify three other diagonals that could have been drawn.

21. Name two segments that are skew to $BH$.

22. State whether $DH$ and $CG$ are parallel, skew, or intersecting.

---

**Standardized Test Practice and Mixed Review**

For Exercises 23 and 24, use the figure at the right.

23. **SHORT RESPONSE** Identify the two polyhedrons that make up the figure.

24. **MULTIPLE CHOICE** Identify the shaded part of the figure.

   - (A) edge
   - (B) face
   - (C) vertex
   - (D) prism

Find the area of each figure. Round to the nearest tenth. (Lesson 7-3)

25.  
26.  
27.  

28. **MANUFACTURING** The label that goes around a jar of peanut butter overlaps itself by $\frac{3}{8}$ inch. If the diameter of the jar is 2 inches, what is the length of the label? (Lesson 7-2)

---

**PREREQUISITE SKILL** Find the area of each triangle described. (Lesson 7-1)

29. base, 3 in.; height, 10 in.  
30. base, 8 ft; height, 7 feet  
31. base, 5 cm; height, 11 cm
Volume of Prisms and Cylinders

**What You'll Learn**
Find the volumes of prisms and cylinders.

**NEW Vocabulary**
- Volume
- Cylinder
- Complex solid

**Hands-On Mini Lab**

The rectangular prism at the right has a volume of 12 cubic units.

**Step 1** Model three other rectangular prisms with a volume of 12 cubic units.

**Step 2** Copy and complete the following table.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Length (units)</th>
<th>Width (units)</th>
<th>Height (units)</th>
<th>Area of Base (units²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

1. Describe how the volume \( V \) of each prism is related to its length \( \ell \), width \( w \), and height \( h \).
2. Describe how the area of the base \( B \) and the height \( h \) of each prism is related to its volume \( V \).

**Volume** is the measure of the space occupied by a solid. Standard measures of volume are cubic units such as cubic inches (in³) or cubic feet (ft³).

**Key Concept**

**Words**
The volume \( V \) of a prism is the area of the base \( B \) times the height \( h \).

**Symbols**

\[ V = Bh \]

**Example**

Find the volume of the prism.

\[ V = Bh \]

Volume of a prism

\[ V = (\ell \cdot w)h \]

The base is a rectangle, so \( B = \ell \cdot w \).

\[ V = (9 \cdot 5)6.5 \quad \ell = 9, \; w = 5, \; h = 6.5 \]

\[ V = 292.5 \quad \text{Simplify.} \]

The volume is 292.5 cubic centimeters.
A cylinder is a solid whose bases are congruent, parallel circles, connected with a curved side. You can use the formula \( V = Bh \) to find the volume of a cylinder, where the base is a circle.

### Find the Volume of a Triangular Prism

Find the volume of the prism.

\[
V = Bh
\]

**Volume of a prism**

\[
V = \left( \frac{1}{2} \cdot 6 \cdot 7 \right) \cdot 10
\]

The base is a triangle, so \( B = \frac{1}{2} \cdot 6 \cdot 7 \).

\[
V = \left( \frac{1}{2} \cdot 6 \cdot 7 \right) \cdot 10
\]

The height of the prism is 10.

\[
V = 210
\]

Simplify.

The volume is 210 cubic inches.

### Key Concept

**Volume of a Cylinder**

**Words**

The volume \( V \) of a cylinder with radius \( r \) is the area of the base \( B \) times the height \( h \).

**Symbols**

\[
V = Bh \quad \text{or} \quad V = \pi r^2 h, \text{ where } B = \pi r^2
\]

### Find the Volumes of Cylinders

**Volume of a Cylinder**

Find the volume of each cylinder.

1. **Volume of a cylinder**

\[
V = \pi r^2h
\]

2. **Replace** \( r \) with 6 and \( h \) with 20.

\[
V \approx 2,261.9
\]

Simplify.

The volume is about 2,261.9 cubic feet.

**diameter of base, 13 m; height, 15.2 m**

Since the diameter is 13 meters, the radius is 6.5 meters.

\[
V = \pi r^2h
\]

3. **Replace** \( r \) with 6.5 and \( h \) with 15.2.

\[
V \approx 2,017.5
\]

Simplify.

The volume is about 2,017.5 cubic meters.

### Your Turn

Find the volume of each solid. Round to the nearest tenth if necessary.

**b.**

\[
V = \pi r^2h
\]

4. **Replace** \( r \) with 8 mm and \( h \) with 5 mm.

\[
V \approx 208.7
\]

Simplify.

The volume is about 208.7 cubic millimeters.

**c.**

\[
V = \pi r^2h
\]

5. **Replace** \( r \) with 2 in. and \( h \) with 7 in.

\[
V \approx 43.9
\]

Simplify.

The volume is about 43.9 cubic inches.
Many objects in real-life are made up of more than one type of solid. Such figures are called complex solids. To find the volume of a complex solid, separate the figure into solids whose volumes you know how to find.

**Find the Volume of a Complex Solid**

**DISPENSERS** Find the volume of the soap dispenser at the right.

The dispenser is made of one rectangular prism and one triangular prism. Find the volume of each prism.

**Rectangular Prism**

![Rectangular Prism Diagram]

\[ V = Bh \]

\[ V = (5 \cdot 7)5 \text{ or } 175 \]

**Triangular Prism**

![Triangular Prism Diagram]

\[ V = \frac{1}{2} \cdot 7 \cdot 3 \]

\[ V = 52.5 \]

The volume of the dispenser is 175 + 52.5 or 227.5 cubic inches.

1. **Write** another formula for the volume of a rectangular prism and explain how it is related to the formula \( V = Bh \).

2. **FIND THE ERROR** Erin and Dulce are finding the volume of the prism shown at the right. Who is correct? Explain.

   **Erin**
   
   \[ A = Bh \]
   \[ A = (10 \cdot 7) \cdot 8 \]
   \[ A = 560 \text{ in}^3 \]

   **Dulce**
   
   \[ A = \frac{1}{2} \cdot 7 \cdot 8 \]
   \[ A = 280 \text{ in}^3 \]

3. **OPEN ENDED** Find the volume of a can or other cylindrical object, being sure to include appropriate units. Explain your method.

**Guided Practice**

Find the volume of each solid. Round to the nearest tenth if necessary.

4. 

![Rectangular Prism Diagram](2 ft 3 ft)

5. 

![Rectangular Prism Diagram](7 m 11 m 14 m)

6. 

![Cylinder Diagram](5 yd)

7. 

![Rectangular Prism Diagram](4 ft 5 ft 6 ft 12 ft)
Find the volume of each solid. Round to the nearest tenth if necessary.

8. rectangular prism: length, 4 in.; width, 6 in.; height, 17 in.
9. 10.
14. rectangular prism: length, 4 in.; width, 6 in.; height, 17 in.
15. triangular prism: base of triangle, 5 ft; altitude, 14 ft; height of prism, \(\frac{8}{2}\) ft
16. cylinder: diameter, 7.2 cm; height, 5.8 cm
17. hexagonal prism: base area 48 mm\(^2\); height, 12 mm

22. **ALGEBRA** Find the height of a rectangular prism with a length of 6.8 meters, a width of 1.5 meters, and a volume of 91.8 cubic meters.

23. **ALGEBRA** Find the height of a cylinder with a radius of 4 inches and a volume of 301.6 cubic inches.

24. Explain how you would find the volume of the hexagonal prism shown at the right. Then find its volume.

**POOLS** For Exercises 25 and 26, use the following information.
A wading pool is to be 20 feet long, 11 feet wide, and 1.5 feet deep.

25. Approximately how much water will the pool hold?
26. The excavated dirt is to be hauled away by wheelbarrow. If the wheelbarrow holds 9 cubic feet of dirt, how many wheelbarrows of dirt must be hauled away from the site?

**CONVERTING UNITS OF MEASURE** For Exercises 27–29, use the cubes at the right.
The volume of the left cube is 1 cubic yard. The right cube is the same size, but the unit of measure has been changed. So, 1 cubic yard = (3)(3)(3) or 27 cubic feet. Use a similar process to convert each measurement.

27. \(1 \text{ ft}^3 = \square \text{ in}^3\)
28. \(1 \text{ cm}^3 = \square \text{ mm}^3\)
29. \(1 \text{ m}^3 = \square \text{ cm}^3\)
30. **PACKAGING** The Cooking Club is selling their own special blends of rice mixes. They can choose from the two containers at the right to package their product. Which container will hold more rice? Explain your reasoning.

![Rice Mix Containers](image)

31. **FARMING** When filled to capacity, a silo can hold 8,042 cubic feet of grain. The circumference \( C \) of the silo is approximately 50.3 feet. Find the height \( h \) of the silo to the nearest foot.

32. **WRITE A PROBLEM** Write about a real-life problem that can be solved by finding the volume of a rectangular prism or a cylinder. Explain how you solved the problem.

**CRITICAL THINKING** For Exercises 33–36, describe how the volume of each solid is affected after the indicated change in its dimension(s).

33. You double one dimension of a rectangular prism.
34. You double two dimensions of a rectangular prism.
35. You double all three dimensions of a rectangular prism.
36. You double the radius of a cylinder.

37. **MULTIPLE CHOICE** A bar of soap in the shape of a rectangular prism has a volume of 16 cubic inches. After several uses, it measures \( 2\frac{1}{4} \) inches by \( 2 \) inches by \( 1\frac{1}{2} \) inches. How much soap was used?

- \( A \) 6\( \frac{3}{4} \) in\(^3\)
- \( B \) 9\( \frac{1}{4} \) in\(^3\)
- \( C \) 10\( \frac{1}{4} \) in\(^3\)
- \( D \) 108 in\(^3\)

38. **MULTIPLE CHOICE** Which is the best estimate of the volume of a cylinder that is 20 meters tall and whose diameter is 10 meters?

- \( A \) 200 m\(^3\)
- \( B \) 500 m\(^3\)
- \( C \) 600 m\(^3\)
- \( D \) 1500 m\(^3\)

39. **PAINTING** You are painting a wall of this room red. Find the area of the red wall to the nearest square foot. (Lesson 7-3)

40. How many edges does an octagonal pyramid have? (Lesson 7-4)

Write each percent as a fraction or mixed number in simplest form. (Lesson 5-1)

41. 0.12\% 42. 225\% 43. 135\% 44. \( \frac{3}{8} \)\%

### PREREQUISITE SKILL
Multiply. (Lesson 2-5)

45. \( \frac{1}{3} \cdot 6 \cdot 10 \)
46. \( \frac{1}{3} \cdot 7 \cdot 15 \)
47. \( \frac{1}{3} \cdot 4^2 \cdot 9 \)
48. \( \frac{1}{3} \cdot 6^2 \cdot 20 \)
1. Draw and label a trapezoid with an area of 20 square centimeters. (Lesson 7-1)

2. Compare and contrast the characteristics of prisms and pyramids. (Lesson 7-4)

3. Find the area of a triangle with a 30-meter base and 12-meter height. (Lesson 7-1)

4. SPORTS A shot-putter must stay inside a circle with a diameter of 7 feet. What is the circumference and area of the region in which the athlete is able to move in this competition? Round to the nearest tenth. (Lesson 7-2)

Find the area of each figure. Round to the nearest tenth. (Lesson 7-3)

5. 3.5 cm

6. 9 m

STORAGE For Exercises 7 and 8, use the diagram of the storage shed at the right.

7. Identify the solid. Name the number and shapes of the faces. Then name the number of edges and vertices. (Lesson 7-4)

8. Find the volume of this storage shed. (Lesson 7-5)

Find the volume of each solid. Round to the nearest tenth. (Lesson 7-5)

9. 7.8 cm

10. 14 yd

11. MULTIPLE CHOICE Which of the following solids is not a polyhedron? (Lesson 7-4)

   - prism
   - cylinder
   - pyramid
   - cube

12. MULTIPLE CHOICE Find the volume of a cube-shaped box with edges 15 inches long. (Lesson 7-5)

   - 225 in³
   - 900 in³
   - 1,350 in³
   - 3,375 in³
Architest

GET READY!

Players: two
Materials: cubes, manila folders, index cards cut in half

GET SET!

- Players each receive 15 cubes and a manila folder.
- Each player designs a structure with some of his or her cubes, using the manila folder to hide the structure from the other player’s view. The player then draws the top, front, back, and side views of the structure on separate index cards. The player also computes the structure’s volume in cubic units, writing this on a fourth index card.

GO!

- Player A tries to guess Player B’s structure. Player A does this by asking Player B for one of the index cards that shows one of the views of the structure. Player A tries to build Player B’s structure.
- Player A receives 4 points for correctly building Players B’s structure after receiving only one piece of information, 3 points for correctly building after only two pieces of information, and so on.
- If Player A cannot build Player B’s structure after receiving all 4 pieces of information, then Player B receives 2 points.
- Player B now tries to build Player A’s structure.
- Who Wins? Play continues for an agreed-upon number of structures. The player with the most points at the end of the game wins.
**Volume of Pyramids and Cones**

**What You’ll Learn**
Find the volumes of pyramids and cones.

**NEW Vocabulary**
- cone

**REVIEW Vocabulary**
- pyramid: a polyhedron with one base that is a polygon and faces that are triangles (Lesson 7-4)

---

**Work with a partner.**

In this Mini Lab, you will investigate the relationship between the volume of a pyramid and the volume of a prism with the same base area and height.

1. Compare the base areas and the heights of the two solids.
2. Fill the pyramid with rice, sliding a ruler across the top to level the amount. Pour the rice into the cube. Repeat until the prism is filled. How many times did you fill the pyramid in order to fill the cube?
3. What fraction of the cube’s volume does one pyramid fill?

---

The volume of a pyramid is one third the volume of a prism with the same base area and height.

---

**Key Concept**

**Volume of a Pyramid**

<table>
<thead>
<tr>
<th>Words</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>The volume $V$ of a pyramid is one third the area of the base $B$ times the height $h$.</td>
<td>![Diagram of a pyramid with volume formula $V = \frac{1}{3}Bh$]</td>
</tr>
</tbody>
</table>

---

**Virginia SOL Standard 8.7** The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.
Lesson 7-6 Volume of Pyramids and Cones

**Find the Volume of a Pyramid**

Find the volume of the pyramid.

\[ V = \frac{1}{3} Bh \]

**Volume of a pyramid**

\[
\begin{align*}
V &= \frac{1}{3} \left( \frac{1}{2} \cdot 8.1 \cdot 6.4 \right) 11 \\
&= \frac{1}{3} \cdot 50.07 \cdot 11 \\
&= 95.04
\end{align*}
\]

Simplify.

The volume is about 95.0 cubic meters.

**Find the Volume of a Cone**

Find the volume of the cone.

\[ V = \frac{1}{3} \pi r^2 h \]

**Volume of a cone**

\[
\begin{align*}
V &= \frac{1}{3} \pi \cdot 3^2 \cdot 14 \\
&= \frac{1}{3} \pi \cdot 9 \cdot 14 \\
&= \frac{1}{3} \cdot 252 \\
&\approx 131.9
\end{align*}
\]

Simplify.

The volume is about 131.9 cubic millimeters.

**Use Volume to Solve a Problem**

**ARCHITECTURE** The area of the base of the Pyramid Arena in Memphis, Tennessee, is 360,000 square feet. If its volume is 38,520,000 cubic feet, find the height of the structure.

\[ V = \frac{1}{3} Bh \]

**Volume of a pyramid**

\[
\begin{align*}
38,520,000 &= \frac{1}{3} \cdot 360,000 \cdot h \\
38,520,000 &= 120,000 \cdot h \\
321 &= h
\end{align*}
\]

Divide each side by 120,000.

The height of the Pyramid Arena is 321 feet.

A cone is a three-dimensional figure with one circular base. A curved surface connects the base and the vertex. The volumes of a cone and a cylinder are related in the same way as those of a pyramid and prism.

**Key Concept**

**Volume of a Cone**

<table>
<thead>
<tr>
<th>Words</th>
<th>The volume ( V ) of a cone with radius ( r ) is one-third the area of the base ( B ) times the height ( h ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>( V = \frac{1}{3} Bh ) or ( V = \frac{1}{3} \pi r^2 h )</td>
</tr>
</tbody>
</table>

**EXAMPLE**

**Find the Volume of a Cone**

Find the volume of the cone.

\[ V = \frac{1}{3} \pi r^2 h \]

**Volume of a cone**

\[
\begin{align*}
V &= \frac{1}{3} \pi \cdot 3^2 \cdot 14 \\
&= \frac{1}{3} \pi \cdot 9 \cdot 14 \\
&= \frac{1}{3} \cdot 252 \\
&\approx 131.9
\end{align*}
\]

Simplify.

The volume is about 131.9 cubic millimeters.
1. **NUMBER SENSE** Which would have a greater effect on the volume of a cone, doubling its radius or doubling its height? Explain your reasoning.

2. **OPEN ENDED** Draw and label a rectangular pyramid with a volume of 48 cubic centimeters.

Find the volume of each solid. Round to the nearest tenth if necessary.

3. [Diagram of a triangular prism]
4. [Diagram of a rectangular prism]
5. [Diagram of a triangular prism]

---

**Guided Practice**

Find the volume of each solid. Round to the nearest tenth if necessary.

6. [Diagram of a cone]
7. [Diagram of a cone]
8. [Diagram of a cone]
9. [Diagram of a triangular pyramid]
10. [Diagram of a rectangular prism]
11. [Diagram of a triangular prism]

---

**Practice and Applications**

Find the volume of each solid. Round to the nearest tenth if necessary.

12. cone: diameter, 12 mm; height, 5 mm
13. cone: radius, \(3\frac{1}{2}\) in.; height, 18 in.
14. octagonal pyramid: base area, 120 ft\(^2\); height, 19 ft
15. triangular pyramid: triangle base, 10 cm; triangle height, 7 cm; prism height, 15 cm
16. [Diagram of a rectangular prism]
17. [Diagram of a cylinder]
18. [Diagram of a triangular prism]
19. [Diagram of a triangular prism]
20. **VOLCANO** A model of a volcano constructed for a science project is cone-shaped with a diameter of 10 inches. If the volume of the model is about 287 cubic inches, how tall is the model?
**ICE CREAM**  For Exercises 21 and 22, use the diagram at the right and the following information. You are filling cone-shaped glasses with frozen custard. Each glass is 8 centimeters wide and 15 centimeters tall.

21. Estimate the volume of custard each glass will hold assuming you fill each one level with the top of the glass.

22. One gallon is equivalent to about 4,000 cubic centimeters. Estimate how many glasses you can fill with one gallon of custard.

23. **WRITE A PROBLEM**  Write about a real-life situation that can be solved by finding the volume of a cone. Then solve the problem.

24. **CRITICAL THINKING**  How could you change the height of a cone so that its volume would remain the same when its radius was tripled?

**EXTENDING THE LESSON**  A sphere is the set of all points in space that are a given distance from a given point, called the center. The volume $V$ of a sphere with radius $r$ is given by the formula $V = \frac{4}{3}\pi r^3$.

Find the volume of each sphere described. Round to the nearest tenth.

25. radius, 3 in.  
26. radius, 6 in.  
27. diameter, 10 m  
28. diameter, 9 ft  
29. How does doubling a sphere’s radius affect its volume? Explain.

**30. MULTIPLE CHOICE**  If each of the following solids has a height of 8 centimeters, which has the greatest volume?

31. **SHORT RESPONSE**  A triangular prism has a volume of 135 cubic centimeters. Find the volume in cubic centimeters of a triangular pyramid with the same base area and height as this prism.

32. **PETS**  Find the volume of a doghouse with a rectangular space that is 3 feet wide, 4 feet deep, and 5 feet high and has a triangular roof 1 1/2 feet higher than the walls of the house.  **(Lesson 7-5)**

33. Name the number and shapes of the faces of a trapezoidal prism. Then name the number of edges and vertices.  **(Lesson 7-4)**

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL**  Find the circumference of each circle. Round to the nearest tenth.  **(Lesson 7-2)**

34. diameter, 9 in.  
35. diameter, $\frac{5}{2}$ ft  
36. radius, 2 m  
37. radius, 3.8 cm
Nets

Work with a partner.

Open the lid of a box and make 5 cuts as shown. Then open the box up and lay it flat. The result is a net. Nets are two-dimensional patterns of three-dimensional figures. You can use a net to build a three-dimensional figure.

ACTIVITY

Copy the net onto a piece of paper, shading the base as shown.

Use scissors to cut out the net.

Fold on the dashed lines and tape the sides together.

Sketch the figure and draw its top, side, and front views.

Your Turn

Use each net to build a figure. Then sketch the figure, and draw and label its top, side, and front views.

Writing Math

1. Describe each shape that makes up the three nets above.
2. Identify each of the solids formed by the three nets above.
7-7

Surface Area of Prisms and Cylinders

**What You’ll Learn**

Find the surface areas of prisms and cylinders.

**NEW Vocabulary**

- surface area

**Mini Lab**

The surface area of a solid is the sum of the areas of all its surfaces, or faces. In this lab, you will find the surface areas of rectangular prisms.

1. Estimate the area in square centimeters of each face for one of your boxes. Then find the sum of these six areas.

2. Now use your ruler to measure the sides of each face. Then find the area of each face to the nearest square centimeter. Find the sum of these areas and compare to your estimate.

3. Estimate and then find the surfaces areas of your other boxes.

One way to easily visualize all of the surfaces of a prism is to sketch a two-dimensional pattern of the solid, called a net, and label all its dimensions.

### Faces

<table>
<thead>
<tr>
<th>Area</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>top and bottom</td>
<td>$(\ell \cdot w) + (\ell \cdot w) = 2\ell w$</td>
</tr>
<tr>
<td>front and back</td>
<td>$(\ell \cdot h) + (\ell \cdot h) = 2\ell h$</td>
</tr>
<tr>
<td>two sides</td>
<td>$(w \cdot h) + (w \cdot h) = 2wh$</td>
</tr>
<tr>
<td>Sum of areas</td>
<td>$2\ell w + 2\ell h + 2wh$</td>
</tr>
</tbody>
</table>

**Surface Area of a Rectangular Prism**

**Words**
The surface area $S$ of a rectangular prism with length $\ell$, width $w$, and height $h$ is the sum of the areas of the faces.

**Symbols**

$$S = 2\ell w + 2\ell h + 2wh$$

**Example**

Find the surface area of the rectangular prism.

- $S = 2\ell w + 2\ell h + 2wh$
- $S = 2(7)(3) + 2(7)(12) + 2(3)(12)$
- $S = 282$

The surface area is 282 square meters.

---

**Virginia SOL Standard 8.7**
The student will investigate and solve practical problems involving volume and surface area of rectangular solids (prisms), cylinders, cones, and pyramids.
SKATEBOARDING  A skateboarding ramp called a wedge is built in the shape of a triangular prism. You plan to paint all surfaces of the ramp. Find the surface area to be painted.

A triangular prism consists of two congruent triangular faces and three rectangular faces.

Draw and label a net of this prism. Find the area of each face.

- **bottom**: \(54 \times 32 = 1,728\)
- **left side**: \(55.3 \times 32 = 1,769.6\)
- **right side**: \(12 \times 32 = 384\)
- **two bases**: \(2 \left( \frac{1}{2} \times 54 \times 12 \right) = 648\)

Add to find the total surface area.

\[1,728 + 1,769.6 + 384 + 648 = 4,529.6\]

The surface area of the ramp is 4,529.6 square inches.

**Your Turn**  Find the surface area of each prism.

a. 3 ft 4 ft 6 ft
   b. 9 yd 21 yd 6 yd
   c. 3.5 m 4 m 7 m

You can find the surface area of a cylinder by finding the area of its two bases and adding the area of its curved side. If you unroll a cylinder, its net is two circles and a rectangle.

\[C = 2\pi r\]
\[S = 2\pi r^2 + 2\pi rh\]

So, the surface area \(S\) of a cylinder is \(2\pi r^2 + 2\pi rh\).
Lesson 7-7
Surface Area of Prisms and Cylinders

Surface Area of a Cylinder

**Words**
The surface area \( S \) of a cylinder with height \( h \) and radius \( r \) is the area of the two bases plus the area of the curved surface.

**Symbols**

\[
S = 2\pi r^2 + 2\pi rh
\]

**Example**
Find the surface area of the cylinder. Round to the nearest tenth.

\[
S = 2\pi r^2 + 2\pi rh
\]

Surface area of a cylinder

Replace \( r \) with 2 and \( h \) with 3.

\[
S \approx 62.8
\]

Simplify.

The surface area is 62.8 square feet.

**Your Turn**
Find the surface area of each cylinder. Round to the nearest tenth.

d. 5 mm

10 mm
e. 6.5 in.

f. 4 in.

14.8 cm

**Skill and Concept Check**

1. **Determine** whether the following statement is true or false. If false, give a counterexample.

   *If two rectangular prisms have the same volume, then they also have the same surface area.*

2. **NUMBER SENSE** If you double the edge length of a cube, explain how this affects the surface area of the prism.

3. **OPEN ENDED** The surface area of a rectangular prism is 96 square feet. Name one possible set of dimensions for this prism.

**Guided Practice**

Find the surface area of each solid. Round to the nearest tenth if necessary.

4. [Image of a rectangular prism with dimensions 4 yd, 5 yd, 3 yd]

5. [Image of a rectangular prism with dimensions 6 in., 10 in., 8 in., 7 in.]

6. [Image of a cylinder with dimensions 8 m, 9.4 m]

7. rectangular prism: length, 12.2 cm; width, 4.8 cm; height, 10.3 cm

8. cylinder: radius, 16 yd; height, 25 yd
Find the surface area of each solid. Round to the nearest tenth if necessary.

9. cube: edge length, 12 m
10. 2 in.
11. 12 ft
12. 6 m
13. 17 yd
14. 4.6 mm

15. cube: edge length, 12 m
16. cylinder: diameter, 18 yd, height, 21 yd
17. cylinder: radius, 7 in.; height, 9 \( \frac{1}{2} \) in.
18. rectangular prism: length, \( \frac{1}{2} \) cm; width, \( \frac{3}{4} \) cm; height, \( \frac{3}{4} \) cm

19. **POOL** A vinyl liner covers the inside walls and bottom of the swimming pool shown below. Find the area of this liner to the nearest square foot.

![Swimming Pool Diagram]

20. **GARDENING** The door of the greenhouse shown below has an area of 4.5 square feet. How many square feet of plastic are needed to cover the roof and sides of the greenhouse?

![Greenhouse Diagram]

21. **MULTI STEP** An airport has changed the carrels used for public telephones. The old carrels consisted of four sides of a rectangular prism. The new carrels are half of a cylinder with an open top. How much less material is needed to construct a new carrel than an old carrel?

![Old Design vs. New Design Diagram]

22. **CAMPING** A camping club has designed a tent with canvas sides and floor as shown below. About how much canvas will the club members need to construct the tent? (Hint: Use the Pythagorean Theorem to find the height of the triangular base.)

![Tent Diagram]
23. **CRITICAL THINKING** Will the surface area of a cylinder increase more if you double the height or double the radius? Explain your reasoning.

**CRITICAL THINKING** The length of each edge of a cube is 3 inches. Suppose the cube is painted and then cut into 27 smaller cubes that are 1 inch on each side.

24. How many of the smaller cubes will have paint on exactly three faces?
25. How many of the smaller cubes will have paint on exactly two faces?
26. How many of the smaller cubes will have paint on only one face?
27. How many of the smaller cubes will have no paint on them at all?
28. Find the answers to Exercises 24–27 if the cube is 10 inches on a side and cut into 1,000 smaller cubes.

**EXTENDING THE LESSON** If you make cuts in a solid, different two-dimensional cross sections result, as shown at the right.

Describe the cross section of each figure cut below.

29.  
30.  
31.  
32.  

---

33. **MULTIPLE CHOICE** The greater the surface area of a piece of ice the faster it will melt. Which block of ice described will be the last to melt?

- A 1 in. by 2 in. by 32 in. block
- B 4 in. by 8 in. by 2 in. block
- C 16 in. by 4 in. by 1 in. block
- D 4 in. by 4 in. by 4 in. block

34. **SHORT RESPONSE** Find the amount of metal needed to construct the mailbox at the right to the nearest tenth of a square inch.

Find the volume of each solid described. Round to the nearest tenth if necessary. (Lesson 7-6)

35. rectangular pyramid: length, 14 m; width, 12 m; height, 7 m
36. cone: diameter 22 cm; height, 24 cm

37. **HEALTH** The inside of a refrigerator in a medical laboratory measures 17 inches by 18 inches by 42 inches. You need at least 8 cubic feet to refrigerate some samples from the lab. Is the refrigerator large enough for the samples? Explain. (Lesson 7-5)

---

**PREREQUISITE SKILL** Multiply. (Lesson 2-5)

38. \( \frac{1}{2} \cdot 2.8 \)
39. \( \frac{1}{2} \cdot 10 \cdot 23 \)
40. \( \frac{1}{2} \cdot 2.5 \cdot 16 \)
41. \( \frac{1}{2} \left( \frac{3}{2} \right) (20) \)
Surface Area of Pyramids and Cones

Find the surface areas of pyramids and cones.

NEW Vocabulary
lateral face
slant height
lateral area

Everyday Meaning of lateral: situated on the side

HISTORY In 1485, Leonardo Da Vinci sketched a pyramid-shaped parachute in the margin of his notebook. In June 2000, using a parachute created with tools and materials available in medieval times, Adrian Nicholas proved Da Vinci’s design worked by descending 7,000 feet.

1. How many cloth faces does this pyramid have? What shape are they?
2. How could you find the total area of the material used for the parachute?

The triangular sides of a pyramid are called lateral faces. The triangles intersect at the vertex. The altitude or height of each lateral face is called the slant height.

The sum of the areas of the lateral faces is the lateral area. The surface area of a pyramid is the lateral area plus the area of the base.

Surface Area of a Pyramid

Find the surface area of the square pyramid.

Find the lateral area and the area of the base.

Area of each lateral face

\[ A = \frac{1}{2}bh \]  
Area of a triangle

\[ A = \frac{1}{2}(8)(15) = 60 \]  
Replace \( b \) with 8 and \( h \) with 15.

There are 4 faces, so the lateral area is 4(60) or 240 square inches.
You can find the surface area of a cone with radius \( r \) and slant height \( \ell \) by finding the area of its bases and adding the area of its curved side. If you unroll a cone, its net is a circle and a portion of a larger circle.

So, the surface area \( S \) of a cone is \( \pi r \ell + \pi r^2 \).

### Surface Area of a Cone

**Words**
The surface area \( S \) of a cone with slant height \( \ell \) and radius \( r \) is the lateral area plus the area of the base.

**Symbols**
\[
S = \pi r \ell + \pi r^2
\]

### Example

**Surface Area of a Cone**

Find the surface area of the cone.

\[
S = \pi r \ell + \pi r^2 \quad \text{Surface area of a cone}
\]

\[
S = \pi(7)(13) + \pi(7)^2 \quad \text{Replace } r \text{ with 7 and } \ell \text{ with 13.}
\]

\[
S \approx 439.8 \quad \text{Simplify.}
\]

The surface area of the cone is about 439.8 square centimeters.

**Your Turn**

Find the surface area of each solid. Round to the nearest tenth if necessary.

**a.**

\[
A = s^2
\]

\[
A = 8^2 \text{ or } 64 \quad \text{Replace } s \text{ with 8.}
\]

The surface area of the pyramid is the sum of the lateral area and the area of the base, 240 + 64 or 304 square inches.
1. **Explain** how the slant height and the height of a pyramid are different.

2. **OPEN ENDED** Draw a square pyramid, giving measures for its slant height and base side length. Then find its lateral area.

---

**GUIDED PRACTICE**

Find the surface area of each solid. Round to the nearest tenth if necessary.

3.

4.

5.

---

**Practice and Applications**

Find the surface area of each solid. Round to the nearest tenth if necessary.

6.

7.

8.

9.

10.

11.

12. cone: diameter, 11.4 ft; slant height, 25 ft

13. square pyramid: base side length, 6 \( \frac{1}{2} \) cm; slant height 8 \( \frac{1}{4} \) cm

14. Find the surface area of the complex solid at the right. Round to the nearest tenth.

15. **ROOFS** A cone-shaped roof has a diameter of 20 feet and a slant height of 16 feet. If roofing material comes in 120 square-foot rolls, how many rolls will be needed to cover this roof? Explain your reasoning.

16. **GLASS** The Luxor Hotel in Las Vegas, Nevada, is a pyramid-shaped building standing 350 feet tall and covered with glass. Its base is a square with each side 646 feet long. Find the surface area of the glass on the Luxor. (Hint: Use the Pythagorean Theorem to find the pyramid’s slant height \( \ell \).)
17. **GEOMETRY** A *frustum* is the part of a solid that remains after the top portion of the solid has been cut off by a plane parallel to the base. The lampshade at the right is a frustum of a cone. Find the surface area of the lampshade.

**CRITICAL THINKING** For Exercises 18–20, use the drawings of the pyramid below, whose lateral faces are equilateral triangles.

18. Find the exact measure of the slant height \( \ell \).
19. Use the slant height to find the exact height \( h \) of the pyramid.
20. Find the exact volume and surface area of the pyramid.

**EXTENDING THE LESSON** The surface area \( S \) of a sphere with radius \( r \) is given by the formula \( S = 4\pi r^2 \).

Find the surface area of each sphere to the nearest tenth.

21. \( r = 3 \text{ m} \)
22. \( r = 10 \text{ in.} \)
23. \( r = 16 \text{ ft} \)
24. \( r = 4.8 \text{ cm} \)

25. **MULTIPLE CHOICE** Which is the best estimate for the surface area of a cone with a radius of 3 inches and a slant height of 5 inches?

- \( A \) 45 in\(^2\)  
- \( B \) 72 in\(^2\)  
- \( C \) 117 in\(^2\)  
- \( D \) 135 in\(^2\)  

26. **MULTIPLE CHOICE** What is the lateral area of the pentagonal pyramid at the right if the slant height is 9 centimeters?

- \( E \) 18 cm\(^2\)  
- \( F \) 72 cm\(^2\)  
- \( G \) 90 cm\(^2\)  
- \( H \) 180 cm\(^2\)  

27. **GEOMETRY** Find the surface area of a cylinder whose diameter is 22 feet and whose height is 7.5 feet. (*Lesson 7-7*)

28. **MULTI STEP** The cylindrical air duct of a large furnace has a diameter of 30 inches and a height of 120 feet. If it takes 15 minutes for the contents of the duct to be expelled into the air, what is the volume of the substances being expelled each hour? (*Lesson 7-5*)

**BASIC SKILL** Find the value of each expression to the nearest tenth.

29. \( 8.35 + 54.2 \)
30. \( 7 - 2.89 \)
31. \( 4.2 \cdot 6.13 \)
32. \( 9.31 \div 5 \)
Investigate the volume and surface area of similar solids.

**What You’ll Learn**

- Investigate the volume and surface area of similar solids.

**Review Vocabulary**

- **Proportion**: An equation stating that two ratios are equivalent (Lesson 4-4)

---

### Similar Solids

The pyramids are **similar solids**, because they have the same shape and their corresponding linear measures are proportional.

The number of times you increase or decrease the linear dimensions of a solid is called the **scale factor**. The heights of pyramid A and pyramid B are 6 meters and 9 meters, respectively. So the scale factor from pyramid A to pyramid B is \( \frac{6}{9} = \frac{2}{3} \).

---

### Activity

Find the surface area and volume of the prism at the right. Then find the surface areas and volumes of similar prisms with scale factors of 2, 3, and 4.

---

### Exercises

1. How many times greater than the surface area of prism A is the surface area of prism B? prism C? prism D?
2. How are the answers to Exercise 1 related to the scale factors?
3. How many times greater than the volume of prism A is the volume of prism B? prism C? prism D?
4. How are the answers to Exercise 3 related to the scale factors?
5. Considering the rectangular prism in the activity above, write expressions for the surface area and volume of a similar prism with scale factor \( x \).
Find the surface area and volume of the cylinder at the right. Then find the surface areas and volumes of similar cylinders with scale factors of 2, 3, and 4.

**EXERCISES**

6. How many times greater than the surface area of cylinder A is the surface area of cylinder B? cylinder C? cylinder D?

7. How are the answers to Exercise 6 related to the scale factors of each cylinder?

8. How many times greater than the volume of cylinder A is the volume of cylinder B? cylinder C? cylinder D?

9. How are the answers to Exercise 8 related to the scale factors of each cylinder?

10. Considering the cylinder in the activity above, write expressions for the surface area and volume of a similar cylinder with scale factor \( x \).

11. **Make a conjecture** about how the volume and surface area of a pyramid are affected when all edges of this solid are multiplied by a scale factor of \( x \).

For Exercises 12 and 13, use the diagram of the two similar prisms at the right.

12. If the surface area of prism A is 52 square feet, find the surface area of prism B.

13. If the volume of prism A is 24 cubic feet, find the volume of prism B.
Analyze measurements.

**NEW Vocabulary**
- precision
- significant digits

**am I ever going to use this?**

CARTOONS Consider the cartoon below.

**Hi & Lois**

DITTO’S PIECE IS BIGGER THAN MINE!

THEY BOTH LOOK ABOUT THE SAME TO ME.

10-10

THERE’S A 1/32” DIFFERENCE.

1. How precisely has the daughter, Dot, measured each piece?
2. Give an example of a situation where this degree of accuracy might be appropriate.

The **precision** of a measurement is the exactness to which a measurement is made. Precision depends upon the smallest unit of measure being used, or the **precision unit**. A measurement is accurate to the nearest precision unit.

**EXAMPLE**

Identify Precision Units

Identify the precision unit of the flask.

There are two spaces between each 50 milliliter-mark, so the precision unit is \( \frac{1}{2} \) of 50 milliliters or 25 milliliters.

Identify the precision unit of each measuring instrument.

a. 

b.

One way to record a measure is to estimate to the nearest precision unit. A more precise method is to include all of the digits that are actually measured, plus one estimated digit. The digits you record when you measure this way are called significant digits. **Significant digits** indicate the precision of the measurement.
Precision

The precision unit of a measuring instrument determines the number of significant digits.

**Example**

- Precision unit: 1 cm
- Actual measure: 14–15 cm
- Estimated measure: 14.3 cm

- Precision unit: 0.1 cm
- Actual measure: 14.3–14.4 cm
- Estimated measure: 14.35 cm

There are special rules for determining significant digits in a given measurement. Numbers are analyzed for significant digits by counting digits from left to right, starting with the first nonzero digit.

<table>
<thead>
<tr>
<th>Number</th>
<th>Significant Digits</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.45</td>
<td>3</td>
<td>All nonzero digits are significant.</td>
</tr>
<tr>
<td>140.06</td>
<td>5</td>
<td>Zeros between two significant digits are significant.</td>
</tr>
<tr>
<td>0.013</td>
<td>2</td>
<td>Zeros used to show place value of the decimal are not significant.</td>
</tr>
<tr>
<td>120.0</td>
<td>4</td>
<td>In a number with a decimal point, all zeros to the right of a nonzero digit are significant.</td>
</tr>
<tr>
<td>350</td>
<td>2</td>
<td>In a number without a decimal point, any zeros to the right of the last nonzero digit are not significant.</td>
</tr>
</tbody>
</table>

**Identify Significant Digits**

Determine the number of significant digits in each measure.

- 10.25 g 4 significant digits
- 0.003 L 1 significant digit

When adding or subtracting measurements, the sum or difference should have the same precision as the least precise measurement.

**Example**

**LIFTING** You are attempting to lift three packages that weigh 5.125 pounds, 6.75 pounds, and 4.6 pounds. Write the combined weight of the packages using the correct precision.

- 6.75 2 decimal places
- 5.125 3 decimal places
- + 4.6 1 decimal place

16.475

The least precise measurement has 1 decimal place, so round the sum to 1 decimal place.

The combined weight of the packages is about 16.5 pounds.
When multiplying or dividing measurements, the product or quotient should have the same number of significant digits as the measurement with the least number of significant digits.

**Multiply Measurements**

**GEOMETRY** Use the correct number of significant digits to find the area of the parallelogram.

\[
\begin{align*}
10.4 \times 6.2 & \quad (3 \text{ significant digits}) \\
64.48 & \quad (2 \text{ significant digits})
\end{align*}
\]

Round the product, 64.48, so that it has 2 significant digits. The area of the parallelogram is about 64 square centimeters.

**Your Turn**

c. Find 3.48 liters − 0.2 liters using the correct precision.
d. Use the correct number of significant digits to calculate 0.45 meter ÷ 0.8 meter.

**Skill and Concept Check**

1. **Determine** which measurement of a bag of dog food would be the most precise: 5 pounds, 74 ounces, or 74.8 ounces. Explain.

2. **OPEN ENDED** Write a 5-digit number with 3 significant digits.

3. **Which One Doesn’t Belong?** Identify the number that does not have the same number of significant digits as the other three. Explain.

4. **Guided Practice**

   Identify the precision of the unit of each measuring instrument.

   4. in.

   5. mm

   Determine the number of significant digits in each measure.

   6. 138.0 g
   7. 0.0037 mm
   8. 50 min
   9. 206.04 cm

   Find each sum or difference using the correct precision.

   10. 45 in. + 12.7 in.
   11. 7.38 m − 5.9 m

   Find each product or quotient using the correct number of significant digits.

   12. 8.2 yd · 4.5 yd
   13. 7.31 s ÷ 5.4 s
Identify the precision unit of each measuring instrument.

14. \[
\begin{array}{ccc}
\text{cm} & 1 & 2 \\
\end{array}
\]

15. \[
\begin{array}{ccc}
\text{in.} & 1 & 2 \\
\end{array}
\]

16. \[
\text{A dropper}
\]

17. \[
\text{A balance}
\]

Determine the number of significant digits in each measure.

18. 0.025 mL 19. 3,450 km 20. 40.03 in. 21. 7.0 kg

22. 104.30 mi 23. 3.06 s 24. 0.009 mm 25. 380 g

Find each sum or difference using the correct precision.

26. 12.85 cm + 5.4 cm 27. 14.003 L − 4.61 L 28. 34 g − 15.2 g

29. 150 m + 44.7 m 30. 100 mi + 63.7 mi 31. 14.37 s − 9.2 s

Find each product or quotient using the correct number of significant digits.

32. 0.8 cm · 9.4 cm 33. 3.82 ft · 3.5 ft 34. 10 mi · 1.2 mi

35. 200 g ÷ 2.6 g 36. 88.5 lb ÷ 0.05 lb 37. 7.50 mL ÷ 0.2 mL

38. **GEOMETRY** A triangle’s sides measure 17.04 meters, 8.2 meters, and 7.375 meters. Write the perimeter using the correct precision.

39. **SURVEYING** A surveyor measures the dimensions of a field and finds that the length is 122.5 meters and the width is 86.4 meters. What is the area of the field? Round to the correct number of significant digits.

**SCHOOL** For Exercises 40–42, refer to the graphic at the right.

40. Are the numbers exact? Explain.

41. How many significant digits are used to describe the number of children enrolled in public school?

42. Find the difference between public and private school enrollment using the correct precision.

---

USA TODAY Snapshots®

**1 in 9 children are in private school**

About 53.5 million children are enrolled in kindergarten through the 12th grade in the USA this year. Private versus public school enrollment:

- **Public** 47.5 million
- **Private** 6 million

Source: U.S. Education Department

By Hilary Wasson and Bob Laird, USA TODAY
43. **CRITICAL THINKING** Find the surface area of the square pyramid at the right. Use the correct precision or number of significant digits as appropriate.

**EXTENDING THE LESSON** The *greatest possible error* is one-half the precision unit. It can be used to describe the actual measure. The cotton swab below appears to be about 7.8 centimeters long.

The possible actual length of the cotton swab is 0.05 centimeter less than or 0.05 centimeter more than 7.8 centimeters. So, it is between 7.75 and 7.85 centimeters long.

44. **SPORTS** An Olympic swimmer won the gold medal in the 100-meter backstroke with a time of 61.19 seconds. Find the greatest possible error of the measurement and use it to determine between which two values is the swimmer’s actual time.

45. **MULTIPLE CHOICE** Choose the measurement that is most precise.

- (A) 54 kg
- (B) 5.4 kg
- (C) 54 g
- (D) 54 mg

46. **GRID IN** Use the correct number of significant digits to find the volume of a cylinder in cubic feet whose radius is 4.0 feet and height is 10.2 feet.

47. **DESSERT** Find the surface area of the waffle cone at the right. (Lesson 7-8)

48. **HISTORY** The great pyramid of Khufu in Egypt was originally 481 feet high, had a square base 756 feet on a side, and slant height of about 611.8 feet. What was its surface area, not including the base? Round to the nearest tenth. (Lesson 7-7)

Solve each equation. Check your solution. (Lesson 2-9)

49. \[ x + 0.26 = -3.05 \]

50. \[ \frac{3}{5} = a - \frac{1}{2} \]

51. \[ -\frac{1}{6} = -\frac{1}{4}n \]

52. \[ \frac{y}{2.4} = -6.5 \]

**Under Construction**

It’s time to complete your project. Use the information and data you have gathered about floor coverings costs and loan rates to prepare a Web page or brochure. Be sure to include a labeled scale drawing with your project.

msmath3.net/webquest
Vocabulary and Concept Check

Choose the letter of the term that best matches each phrase.

1. a flat surface of a prism
2. the measure of the space occupied by a solid
3. a figure that has two parallel, congruent circular bases
4. any three-dimensional figure
5. the sides of a pyramid
6. the distance around a circle
7. the exactness to which a measurement is made
8. any side of a parallelogram
9. a solid figure with flat surfaces that are polygons

Lesson-by-Lesson Exercises and Examples

Example 1
Find the area of the trapezoid.

height: 5 inches
bases: 6 inches and 13 inches

\[ A = \frac{1}{2} h(b_1 + b_2) \]

Area of a trapezoid

\[ A = \frac{1}{2}(5)(6 + 13) \]

\[ h = 5, b_1 = 6, b_2 = 13 \]

\[ A = \frac{1}{2}(5)(19) \text{ or 47.5} \]

Simplify.

The area is 47.5 square inches.
7-2 **Circumference and Area of Circles** (pp. 319–323)

Find the circumference and area of each circle. Round to the nearest tenth.

14. \[ \text{Circumference} = \pi \times \text{diameter} \]
15. \[ \text{Area} = \pi \times \left(\frac{\text{diameter}}{2}\right)^2 \]

16. The diameter is $4\frac{1}{3}$ feet.
17. The radius is 2.6 meters.

**Example 2** Find the circumference and area of the circle.

The radius of the circle is 5 yards.

\[ C = 2\pi r \quad A = \pi r^2 \]

\[ C = 2 \times \pi \times 5 \quad A = \pi \times 5^2 \]

\[ C \approx 31.4 \text{ yd} \quad A \approx 78.5 \text{ yd}^2 \]

7-3 **Area of Complex Figures** (pp. 326–329)

Find the area of each figure. Round to the nearest tenth if necessary.

18. \[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]
19. \[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

20. \[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]
21. \[ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} \]

**Example 3** Find the area of the complex figure.

Area of semicircle \[ A = \frac{1}{2} \times \pi \times r^2 \]
Area of trapezoid \[ A = \frac{1}{2} (b_1 + b_2)h \]

\[ A \approx 6.3 \quad A = 42 \]

The area is about $6.3 + 42 = 48.3$ square meters.

7-4 **Three-Dimensional Figures** (pp. 331–334)

Identify each solid. Name the number and shapes of the faces. Then name the number of edges and vertices.

22. \[ \text{Solid} \]
23. \[ \text{Solid} \]

**Example 4** Name the number and shapes of the faces of a rectangular prism. Then name the number of edges and vertices.

8 vertices \[ 6 \text{ rectangular faces} \]
12 edges
7-5 Volume of Prisms and Cylinders (pp. 335–339)

Find the volume of each solid.

24.

25.

26. FOOD A can of green beans has a diameter of 10.5 centimeters and a height of 13 centimeters. Find its volume.

Example 5 Find the volume of the solid.

The base of this prism is a triangle.

\[ V = Bh \]

\[ B = \text{area of base}, \ h = \text{height of prism} \]

\[ V = \left( \frac{1}{2} \cdot 13 \cdot 10 \right) \cdot 18 \]

\[ V = 1,170 \text{ ft}^3 \]

7-6 Volume of Pyramids and Cones (pp. 342–345)

Find the volume of each solid. Round to the nearest tenth if necessary.

27.

28.

29. cone: diameter, 9 yd; height, 21 yd

Example 6 Find the volume of the pyramid.

The base of the rectangle is a triangle.

\[ V = \frac{1}{3}Bh \]

Volume of pyramid or cone

\[ V = \frac{1}{3}(12 \cdot 6)8 \]

\[ V = 192 \text{ in}^3 \]

7-7 Surface Area of Prisms and Cylinders (pp. 347–351)

Find the surface area of each solid. Round to the nearest tenth if necessary.

30.

31.

32. SET DESIGN All but the bottom of a platform 15 feet long, 8 feet wide, and 3 feet high is to be painted. Find the surface area of the platform to be painted.

Example 7 Find the surface area of the cylinder.

Find the area of the two circular bases and add the area of the curved surface.

\[ S = 2\pi r^2 + 2\pi rh \]

\[ r = 8 \text{ and } h = 11 \]

\[ S \approx 955.0 \]

Use a calculator.

The surface area is about 955.0 square millimeters.
# 7-8 Surface Area of Pyramids and Cones (pp. 352–355)

Find the surface area of each solid. Round to the nearest tenth if necessary.

33. \(7 \text{ ft} \)
34. \(3.4 \text{ mm} \)
35. \(13 \text{ cm} \)
36. \(9 \text{ yd} \)

37. **DECORATING** All but the underside of a 10-foot tall conical-shaped tree is to be covered with fake snow. The base of the tree has a radius of 5 feet, and its slant height is about 11.2 feet. How much area is to be covered with fake snow?

## Example 8
Find the surface area of the square pyramid.

\[ A = \frac{1}{2}bh \]

**Area of triangle**

\[ A = \frac{1}{2}(3)(7) \text{ or } 10.5 \]

The total lateral area is \(4(10.5)\) or 42 square meters. The area of the base is \(3(3)\) or 9 square meters. So the total surface area of the pyramid is \(42 + 9\) or 51 square meters.

## Example 9
Find the surface area of the cone.

\[ S = \pi r\ell + \pi r^2 \]

**Surface area of a cone**

\[ S = \pi(4)(13) + \pi(4)^2 \]

\[ r = 4 \text{ and } \ell = 13 \]

\[ S \approx 213.6 \]

Use a calculator.

The surface area is about 213.6 square inches.

# 7-9 Precision and Significant Digits (pp. 358–362)

38. **MEASUREMENT** Order the following measures from least precise to most precise.

- 0.50 cm
- 0.005 cm
- 0.5 cm
- 50 cm

Determine the number of significant digits in each measure.

39. 0.14 ft  
40. 7.0 L  
41. 9.04 s

Find each sum or difference using the correct precision.

42. \(40 \text{ g} + 15.7 \text{ g} \)
43. \(45.3 \text{ lb} - 0.02 \text{ lb} \)

Find each product or quotient using the correct number of significant digits.

44. \(6.4 \text{ yd} \times 2 \text{ yd} \)
45. \(200.8 \text{ m} \div 12.0 \text{ m} \)

## Example 10
Determine the number of significant digits in a measure of 180 miles.

In a number without a decimal point, any zeros to the right of the last nonzero digit are not significant. Therefore, 180 miles has 2 significant digits, 1 and 8.

## Example 11
Use the correct number of significant digits to find

\[
701 \text{ feet} \times 0.04 \text{ feet}.
\]

\[
\frac{701}{28.04} \leftarrow 2 \text{ significant digits}
\]

\[
\frac{701}{28.04} \leftarrow 1 \text{ significant digit}
\]

The product, rounded to 1 significant digit, is 30 square feet.
1. Explain how to find the volume of any prism.
2. Explain how to find the surface area of any prism.

Find the area of each figure. Round to the nearest tenth if necessary.

3. 4. 5. 6.

7. **CIRCUS** The elephants at a circus are paraded around the edge of the center ring two times. If the ring has a radius of 25 yards, about how far do the elephants walk during this part of the show?

8. **CAKE DECORATION** Mrs. Chávez designed the flashlight birthday cake shown at the right. If one container of frosting covers 250 square inches of cake, how many containers will she need to frost the top of this cake? Explain your reasoning.

Find the volume and surface area of each solid. Round to the nearest tenth if necessary.

9. 10. 11. 12.

13. Determine the number of significant digits in 0.089 milliliters.
14. Find 18.2 milligrams − 7.34 milligrams using the correct precision.
15. Find 0.5 yards · 18.3 yards using the correct number of significant digits.

16. **MULTIPLE CHOICE** Find the volume of the solid at the right.

- A 2,160 ft³
- B 2,520 ft³
- C 3,600 ft³
- D 7,200 ft³
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Unleaded gasoline costs \( \frac{1.49}{10} \) per gallon. What is the best estimate of the cost of 8.131 gallons of unleaded gasoline? (Prerequisite Skill, pp. 600–601)
   - A $8  
   - B $9  
   - C $12  
   - D $16

2. Which equation is equivalent to \( \frac{n}{7} = \frac{7}{4} \)? (Lesson 1-8)
   - F \( n = 3 \)  
   - G \( n + 7 - 7 = -4 + 7 \)  
   - H \( n + 14 = -8 \)  
   - I \( n + 7 - 7 = -4 - 7 \)

3. Jamie started at point \( F \) and drove 28 miles due north to point \( G \). He then drove due west to point \( H \). He was then 35 miles from his starting point. What was the distance from point \( G \) to point \( H \)? (Lesson 3-5)
   - A 7 mi  
   - B 14 mi  
   - C 21 mi  
   - D 31.5 mi

4. In 1990, the population of Tampa, Florida, was about 281,000. In 2000, the population was about 303,000. What was the approximate percent of increase in population over this ten-year period? (Lesson 5-7)
   - F 7%  
   - G 8%  
   - H 22%  
   - I 93%

5. In the diagram, \( \angle A \equiv \angle B \). Find the measure of \( \angle A \). (Lessons 6-1, 6-2)
   - A 35°  
   - B 55°  
   - C 70°  
   - D 110°

6. Keisha needed to paint a triangular wall that was 19 feet long and 8 feet tall. When she stopped to rest, she still had 25 square feet of wall unpainted. How many square feet of wall did she paint before she stopped to rest? (Lesson 7-1)
   - F 51 ft²  
   - G 76 ft²  
   - H 101 ft²  
   - I 127 ft²

7. If a circle’s circumference is 28 yards, what is the best estimate of its diameter? (Lesson 7-2)
   - A 9 yd  
   - B 14 yd  
   - C 21 yd  
   - D 84 yd

8. The drawing shows a solid figure built with cubes. Which drawing represents a view of this solid from directly above? (Lesson 7-4)

9. The volume of the pyramid at the right is 54 cubic meters. Find the height of the pyramid. (Lesson 7-6)
   - A 3 m  
   - B 9 m  
   - C 18 m  
   - D 36 m

**Question 9** Most standardized tests will include any commonly used formulas at the front of the test booklet, but it will save you time to memorize many of these formulas. For example, you should memorize that the volume of a pyramid is one-third the area of the base times the height of the pyramid.
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. You need $\frac{2}{3}$ cups of chocolate chips to make one batch of chocolate chip cookies. How many $\frac{1}{3}$-cups of chocolate chips is this?  \((\text{Lesson 2-4})\)

11. Four days ago, Evan had completed 5 pages of his term paper. Today he has completed a total of 15 pages. Find the rate of change in his progress in pages per day.  \((\text{Lesson 4-2})\)

12. A boy who is $5\frac{1}{2}$ feet tall casts a shadow 4 feet long. A nearby tree casts a shadow 10 feet long.

What is the height of the tree in feet?  \((\text{Lesson 4-7})\)

13. Find the area of the top of a compact disc if its diameter is 12 centimeters and the diameter of the hole is 1.5 centimeters.  \((\text{Lesson 7-2})\)

14. Mr. Brauen plans to carpet the part of his house shown on the floor plan below. How many square feet of carpet does he need?  \((\text{Lesson 7-3})\)

15. The curved part of a can will be covered by a label. What is the area of the label to the nearest tenth of a square centimeter?  \((\text{Lesson 7-7})\)

16. A prism with a triangular base has 9 edges, and a prism with a rectangular base has 12 edges.

Explain in words or symbols how to determine the number of edges for a prism with a 9-sided base. Be sure to include the number of edges in your explanation.  \((\text{Lesson 7-4})\)

17. The diagrams show the design of the trashcans in the school cafeteria.  \((\text{Lessons 7-5 and 7-7})\)

a. Find the volume of trash each can is designed to hold to the nearest tenth.
b. The top and sides of the cans need to be painted. Find the surface area of each can to the nearest tenth.
c. The paint used by the school covers 200 square feet per gallon. How many trashcans can be covered with 1 gallon of paint?