Our world is made up of lines, angles, and shapes, both two- and three-dimensional. In this unit, you will learn about the properties and measures of geometric figures.
Can you build it? Yes, you can! You’ve been selected to head the architectural and construction teams on a house of your own design. You’ll create the uniquely shaped floor plan, research different floor coverings for the rooms in your house, and finally research different loans to cover the cost of purchasing these floor coverings. So grab a hammer and some nails, and don’t forget your geometry and measurement tool kits. You’re about to construct a cool adventure!

Log on to msmath3.net/webquest to begin your WebQuest.
How is geometry used in the game of pool?

A billiard ball is struck so that it bounces off the cushion of a pool table and heads for a corner pocket. The three angles created by the path of the ball and the cushion together form a straight angle that measures $180^\circ$. Pool players use such angle relationships and the properties of reflections to make their shots.

You will solve problems about angle relationships in Lesson 6-1.
Take this quiz to see whether you are ready to begin Chapter 6. Refer to the lesson number in parentheses if you need more review.

**Vocabulary Review**
State whether each sentence is true or false. If false, replace the underlined word or number to make a true sentence.

1. For the right triangle shown, the Pythagorean Theorem states that \( a^2 + c^2 = b^2 \). (Lesson 3-4)
2. A rectangle is also a polygon. (Lesson 4-5)

**Prerequisite Skills**
Solve each equation. (Lesson 1-8)

3. \( 49 + b + 45 = 180 \)  
4. \( t + 98 + 55 = 180 \)  
5. \( 15 + 67 + k = 180 \)

Find the missing side length of each right triangle. Round to the nearest tenth, if necessary. (Lesson 3-4)

6. \( a, 8 \text{ m}; b, 6 \text{ m} \)  
7. \( b, 9 \text{ ft}; a, 7 \text{ ft} \)  
8. \( a, 4 \text{ in.}; c, 5 \text{ in.} \)  
9. \( c, 10 \text{ yd}; a, 3 \text{ yd} \)

Decide whether the figures are congruent. Write yes or no and explain your reasoning. (Lesson 4-5)

10. \begin{tikzpicture}  
\draw[blue] (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;  
\draw[blue] (0,0) -- (3,0) -- (3,3) -- (0,3) -- cycle;  
\draw[blue] (0,0) -- (3,3);  
\end{tikzpicture} \quad 3 \text{ cm} \quad 3 \text{ cm} \quad 60^\circ \quad 45^\circ

11. \begin{tikzpicture}  
\draw[blue] (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;  
\draw[blue] (0,0) -- (1,1);  
\end{tikzpicture} \quad 3 \text{ cm} \quad 3 \text{ cm} \quad 60^\circ \quad 45^\circ

**Geometry** Make this Foldable to help you organize your notes. Begin with a plain piece of 11" × 17" paper.

1. **Fold**
Fold the paper in fifths lengthwise.

2. **Open and Fold**
Fold a 2 1/2" tab along the short side. Then fold the rest in half.

3. **Label**
Draw lines along folds and label each section as shown.

<table>
<thead>
<tr>
<th>Foldable</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lines</td>
<td></td>
</tr>
<tr>
<td>Polygons</td>
<td></td>
</tr>
<tr>
<td>Symmetry</td>
<td></td>
</tr>
<tr>
<td>Transformations</td>
<td></td>
</tr>
</tbody>
</table>

**Reading and Writing** As you read and study the chapter, complete the table by writing down important definitions and key concepts for each heading.
Line and Angle Relationships

**What You’ll Learn**
Identify special pairs of angles and relationships of angles formed by two parallel lines cut by a transversal.

**New Vocabulary**
aacute angle
bright angle
obtuse angle
straight angle
vertical angles
adjacent angles
complementary angles
supplementary angles
perpendicular lines
parallel lines
transversal

**Hands-On Mini Lab**

Work with a partner.

**STEP 1** Draw two different pairs of intersecting lines and label the angles formed as shown.

**STEP 2** Find and record the measure of each angle.

**STEP 3** Color angles that have the same measure.

1. For each set of intersecting lines, identify the pairs of angles that have the same measure.
2. What is true about the sum of the measures of the angles sharing a side?

Angles can be classified by their measures.
- **Acute angles** have measures less than 90°.
- **Right angles** have measures equal to 90°.
- **Obtuse angles** have measures between 90° and 180°.
- **Straight angles** have measures equal to 180°.

Pairs of angles can be classified by their relationship to each other. Recall that angles with the same measure are congruent.

**Key Concept**

<table>
<thead>
<tr>
<th>Vertical angles</th>
<th>Adjacent angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>are opposite angles formed by intersecting lines. Vertical angles are congruent.</td>
<td>have the same vertex, share a common side, and do not overlap.</td>
</tr>
<tr>
<td><img src="image" alt="Diagram of vertical angles" /></td>
<td><img src="image" alt="Diagram of adjacent angles" /></td>
</tr>
<tr>
<td>∠1 and ∠2 are vertical angles. ∠1 = ∠2</td>
<td>∠1 and ∠2 are adjacent angles. m∠ABC = m∠1 + m∠2</td>
</tr>
<tr>
<td>The sum of the measures of complementary angles is 90°.</td>
<td>The sum of the measures of supplementary angles is 180°.</td>
</tr>
<tr>
<td><img src="image" alt="Diagram of complementary angles" /></td>
<td><img src="image" alt="Diagram of supplementary angles" /></td>
</tr>
<tr>
<td>m∠ABD + m∠DBC = 90°</td>
<td>m∠C + m∠D = 180°</td>
</tr>
</tbody>
</table>

**Virginia SOL Standard 8.6** The student will verify by measuring and describe the relationships among vertical angles, supplementary angles, and complementary angles and will measure and draw angles of less than 360°.
You can use the relationships between pairs of angles to find missing measures.

**Classify Angles and Angle Pairs**

Classify each angle or angle pair using all names that apply.

1. \( \angle 1 \) is greater than 90°
   So, \( \angle 1 \) is an obtuse angle.

\[ \angle 1 \] and \( \angle 2 \) are adjacent angles since they have the same vertex, share a common side, and do not overlap. Together, they form a straight angle measuring 180°. So, \( \angle 1 \) and \( \angle 2 \) are also supplementary angles.

**Your Turn**

Classify each angle or angle pair using all names that apply.

a. 

b. 

c. 

You can use the relationships between pairs of angles to find missing measures.

**Find a Missing Angle Measure**

In the figure, \( m \angle ABC = 90° \). Find the value of \( x \).

\[
m \angle ABD + m \angle DBC = 90^
\]

Write an equation.

\[
x + 65 = 90 \quad m \angle ABD = x \text{ and } m \angle DBC = 65
\]

Subtract 65 from each side.

\[
x = 25 \quad \text{Simplify.}
\]

**Your Turn**

Find the value of \( x \) in each figure.

d. 

e. 

Lines that intersect at right angles are called **perpendicular lines**. Two lines in a plane that never intersect or cross are called **parallel lines**.

\[ \text{Symbols: } m \perp n \]

\[ \text{Symbols: } p \parallel q \]

A red right angle symbol indicates that lines \( m \) and \( n \) are perpendicular.

Red arrowheads indicate that lines \( p \) and \( q \) are parallel.
A line that intersects two or more other lines is called a **transversal**. When a transversal intersects two lines, eight angles are formed that have special names.

If the two lines cut by a transversal are **parallel**, then these special pairs of angles are congruent.

---

**Key Concept**

**Parallel Lines**

If two parallel lines are cut by a transversal, then the following statements are true.

- **Alternate interior angles**, those on opposite sides of the transversal and inside the other two lines, are congruent.
  
  **Example:** $\angle 2 \cong \angle 8$

- **Alternate exterior angles**, those on opposite sides of the transversal and outside the other two lines, are congruent.
  
  **Example:** $\angle 4 \cong \angle 6$

- **Corresponding angles**, those in the same position on the two lines in relation to the transversal, are congruent.
  
  **Example:** $\angle 3 \cong \angle 7$

---

You can use congruent angle relationships to solve real-life problems.

**EXAMPLE**

**Find an Angle Measure**

**CARPENTRY** You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If $m\angle 1 = 148^\circ$, find $m\angle 2$ and $m\angle 3$.

Since $\angle 1$ and $\angle 2$ are alternate interior angles, they are congruent. So, $m\angle 2 = 148^\circ$.

Since $\angle 2$ and $\angle 3$ are supplementary, the sum of their measures is $180^\circ$. Therefore, $m\angle 3 = 180^\circ - 148^\circ = 32^\circ$.

---

**Your Turn** For Exercises f–h, use the figure at the right.

f. Find $m\angle 2$ if $m\angle 1 = 63^\circ$.

g. Find $m\angle 3$ if $m\angle 8 = 100^\circ$.

h. Find $m\angle 4$ if $m\angle 7 = 82^\circ$.
1. **OPEN ENDED** Draw a pair of complementary angles.

2. **Draw** a pair of parallel lines and a third line intersecting them. Choose one angle and mark it with a ✔. Then mark all other angles that are congruent to that angle with a ✔. Explain.

**Guided Practice**

Classify each angle or angle pair using all names that apply.

3.        4.        5.

Find the value of $x$ in each figure.

6.        7.        8.

For Exercises 9–12, use the figure at the right.

9. Find $m\angle 4$ if $m\angle 5 = 43^\circ$.

10. Find $m\angle 1$ if $m\angle 3 = 135^\circ$.

11. Find $m\angle 6$ if $m\angle 8 = 126^\circ$.

12. Find $m\angle A$ if $m\angle B = 15^\circ$ and $\angle A$ and $\angle B$ are supplementary.

**Practice and Applications**

Classify each angle or angle pair using all names that apply.


16.        17.        18.

Find the value of $x$ in each figure.

19.        20.        21.

22.        23.        24.


**Homework Help**

For Exercises 13–18

19–29

30–38

See Examples

1, 2

3

4

Extra Practice

See pages 629, 653.
27. **ALGEBRA** Angles $P$ and $Q$ are vertical angles. If $\angle P = 45^\circ$ and $\angle Q = (x + 25)^\circ$, find the value of $x$.

28. **ALGEBRA** Angles $A$ and $B$ are supplementary. If $\angle A = 2x^\circ$ and $\angle B = 80^\circ$, find the value of $x$.

29. **POOL** Aaron is trying a complicated pool shot. He wants to hit the number 8 ball into the corner pocket. If Aaron knows the angle measures shown in the diagram, what angle $x$ must the path of the ball take to go into the corner pocket?

For Exercises 30–37, use the figure at the right.

30. Find $m \angle 2$ if $m \angle 3 = 108^\circ$.  
31. Find $m \angle 6$ if $m \angle 7 = 111^\circ$.
32. Find $m \angle 5$ if $m \angle 8 = 85^\circ$.  
33. Find $m \angle 8$ if $m \angle 1 = 63^\circ$.
34. Find $m \angle 8$ if $m \angle 2 = 50^\circ$.  
35. Find $m \angle 4$ if $m \angle 1 = 59^\circ$.
36. Find $m \angle 5$ if $m \angle 4 = 72^\circ$.  
37. Find $m \angle 5$ if $m \angle 7 = 98^\circ$.

38. **PARKING** Engineers angled the parking spaces along a downtown street so that cars could park and back out easily. All of the lines marking the parking spaces are parallel. If $m \angle 1 = 55^\circ$, find $m \angle 2$. Explain your reasoning.

39. **CRITICAL THINKING** Suppose two parallel lines are cut by a transversal. How are the interior angles on the same side of the transversal related? Use a diagram to explain your reasoning.

40. **SHORT RESPONSE** If $m \angle A = 81^\circ$ and $\angle A$ and $\angle B$ are complementary, what is $m \angle B$?

41. **MULTIPLE CHOICE** Find the value of $x$ in the figure at the right.

   - $A$ 30  
   - $B$ 40  
   - $C$ 116  
   - $D$ 124

42. A savings account starts with $560. If the simple interest rate is 3%, find the total amount after 18 months. *(Lesson 5-8)*

Find each percent of change. Round to the nearest tenth if necessary. State whether the percent of change is an increase or a decrease. *(Lesson 5-7)*

43. original: 20  
   new: 27
44. original: 45  
   new: 18
45. original: 620  
   new: 31
46. original: 260  
   new: 299

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Solve each equation. Check your solution. *(Lesson 1-8)*

47. $n + 32 + 67 = 180$  
48. $45 + 89 + x = 180$  
49. $180 = 120 + a + 15$
Constructing Parallel Lines

In this lab, you will construct a line parallel to a given line.

**ACTIVITY**

**STEP 1**
Draw a line and label it \( p \). Then draw and label a point \( A \) not on line \( p \).

**STEP 2**
Draw a line through point \( A \) so that it intersects line \( p \). Label the point of intersection point \( B \).

**STEP 3**
Place the compass at point \( B \) and draw a large arc. Label the point where the arc crosses line \( p \) as point \( C \), and label where it crosses line \( AB \) as point \( D \).

**STEP 4**
With the same compass opening, place the compass at point \( A \) and draw a large arc. Label the point of intersection with line \( AB \) as point \( E \).

**STEP 5**
Use your compass to measure the distance between points \( D \) and \( C \).

**STEP 6**
With the compass opened the same amount, place the compass at point \( A \) and draw an arc to intersect the arc already drawn. Label this point \( F \).

**STEP 7**
Draw a line through points \( A \) and \( F \). Label this line \( q \). You have drawn \( q \parallel p \).

**Your Turn**
Draw a line. Then construct a line parallel to it.

---

**Writing Math**

Work with a partner. Use the information in the activity above.

1. **Classify** \( \angle DBC \) and \( \angle FAE \) in relationship to lines \( p, q, \) and transversal \( AB \).

2. **Explain** why you should expect \( \angle ABC \) to be congruent to \( \angle FAE \).
Find missing angle measures in triangles and classify triangles by their angles and sides.

NEW Vocabulary
triangle
acute triangle
obtuse triangle
right triangle
scalene triangle
isosceles triangle
equilateral triangle

A **triangle** is a figure formed by three line segments that intersect only at their endpoints. Recall that triangles are named by the letters at their vertices.

**Triangle** $LMN$ is written $\triangle LMN$.

**Key Concept**

**Angles of a Triangle**

**Words**
The sum of the measures of the angles of a triangle is $180^\circ$.

**Symbols**

$$x + y + z = 180$$

**Find a Missing Angle Measure**

Find the value of $x$ in $\triangle RST$.

$$m \angle R + m \angle S + m \angle T = 180$$

$$x + 72 + 74 = 180$$

$$x + 146 = 180$$

$$x = 34$$

The value of $x$ is 34.
All triangles have at least two acute angles. Triangles can be classified by the measure of the third angle.

### Classify Triangles by Angles

<table>
<thead>
<tr>
<th>Acute Triangle</th>
<th>Obtuse Triangle</th>
<th>Right Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>three acute angles</td>
<td>one obtuse angle</td>
<td>one right angle</td>
</tr>
</tbody>
</table>

In an *equiangular* triangle, all angles have the same measure, $60^\circ$.

Triangles can also be classified by the number of congruent sides. Congruent sides are often marked with tick marks.

### Classify Triangles by Sides

<table>
<thead>
<tr>
<th>Scalene Triangle</th>
<th>Isosceles Triangle</th>
<th>Equilateral Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>no congruent sides</td>
<td>at least two sides congruent</td>
<td>three sides congruent</td>
</tr>
</tbody>
</table>

### Classify Triangles

Classify each triangle by its angles and by its sides.

**Example 1**

- **Angles**: $\triangle ABC$ has all acute angles.
- **Sides**: $\triangle ABC$ has two congruent sides.
- **Result**: $\triangle ABC$ is an acute isosceles triangle.

**Example 2**

- **Angles**: $\triangle XYZ$ has one right angle.
- **Sides**: $\triangle XYZ$ has no congruent sides.
- **Result**: $\triangle XYZ$ is a right scalene triangle.

### Your Turn

Classify each triangle by its angles and by its sides.

**a.**

- **Angles**: $78^\circ$, $57^\circ$, $45^\circ$.
- **Sides**: No congruent sides.
- **Result**: Acute scalene triangle.

**b.**

- **Angles**: $110^\circ$, $35^\circ$, $35^\circ$.
- **Sides**: At least two congruent sides.
- **Result**: Acute isosceles triangle.

**c.**

- **Angles**: $60^\circ$, $60^\circ$, $60^\circ$.
- **Sides**: Three sides congruent.
- **Result**: Equilateral triangle.
1. **OPEN ENDED** Name a real-life object that is shaped like an isosceles triangle. Explain.

2. **Describe** the types of angles that are in a right triangle.

**GUIDED PRACTICE**

Find the value of $x$ in each triangle.

3. 

4. 

5. 

Classify each triangle by its angles and by its sides.

6. 

7. 

8. 

9. 

**Practice and Applications**

Find the value of $x$ in each triangle.

10. 

11. 

12. 

13. 

14. 

15. 

Classify each triangle by its angles and by its sides.

16. 

17. 

18. 

19. 

20. 

21. 

22. 

23. 

**EXTRA PRACTICE**

See pages 629, 653.
24. **BRIDGE BUILDING** At a Science Olympiad tournament, your team is to design and construct a bridge that will hold the most weight for a given span. Your team knows that triangles add stability to bridges. Below is a side view of your team’s design. Name and classify three differently-shaped triangles in your design.

![Bridge Design](image)

Draw each triangle. If it is not possible to draw the triangle, write *not possible*.

25. three acute angles
26. two obtuse angles
27. obtuse isosceles with two acute angles
28. obtuse equilateral
29. right equilateral
30. right scalene

Determine whether each statement is *sometimes*, *always*, or *never* true.

31. Isosceles triangles are equilateral.
32. Equilateral triangles are isosceles.

33. **CRITICAL THINKING** Explain why all triangles have at least two acute angles.

34. **SHORT RESPONSE** Triangle $ABC$ is isosceles. What is the value of $x$?

35. **MULTIPLE CHOICE** Which term describes the relationship between the two acute angles of a right triangle?

   A. adjacent
   B. complementary
   C. vertical
   D. supplementary

Find the measure of each angle in the figure if $m \parallel n$ and $m \angle 7 = 95^\circ$.

36. $\angle 4$
37. $\angle 3$
38. $\angle 1$
39. $\angle 2$

40. **SAVINGS** Shala’s savings account earned $4.56 in 6 months at a simple interest rate of 4.75%. How much was in her account at the beginning of that 6-month period? (Lesson 5-8)

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Find the missing side length of each right triangle. Round to the nearest tenth if necessary. (Lesson 3-4)

41. $a$, 5 ft; $b$, 8 ft
42. $b$, 10 m; $c$, 12 m
43. $a$, 6 in.; $c$, 13 in.
44. $a$, 7 yd; $b$, 7 yd
**Bisecting Angles**

In this lab, you will learn to bisect an angle.

**ACTIVITY**

Draw $\angle JKL$.

**STEP 1** Place the compass at point $K$ and draw an arc that intersects both sides of the angle. Label the intersections $X$ and $Y$.

**STEP 2** With the compass at point $X$, draw an arc in the interior of $\angle JKL$.

**STEP 3** Using this setting, place the compass at point $Y$. Draw another arc.

**STEP 4** Label the intersection of these arcs $H$. Then draw $KH$. $KH$ is the bisector of $\angle JKL$.

**Your Turn**

Draw each kind of angle. Then bisect it.

a. acute  

b. obtuse

---

**Writing Math**

Work with a partner. Use the information in the activity above.

1. Describe what is true about the measures of $\angle JKH$ and $\angle HKL$.

2. Explain why we say that $KH$ is the bisector of $\angle JKL$.

3. The point where the bisectors of all three angles of a triangle meet is called the incenter. Draw a triangle. Then locate its incenter using only a compass and straightedge.
Find missing measures in $30^\circ$-$60^\circ$ right triangles and $45^\circ$-$45^\circ$ right triangles.

**Hands-On Mini Lab**

Work with a partner.

**Step 1** Trace the equilateral triangle and square below and cut them out.

**Step 2** Measure each angle.

**Step 3** Fold the triangle so that one half matches the other. Fold the square in half along a diagonal.

1. What type of triangles have you formed?
2. What are the measures of the angles of the folded triangle?
3. Measure and describe the relationship between the shortest and longest sides of this triangle.
4. What are the measures of the angles of the triangle formed by folding the square?
5. Measure and describe the relationship between the legs of this triangle.

The sides of a triangle whose angles measure $30^\circ$, $60^\circ$, and $90^\circ$ have a special relationship. The hypotenuse is always twice as long as the side opposite the $30^\circ$ angle.

**Example** Find Lengths of a $30^\circ$-$60^\circ$ Right Triangle

Find each missing length.

**Step 1** Find $a$.

\[ a = \frac{1}{2}c \]

Write the equation.

\[ a = \frac{1}{2}(10) \text{ or } 5 \]

Replace $c$ with 10.

(continued on the next page)
Step 2 Find $b$.

\[ \begin{align*}
10^2 &= a^2 + b^2 \\
100 &= 5^2 + b^2 \\
100 - 25 &= 25 + b^2 - 25 \\
75 &= b^2 \\
\sqrt{75} &= \sqrt{b^2} \\
8.7 &\approx b
\end{align*} \]

The length of $a$ is 5 feet, and the length of $b$ is about 8.7 feet.

Your Turn

Find each missing length. Round to the nearest tenth if necessary.

a. 

\[
\begin{align*}
\text{a} &= 12 \text{ in.} \\
\text{b} &= 30^\circ \\
\text{c} &= 60^\circ
\end{align*}
\]

b. 

\[
\begin{align*}
\text{a} &= 15 \text{ cm} \\
\text{b} &= 60^\circ \\
\text{c} &= 30^\circ
\end{align*}
\]

c. 

\[
\begin{align*}
\text{a} &= 7 \text{ m} \\
\text{b} &= 60^\circ \\
\text{c} &= 45^\circ
\end{align*}
\]

A 45°-45° right triangle is also an isosceles triangle because two angle measures are the same. Thus, the legs are always congruent.

Example

Find Lengths of a 45°-45° Right Triangle

ART The ancient Greeks sometimes used 45°-45° right triangles in their art. The sculpture on the right is based on such a triangle. Suppose the base of a reproduction of the sculpture shown is 15 feet long. Find each missing length.

Step 1 Find $a$.

Sides $a$ and $b$ are the same length. Since $b = 15$ feet, $a = 15$ feet.

Step 2 Find $c$.

\[
\begin{align*}
c^2 &= a^2 + b^2 \\
c^2 &= 15^2 + 15^2 \\
c^2 &= 225 + 225 \\
c^2 &= 450 \\
\sqrt{c^2} &= \sqrt{450} \\
c &\approx 21.2
\end{align*}
\]
1. **Write** a sentence describing the relationship between the hypotenuse of a $30^\circ$-$60^\circ$ right triangle and the leg opposite the $30^\circ$ angle.

2. **OPEN ENDED** Give a real-life example of a $45^\circ$-$45^\circ$ right triangle.

### GUIDED PRACTICE

Find each missing length. Round to the nearest tenth if necessary.

- **Exercise 1**

### Practice and Applications

Find each missing length. Round to the nearest tenth if necessary.

- **Exercise 2**

12. The length of the hypotenuse of a $30^\circ$-$60^\circ$ right triangle is 7.5 meters. Find the length of the side opposite the $30^\circ$ angle.

13. In a $30^\circ$-$60^\circ$ right triangle, the length of the side opposite the $30^\circ$ angle is 5.8 centimeters. What is the length of the hypotenuse?

14. The length of one of the legs of a $45^\circ$-$45^\circ$ right triangle is 6.5 inches. Find the lengths of the other sides.

15. In a $45^\circ$-$45^\circ$ right triangle, the length of one leg is 7.5 feet. What are the lengths of the other sides?

16. **HISTORY** Redan lines such as the one below were used at many battlefields in the Civil War. A redan is a triangular shape that goes out from the main line of defense. What is the distance $h$ from the base of each redan to its farthest point?
17. **QUILTING** Refer to the photograph at the right. Each triangle in the Flying Geese pattern is a 45°-45° right triangle. If the length of a leg is $2 \frac{1}{2}$ inches, find the length of each hypotenuse.

18. **SKIING** A ski jump is constructed so that the length of the board necessary for the surface of the ramp is twice as long as the ramp is high $h$. If the ramp forms a right triangle, what is the measure of $\angle 1$? Explain your reasoning.

19. **WRITE A PROBLEM** Write a real-life problem involving a 30°-60° right triangle or a 45°-45° right triangle. Then solve the problem.

20. **CRITICAL THINKING** Find the length of each leg of a 45°-45° right triangle whose hypotenuse measures $\sqrt{162}$ centimeters.

21. **MULTIPLE CHOICE** The midpoints of the sides of the square at the right are joined to form a smaller square. What is the area of the smaller square?

- A. 196 in²
- B. 98 in²
- C. 49 in²
- D. 9.9 in²

22. **MULTIPLE CHOICE** Which values represent the sides of a 30°-60° right triangle?

- A. 3 cm, 4 cm, 5 cm
- B. 8 cm, 8 cm, $\sqrt{128}$ cm
- C. 4 cm, 8 cm, $\sqrt{48}$ cm
- D. 5 cm, 12 cm, 13 cm

23. Two sides of a triangle are congruent. The angles opposite these sides each measure 70°. Classify the triangle by its angles and by its sides. (Lesson 6-2)

Classify each angle or angle pair using all names that apply. (Lesson 6-1)

24. 25. 26. 27.

28. $\frac{2}{3} \cdot \frac{5}{8}$
29. $\frac{-2}{5} \cdot \frac{3}{4}$
30. $-1 \frac{1}{2} \left(-2 \frac{1}{3}\right)$
31. $2 \frac{2}{3} \left(-2 \frac{1}{4}\right)$

**PREREQUISITE SKILL** Solve each equation. Check your solution. (Lesson 1-8)

32. $x + 90 + 50 + 100 = 360$
33. $45 + 150 + x + 85 = 360$
Constructing Perpendicular Bisectors

In this lab, you will learn to construct a line perpendicular to a segment so that it bisects that segment.

**ACTIVITY**

**STEP 1**
Draw $\overline{AB}$. Then place the compass at point $A$. Using a setting greater than one half the length of $\overline{AB}$, draw an arc above and below $\overline{AB}$.

**STEP 2**
Using this setting, place the compass at point $B$. Draw another set of arcs above and below $\overline{AB}$ as shown.

**STEP 3**
Label the intersection of these arcs $X$ and $Y$ as shown. Then draw $\overline{XY}$. $\overline{XY}$ is the **perpendicular bisector** of $\overline{AB}$. Label the intersection of $\overline{AB}$ and this new line segment $M$.

**Your Turn**
Draw a line segment. Then construct the perpendicular bisector of the segment.

**Writing Math**

Work with a partner. Use the information in the activity above.
1. **Describe** what is true about the measures of $\overline{AM}$ and $\overline{MB}$.
2. **Find** $m\angle XMB$. Then describe the relationship between $\overline{AB}$ and $\overline{XY}$.
3. **Explain** how to construct a $45^\circ$-$45^\circ$ right triangle with legs half as long as the segment below. Then construct the triangle.
Classifying Quadrilaterals

**Hands-On Mini Lab**

Work with a partner.

The polygon at the right is a quadrilateral, since it has four sides and four angles.

1. **Draw a quadrilateral.**

2. **Pick one vertex and draw the diagonal to the opposite vertex.**

3. Name the shape of the figures formed when you drew the diagonal. How many figures were formed?

4. You know that the sum of the angle measures of a triangle is 180°. Use this fact to find the sum of the angle measures in a quadrilateral. Explain your reasoning.

5. Find the measure of each angle of your quadrilateral. Compare the sum of these measures to the sum you found in Exercise 2.

The angles of a quadrilateral have a special relationship.

**Key Concept**

**Angles of a Quadrilateral**

**Words** The sum of the measures of the angles of a quadrilateral is 360°.

**Model**

**Symbols** \[ w + x + y + z = 360 \]

**Example**

Find the value of \( w \) in quadrilateral \( WXYZ \).

\[
\begin{align*}
m\angle W + m\angle X + m\angle Y + m\angle Z &= 360 \\
w + 45 + 110 + 65 &= 360 \\
w + 220 &= 360 \\
-220 &= -220 \\
w &= 140
\end{align*}
\]

The sum of the measures is 360. Replace \( m\angle W \) with \( w \), \( m\angle X \) with 45, \( m\angle Y \) with 110, and \( m\angle Z \) with 65. Subtract 220 from each side. Simplify.
The concept map below shows how quadrilaterals are classified. Notice that the diagram goes from the most general type of quadrilateral to the most specific.

The best description of a quadrilateral is the one that is the most specific.

Classify Quadrilaterals

Classify each quadrilateral using the name that best describes it.

a. The quadrilateral has one pair of parallel sides. It is a trapezoid.

b. The quadrilateral is a parallelogram with four congruent sides. It is a rhombus.
1. **Explain** why a square is a type of rhombus.

2. **OPEN ENDED** Give a real-life example of a parallelogram.

3. **Which One Doesn’t Belong?** Identify the quadrilateral that does not belong with the other three. Explain your reasoning.

   - rhombus
   - rectangle
   - square
   - trapezoid

**Guided Practice**

Find the value of \( x \) in each quadrilateral.

4. \[
\begin{array}{c}
\text{80}^\circ \\
\text{125}^\circ \\
\text{110}^\circ \\
x^\circ
\end{array}
\]

5. \[
\begin{array}{c}
\text{30}^\circ \\
\text{150}^\circ \\
\text{150}^\circ \\
x^\circ
\end{array}
\]

6. \[
\begin{array}{c}
\text{50}^\circ
\end{array}
\]

Classify each quadrilateral using the name that *best* describes it.

7. \[
\begin{array}{c}
\text{x}^\circ
\end{array}
\]

8. \[
\begin{array}{c}
\text{x}^\circ
\end{array}
\]

9. \[
\begin{array}{c}
\text{x}^\circ
\end{array}
\]

**Practice and Applications**

Find the value of \( x \) in each quadrilateral.

10. \[
\begin{array}{c}
\text{60}^\circ \\
\text{106}^\circ \\
\text{120}^\circ \\
x^\circ
\end{array}
\]

11. \[
\begin{array}{c}
\text{x}^\circ \\
\text{95}^\circ \\
\text{112}^\circ \\
\text{58}^\circ
\end{array}
\]

12. \[
\begin{array}{c}
\text{103}^\circ \\
\text{84}^\circ \\
\text{61}^\circ \\
x^\circ
\end{array}
\]

13. \[
\begin{array}{c}
\text{x}^\circ \\
\text{170}^\circ \\
\text{35}^\circ \\
\text{90}^\circ
\end{array}
\]

14. \[
\begin{array}{c}
\text{x}^\circ \\
\text{145}^\circ \\
\text{x}^\circ \\
\text{45}^\circ
\end{array}
\]

15. \[
\begin{array}{c}
\text{x}^\circ \\
\text{52}^\circ \\
\text{99}^\circ \\
\text{67}^\circ
\end{array}
\]

Classify each quadrilateral using the name that *best* describes it.

16. \[
\begin{array}{c}
\text{16}
\end{array}
\]

17. \[
\begin{array}{c}
\text{17}
\end{array}
\]

18. \[
\begin{array}{c}
\text{18}
\end{array}
\]

19. \[
\begin{array}{c}
\text{19}
\end{array}
\]

20. \[
\begin{array}{c}
\text{20}
\end{array}
\]

21. \[
\begin{array}{c}
\text{21}
\end{array}
\]

22. \[
\begin{array}{c}
\text{22}
\end{array}
\]

23. \[
\begin{array}{c}
\text{23}
\end{array}
\]
24. **INTERIOR DESIGN** The stained glass window shown is an example of how geometric figures can be used in decorating. Identify all of the quadrilaterals within the print.

25. **ALGEBRA** In parallelogram $WXYZ$, $m \angle W = 45^\circ$, $m \angle X = 135^\circ$, $m \angle Y = 45^\circ$, and $m \angle Z = (x + 15)^\circ$. Find the value of $x$.

26. **ALGEBRA** In trapezoid $ABCD$, $m \angle A = 2a^\circ$, $m \angle B = 40^\circ$, $m \angle C = 110^\circ$, and $m \angle D = 70^\circ$. Find the value of $a$.

Name all quadrilaterals with the given characteristic.

27. only one pair of parallel sides  
28. opposite sides congruent  
29. all sides congruent  
30. all angles are right angles

**CRITICAL THINKING** Determine whether each statement is true or false. If false, draw a counterexample.

31. All trapezoids are quadrilaterals.  
32. All squares are rectangles.  
33. All rhombi (plural of rhombus) are squares.  
34. A trapezoid can have only one right angle.

35. **MULTIPLE CHOICE** Which of the following does not describe the quadrilateral at the right?

- A parallelogram  
- B square  
- C trapezoid  
- D rhombus

36. **SHORT RESPONSE** In rhombus $WXYZ$, $m \angle Z = 70^\circ$, $m \angle X = 70^\circ$, and $m \angle Y = 110^\circ$. Find the measure of $\angle W$.

37. The length of the hypotenuse of a $30^\circ$-$60^\circ$ right triangle is 16 feet. Find the length of the side opposite the $60^\circ$ angle. Round to the nearest tenth. (Lesson 6-3)

38. The length of one of the legs of a $45^\circ$-$45^\circ$ right triangle is 8 meters. Find the length of the hypotenuse. Round to the nearest tenth. (Lesson 6-3)

Classify each triangle by its angles and by its sides. (Lesson 6-2)

39.  
40.  
41.  

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Decide whether the figures are congruent. Write yes or no and explain your reasoning. (Lesson 4-5)

42. yes  
43. no  
44. yes
Problem-Solving Strategy
A Follow-Up of Lesson 6-4

What You’ll Learn
Solve problems using the logical reasoning strategy.

Use Logical Reasoning

Jacy, how can we be sure this playing field we’ve marked out is a rectangle? We don’t have anything we can use to measure its angles.

Someone told me that there is something special about the diagonals of a rectangle. Zach, let’s see if we can **use logical reasoning** to figure out what that is.

**Explore**
The playing field is a parallelogram because its opposite sides are the same length. Our math teacher said that means they are also parallel. We need to see what the relationship is between the diagonals of a rectangle.

**Plan**
Let’s draw several different rectangles, measure the diagonals, and see if there is a pattern.

**Solve**

```
\[ AC = BD \]
```

It appears that the diagonals of a rectangle are congruent. If the diagonals of our field are congruent, then we can reason that it is a rectangle.

**Examine**
Do all parallelograms, not just rectangles, have congruent diagonals? The counterexample at the right suggests that this statement is false.

1. **Deductive reasoning** uses an existing rule to make a decision. **Determine** where Zach and Jacy used deductive reasoning. Explain.

2. **Inductive reasoning** is the process of making a rule after observing several examples and using that rule to make a decision. **Determine** where Zach and Jacy used inductive reasoning. Explain.

3. **Write** about a situation in which you use inductive reasoning to solve a problem. Then solve the problem.
Solve. Use logical reasoning.

4. **GEOMETRY** Draw several parallelograms and measure their angles. What can you conclude about opposite angles of parallelograms? Did you use deductive or inductive reasoning?

5. **SPORTS** Noah, Brianna, Mackenzie, Antoine, and Bianca were the first five finishers of a race. From the given clues, give the order in which they finished.
   - Noah passed Mackenzie just before the finish line.
   - Bianca finished 5 seconds ahead of Noah.
   - Brianna crossed the finish line after Mackenzie.
   - Antoine was fifth at the finish line.

Solve. Use any strategy.

6. **GEOMETRY** If the sides of the pentagons shown are 1 unit long, find the perimeter of 8 pentagons arranged according to the pattern below.

7. **MONEY** After a trip to the mall, Alex and Marcus counted their money to see how much they had left. Alex said, “If I had $4 more, I would have as much as you.” Marcus replied, “If I had $4 more, I would have twice as much as you.” How much does each boy have?

8. **WEATHER** Based on the data shown, what is a reasonable estimate for the difference in the July high and low temperatures in Statesboro?

9. **MEASUREMENT** You have a large container of pineapple juice, an empty 4-pint container, and an empty 5-pint container. Explain how you can use these containers to measure 2 pints of juice for a punch recipe.

10. **LAUNDRY** You need two clothespins to hang one towel on a clothesline. One clothespin can be used on a corner of one towel and a corner of the towel next to it. What is the least number of clothespins you need to hang 8 towels?

11. **STANDARDIZED TEST PRACTICE** Vanessa and Ashley varied the length of a pendulum and measured the time it took for the pendulum to complete one swing back and forth. Based on their data, how long do you think a pendulum with a swing of 5 seconds is?

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (ft)</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

   - A 21 ft
   - B 23 ft
   - C 24 ft
   - D 25 ft
Angles of Polygons

In this lab, you will use the fact that the sum of the angle measures of a triangle is 180° to find the sum of the angle measures of any polygon.

INVESTIGATE  Work with a partner.

Copy and complete the table below.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Sketch of Figure</th>
<th>Number of Triangles</th>
<th>Sum of Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td>1</td>
<td>1(180°) = 180°</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>2</td>
<td>2(180°) = 360°</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. **Predict** the number of triangles in an octagon and the sum of its angle measures. Check your prediction by drawing a figure.

2. **Write** an algebraic expression that tells the number of triangles in an \( n \)-sided polygon. Then write an expression for the sum of the angle measures in an \( n \)-sided polygon.

**REGULAR POLYGONS** A regular polygon is one that is equilateral (all sides congruent) and equiangular (all angles congruent). Polygons that are not regular are said to be irregular.

3. Use your results from Exercise 2 to find the measure of each angle in the four regular polygons shown above. Check your results by using a protractor to measure one angle of each polygon.

4. **Write** an algebraic expression that tells the measure of each angle in an \( n \)-sided regular polygon. Use it to predict the measure of each angle in a regular octagon.
Congruent Polygons

Identify congruent polygons.

QUILTING
A template, or pattern, for a quilt block contains the minimum number of shapes needed to create the pattern.

1. How many different kinds of triangles are shown in the Winter Stars quilt at the right? Explain your reasoning and draw each triangle.
2. Copy the quilt and label all matching triangles with the same number, starting with 1.

Polygons that have the same size and shape are called **congruent polygons**. Recall that the parts of polygons that “match” are called corresponding parts.

In a congruence statement, the letters identifying each polygon are written so that corresponding vertices appear in the same order. For example, for the diagram below, write \( \triangle CBD \cong \triangle PQR \).

Two polygons are congruent if all pairs of corresponding angles are congruent and all pairs of corresponding sides are congruent.
Identify Congruent Polygons

Determine whether the triangles shown are congruent. If so, name the corresponding parts and write a congruence statement.

**Angles**
The arcs indicate that $\angle X \cong \angle M$, $\angle Y \cong \angle N$, and $\angle Z \cong \angle L$.

**Sides**
The side measures indicate that $XY \cong MN$, $YZ \cong NL$, and $XZ \cong ML$.

Since all pairs of corresponding angles and sides are congruent, the two triangles are congruent. One congruence statement is $\triangle XYZ \cong \triangle MNL$.

Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.

a. 

b. 

You can use corresponding parts to find the measures of an angle or side in a figure that is congruent to a figure with known measures.

**Find Missing Measures**

In the figure, $\triangle AFH \cong \triangle QRN$.

**Find $m\angle Q$.**

According to the congruence statement, $\angle A$ and $\angle Q$ are corresponding angles. So, $\angle A \cong \angle Q$. Since $m\angle A = 40^\circ$, $m\angle Q = 40^\circ$.

**Find $NR$.**

$FH$ corresponds to $NR$. So, $FH \cong NR$. Since $FH = 9$ inches, $NR = 9$ inches.

**Your Turn**

In the figure, quadrilateral $ABCD$ is congruent to quadrilateral $WXYZ$. Find each measure.

c. $m\angle X$
d. $YX$
e. $m\angle Y$
1. **OPEN ENDED** Draw and label a pair of congruent polygons. Be sure to indicate congruent angles and sides on your drawing.

2. **FIND THE ERROR** Justin and Amanda are writing a congruence statement for the triangles at the right. Who is correct? Explain.

   Justin
   \[ \triangle ABC \cong \triangle XYZ \]

   Amanda
   \[ \triangle ABC \cong \triangle YXZ \]

**Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.**

3. 
   \[ \triangle ABC \cong \triangle DEF \]

   \[ \angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F \]

4. 
   \[ \triangle JKL \cong \triangle MNP \]

   \[ JK \cong LM, KL \cong MN, LJ \cong PN \]

**In the figure, \[ \triangle PQR \cong \triangle YWX \]. Find each measure.**

5. \[ m\angle X \]

6. \[ YW \]

7. \[ XY \]

8. \[ m\angle W \]

**Determine whether the polygons shown are congruent. If so, name the corresponding parts and write a congruence statement.**

9. 
   \[ \triangle ABC \cong \triangle DEF \]

10. 
    \[ \triangle CDE \cong \triangle FGH \]

11. 
    \[ \text{BIRDS} \quad \text{The wings of a hummingbird are shaped like triangles. Determine whether these triangles are congruent. If so, name the corresponding parts and write a congruence statement.} \]

12. 
    \[ \triangle VST \cong \triangle WYZ \]

13. 
    \[ \angle V \cong \angle W, \angle S \cong \angle Z, \angle T \cong \angle Y \]
In the figure, \( \triangle JKL \cong \triangle PNM \). Find each measure.

14. \( PN \)  
15. \( PM \)  
16. \( m\angle P \)  
17. \( m\angle N \)  

22. **ALGEBRA** Find the value of \( x \) in the two congruent triangles.

23. **TRAVEL** An overhead sign on an interstate highway is shown at the right. In the scaffolding, \( \triangle ABC \cong \triangle DCB \), \( AC = 2.5 \) meters, \( BC = 1 \) meter, and \( AB = 2.7 \) meters. What is the length of \( BD \)?

24. **CRITICAL THINKING** Tell whether the following statement is sometimes, always or never true. Explain your reasoning.

If the perimeters of two triangles are equal, then the triangles are congruent.

25. **SHORT RESPONSE** Which of the following polygons appear congruent?

   a.  
   b.  
   c.  
   d.  

26. **MULTIPLE CHOICE** If \( \triangle AFG \cong \triangle PQR \), which statement is not true?

   A. \( \angle G \equiv \angle R \)  
   B. \( \overline{AG} \equiv \overline{PQ} \)  
   C. \( \angle P \equiv \angle A \)  
   D. \( \overline{AG} \equiv \overline{PR} \)

   Classify each quadrilateral using the name that best describes it. (Lesson 6-4)

27.  
28.  
29.  

30. The length of each leg of a \( 45^\circ-45^\circ \) right triangle is 14 feet. Find the length of the hypotenuse. (Lesson 6-3)

**BASIC SKILL** Which figure cannot be folded so one half matches the other?

31.  
32.  

---

282 Chapter 6 Geometry
Doug Martin
Construct congruent triangles.

**Materials**
- compass
- straightedge
- protractor
- paper

**ACTIVITY**

Use a straightedge to draw a line. Put a point on it labeled $X$.

Open your compass to the same width as the length of $\overline{AB}$. Put the compass point at $X$. Draw an arc that intersects the line. Label this point of intersection $Y$.

Open your compass to the same width as the length of $\overline{AC}$. Place your compass point at $X$ and draw an arc above the line.

Open your compass to the same width as the length of $\overline{BC}$. Place the compass point at $Y$ and draw an arc above the line so that it intersects the arc drawn in Step 3. Label this point $Z$.

Draw $\overline{YZ}$ and $\overline{XZ}$. $\triangle ABC \cong \triangle XYZ$.

**Writing Math**

1. Explain why the corresponding sides of $\triangle ABC$ and $\triangle XYZ$ are congruent.
2. Draw three different triangles. Then construct a triangle that is congruent to each one.
1. Describe three ways to classify triangles by their sides. (Lesson 6-2)
2. List and define five types of quadrilaterals. (Lesson 6-4)

For Exercises 3–5, use the figure at the right. (Lesson 6-1)
3. Find \( m \angle 6 \) if \( m \angle 7 = 84^\circ \).
4. Find \( m \angle 5 \) if \( m \angle 1 = 35^\circ \).

Find the value of \( x \) in each figure. (Lessons 6-2 and 6-4)

8. **FLAGS** The “Union Jack”, a common name for the flag of the United Kingdom, is shown at the right. The blue portions of the flag are triangular. Determine whether the triangles indicated are congruent. If so, write a congruence statement. (Lesson 6-5)

9. **MULTIPLE CHOICE** How many pairs of congruent triangles are formed by the diagonals of a rectangle? (Lesson 6-5)
   - A 2
   - B 3
   - C 4
   - D 5

10. **GRID IN** Find the value of \( a \) and \( b \). (Lesson 6-1)
**Polygon Bingo**

**Get Ready!**

- **Players:** two
- **Materials:** 10 counters, 1 number cube, marker, 1 large red cube, 1 large blue cube, 2 square sheets of paper

**Get Set!**

- Write quadrilateral, trapezoid, parallelogram, rectangle, rhombus, and square on different faces of the red cube.
- In the same manner, write scalene, isosceles, equilateral, acute, right, and obtuse on different faces of the blue cube.
- Create two boards like the one shown by drawing a different polygon in each square. Use no shape more than once.

**Go!**

- The starting player rolls the number cube. If an even number is rolled, the player rolls the red cube. If an odd number is rolled, the player rolls the blue cube.
- The player covers with a counter any shape that matches the information on the top face of the cube. If a player cannot find a figure matching the information, he or she loses a turn.
- **Who Wins?** The first player to get three counters in a row wins.
Identify line symmetry and rotational symmetry.

**NEW Vocabulary**
- line symmetry
- line of symmetry
- rotational symmetry
- angle of rotation

**Work with a partner.**
Trace the outline of the starfish shown onto both a piece of tracing paper and a transparency.

1. Draw a line down the center of your starfish outline. Then fold your paper across this line. What do you notice about the two halves?
2. Are there other lines you can draw on your outline that will produce the same result? If so, how many?
3. Place the transparency over the outline on your tracing paper. Use your pencil point at the centers of the starfish to hold the transparency in place. How many times can you rotate the transparency from its original position so that the two figures match? Do not count the original position.
4. Find the first angle of rotation by dividing $360^\circ$ by the number of times the figures matched.
5. List the other angles of rotation by adding the first angle of rotation to the previous angle. Stop when you reach $360^\circ$.

A figure has **line symmetry** if it can be folded over a line so that one half of the figure matches the other half. This fold line is called the **line of symmetry**.

Some figures, such as the starfish in the Mini Lab above, have more than one line of symmetry. The figure at the right has one vertical, one horizontal, and two diagonal lines of symmetry.
A figure has **rotational symmetry** if it can be rotated or turned less than 360° about its center so that the figure looks exactly as it does in its original position. The degree measure of the angle through which the figure is rotated is called the **angle of rotation**. Some figures have just one angle of rotation, while others, like the starfish, have several.

**EXAMPLE**

**Identify Rotational Symmetry**

**LOGOS** Determine whether each figure has rotational symmetry. Write yes or no. If yes, name its angle(s) of rotation.

Yes, this figure has rotational symmetry. It will match itself after being rotated 180°.

Yes, this figure has rotational symmetry. It will match itself after being rotated 120° and 240°.
1. **OPEN ENDED** Draw a figure that has rotational symmetry.

2. **Which One Doesn’t Belong?** Identify the capital letter that does not have the type of symmetry as the other three. Explain your reasoning.

   ![Letter Images](A B M S)

**GUIDED PRACTICE**

**SPORTS** For Exercises 3–6, complete parts a and b for each figure.

a. Determine whether the logo has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write **none**.

b. Determine whether the logo has rotational symmetry. Write **yes** or **no**. If yes, name its angle(s) of rotation.

3. ![Horseshoe Logo](image)
4. ![Bull Logo](image)
5. ![B Logo](image)
6. ![Star Logo](image)

**Practice and Applications**

**JAPANESE FAMILY Crests** For Exercises 7–14, complete parts a and b for each figure.

a. Determine whether the figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write **none**.

b. Determine whether the figure has rotational symmetry. Write **yes** or **no**. If yes, name its angle(s) of rotation.

7. ![Red Crest](image)
8. ![Green Crest](image)
9. ![Flower Crest](image)
10. ![Blue Crest](image)
11. ![Brown Crest](image)
12. ![Purple Crest](image)
13. ![Red and Blue Crest](image)
14. ![Red and Green Crest](image)

15. **TRIANGLES** Which types of triangles—scalene, isosceles, equilateral—have line symmetry? Which have rotational symmetry?

16. **ALPHABET** What capital letters of the alphabet produce the same letter after being rotated 180°?
**ROAD SIGNS** For Exercises 17 and 18, use the diagrams below.

a. 

b. 

c. 

d. 

17. Determine whether each sign has line symmetry. If it does, trace the sign and draw all lines of symmetry. If not, write none.

18. Which of the signs above could be rotated and still look the same?

19. **RESEARCH** Use the Internet or other resource to find other examples of road signs that have line and/or rotational symmetry.

20. **ART** Artist Scott Kim uses reflections of words or names as part of his art. Patricia’s reflected name is at the right. Create a reflection design for your name using tracing paper.

**CRITICAL THINKING** Determine whether each statement is true or false. If false, give a counterexample.

21. If a figure has one horizontal and one vertical line of symmetry, then it also has rotational symmetry.

22. If a figure has rotational symmetry, it also has line symmetry.

---

23. **MULTIPLE CHOICE** Which shape has only two lines of symmetry?

   - A
   - B
   - C
   - D

24. **SHORT RESPONSE** Copy the figure at the right. Then shade two squares so that the figure has rotational symmetry.

25. **DESIGN** The former symbol for the National Council of Teachers of Mathematics is shown at the right. Which triangles in the symbol appear to be congruent? (Lesson 6-5)

26. In parallelogram $ABCD$, $m\angle A = 55^\circ$, $m\angle B = 125^\circ$, $m\angle C = x^\circ$, and $m\angle D = 125^\circ$. Find the value of $x$. (Lesson 6-4)

---

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Graph each point on a coordinate plane. (Page 614)

27. $A(3, 2)$
28. $B(-1, 4)$
29. $C(-2, -1)$
30. $D(0, 3)$
**Reflections**

*Virginia SOL* The student will apply transformations (rotate or turn, reflect or flip, translate or slide, and dilate or scale) to geometric figures represented on graph paper. The student will identify applications of transformations, such as tiling, fabric design, art, and scaling.

**Photography** The undisturbed surface of a pond acts like a mirror and can provide the subject for beautiful photographs.

1. Compare the shape and size of the bird to its image in the water.

2. Compare the perpendicular distance from the water line to each of the points shown. What do you observe?

3. The points A, B, and C appear counterclockwise on the bird. How are these points oriented on the bird’s image?

The mirror image produced by flipping a figure over a line is called a *reflection*. This line is called the *line of reflection*. A reflection is one type of *transformation* or mapping of a geometric figure.

**Key Concept**

**Properties of Reflections**

1. Every point on a reflection is the same distance from the line of reflection as the corresponding point on the original figure.

2. The image is congruent to the original figure, but the orientation of the image is different from that of the original figure.

**Example**

**Draw a Reflection**

Copy ΔJKL at the right on graph paper. Then draw the image of the figure after a reflection over the given line.

**Step 1** Count the number of units between each vertex and the line of reflection.

**Step 2** Plot a point for each vertex the same distance away from the line on the other side.

**Step 3** Connect the new vertices to form the image of ΔJKL, ΔJ′K′L′.
Lesson 6-7

Reflections

291

If a figure touches the line of reflection as it does in Example 3, then the figure and its image form a new figure that has line symmetry. The line of reflection is a line of symmetry.

**Reflect a Figure over the x-axis**

Graph \( \triangle PQR \) with vertices \( P(-3, 4) \), \( Q(4, 2) \), and \( R(-1, 1) \). Then graph the image of \( \triangle PQR \) after a reflection over the x-axis, and write the coordinates of its vertices.

The coordinates of the vertices of the image are \( P'(-3, -4) \), \( Q'(4, -2) \), and \( R'(-1, 1) \).

Examine the relationship between the coordinates of each figure.

Notice that the \( y \)-coordinate of a point reflected over the x-axis is the opposite of the \( y \)-coordinate of the original point.

**Points on Line of Reflection**

Notice that if a point lies on the line of reflection, the image of that point has the same coordinates as those of the point on the original figure.

**Reflect a Figure over the y-axis**

Graph quadrilateral \( ABCD \) with vertices \( A(-4, 1) \), \( B(-2, 3) \), \( C(0, -3) \), and \( D(-3, -2) \). Then graph the image of \( ABCD \) after a reflection over the y-axis, and write the coordinates of its vertices.

The coordinates of the vertices of the image are \( A'(4, 1) \), \( B'(2, 3) \), \( C'(0, -3) \), and \( D'(3, -2) \).

Examine the relationship between the coordinates of each figure.

Notice that the \( x \)-coordinate of a point reflected over the y-axis is the opposite of the \( x \)-coordinate of the original point.

**Your Turn**

Graph \( \triangle FGH \) with vertices \( F(1, -1) \), \( G(5, -3) \), and \( H(2, -4) \). Then graph the image of \( \triangle FGH \) after a reflection over the given axis, and write the coordinates of its vertices.

a. \( x \)-axis

b. \( y \)-axis

If a figure touches the line of reflection as it does in Example 3, then the figure and its image form a new figure that has line symmetry. The line of reflection is a line of symmetry.
1. **OPEN ENDED** Draw a triangle on grid paper. Then draw a horizontal line below the triangle. Finally, draw the image of the triangle after it is reflected over the horizontal line.

2. **Explain** how a reflection and line symmetry are related.

3. **Which One Doesn’t Belong?** Identify the transformation that is not the same as the other three. Explain your reasoning.

4. Copy the figure at the right on graph paper. Then draw the image of the figure after a reflection over the given line.

Graph the figure with the given vertices. Then graph its image after a reflection over the given axis, and write the coordinates of its vertices.

5. parallelogram $QRST$ with vertices $Q(-3, 3), R(2, 4), S(3, 2),$ and $T(-2, 1);$ $x$-axis

6. triangle $JKL$ with vertices $J(-2, 3), K(-1, -4),$ and $L(-4, -2);$ $y$-axis
Copy each figure onto graph paper. Then draw the image of the figure after a reflection over the given line.

7. 8. 9.
10. 11. 12.

For Exercises 13–16, determine whether the figure in green is a reflection of the figure in blue over the line \( n \). Write yes or no. Explain.

13. 14. 15. 16.

Graph the figure with the given vertices. Then graph its image after a reflection over the given axis, and write the coordinates of its vertices.

17. triangle \( ABC \) with vertices \( A(-1, -1), B(-2, -4), \) and \( C(-4, -1); x \)-axis
18. triangle \( FGH \) with vertices \( F(3, 3), G(4, -3), \) and \( H(2, 1); y \)-axis
19. square \( JKLM \) with vertices \( J(-2, 0), K(-1, -2), L(-3, -3), \) and \( M(-4, -1); y \)-axis
20. quadrilateral \( PQRS \) with vertices \( P(1, 3), Q(3, 5), R(5, 2), \) and \( S(3, 1); x \)-axis

Name the line of reflection for each pair of figures.

21. 22. 23. 24.

25. **DESIGN** Does the rug below have line symmetry? If so, sketch the rug and draw the line(s) of symmetry.

26. **DESIGN** Copy and complete the rug pattern shown so that the completed figure has line symmetry.
ALPHABET For Exercises 27 and 28, use the figure at the right. It shows that the capital letter A looks the same after a reflection over a vertical line. It does not look the same after a reflection over a horizontal line.

27. What other capital letters look the same after a reflection over a vertical line?

28. Which capital letters look the same after a reflection over a horizontal line?

29. CRITICAL THINKING Suppose a point \( P \) with coordinates \((-4, 5)\) is reflected so that the coordinates of its image are \((-4, -5)\). Without graphing, which axis was this point reflected over? Explain.

SHORT RESPONSE For Exercises 30 and 31, use the drawing at the right.

30. The drawing shows the pattern for the left half of the front of the shirt. Copy the pattern onto grid paper. Then draw the outline of the pattern after it has been flipped over a vertical line. Label it “Right Front.”

31. Use two geometric terms to explain the relationship between the left and right fronts of the shirt.

32. MULTIPLE CHOICE Which of the following is the reflection of \( \triangle ABC \) with vertices \( A(1, -1), B(4, -1), \) and \( C(2, -4) \) over the \( x \)-axis?

33. CARDS Determine whether each card has rotational symmetry. Write yes or no. If yes, name its angle(s) of rotation. (Lesson 6-6)

37. Find the value of \( x \) if the triangles at the right are congruent. (Lesson 6-5)

PREREQUISITE SKILL Add. (Lesson 1-4)

38. \(-4 + (-1)\) 39. \(-5 + 3\) 40. \(-1 + 4\) 41. \(2 + (-2)\)
A definition map can help you visualize the parts of a good definition. Ask yourself these questions about the vocabulary terms.

- What is it? (Category)
- What can it be compared to? (Comparisons)
- What is it like? (Properties)
- What are some examples? (Illustrations)

Here’s a definition map for reflection.

**Comparisons**
- What can it be compared to?
  - dilation
  - translation
  - rotation

**Category**
- What is it?
  - transformation or mapping of figure
  - reflection

**Properties**
- What is it like?
  - a flip
  - mirror image
  - can produce figure with line symmetry

**Illustrations**
- What are some examples?
  - image of self in mirror
  - left and right hands

**SKILL PRACTICE**
Make a definition map for each term.

1. complementary angles (Page 256)  
2. perpendicular lines (Page 257)
3. isosceles triangle (Page 263)  
4. square (Page 273)
In chess, there are rules governing how many spaces and in what direction each game piece can be moved during a player’s turn. The diagram at the right shows one legal move of a knight.

1. Describe the motion involved in moving the knight.
2. Compare the shape, size, and orientation of the knight in its original position to that of the knight in its new position.

A translation (sometimes called a slide) is the movement of a figure from one position to another without turning it.

**Key Concept**

1. Every point on the original figure is moved the same distance and in the same direction.
2. The image is congruent to the original figure, and the orientation of the image is the same as that of the original figure.

**Properties of Translations**

**Model**

![Diagram of a translation]

**Example**

**Draw a Translation**

Copy trapezoid WXYZ at the right on graph paper. Then draw the image of the figure after a translation 4 units left and 2 units down.

**Step 1** Move each vertex of the trapezoid 4 units left and 2 units down.

**Step 2** Connect the new vertices to form the image.
**Translations** In the coordinate plane, a translation can be described using an ordered pair. A translation up or to the right is positive. A translation down or to the left is negative. 

(2, −5) means a translation 2 units right and 5 units down.

**Example**

Graph ΔJKL with vertices J(−3, 4), K(1, 3), and L(−4, 1). Then graph the image of ΔJKL after a translation 2 units right and 5 units down. Write the coordinates of its vertices.

The coordinates of the vertices of the image are J′(−1, −1), K′(3, −2), and L′(−2, −4). Notice that these vertices can also be found by adding 2 to the x-coordinates and −5 to the y-coordinates, or (2, −5).

<table>
<thead>
<tr>
<th>Original</th>
<th>Add (2, −5).</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>J(−3, 4)</td>
<td>(−3 + 2, 4 + (−5))</td>
<td>J′(−1, −1)</td>
</tr>
<tr>
<td>K(1, 3)</td>
<td>(1 + 2, 3 + (−5))</td>
<td>K′(3, −2)</td>
</tr>
<tr>
<td>L(−4, 1)</td>
<td>(−4 + 2, 1 + (−5))</td>
<td>L′(−2, −4)</td>
</tr>
</tbody>
</table>

**Your Turn**

Graph ΔABC with vertices A(4, −3), B(0, 2), and C(5, 1). Then graph its image after each translation, and write the coordinates of its vertices.

- a. 2 units down
- b. 4 units left and 3 units up

**Use a Translation**

**MULTIPLE-CHOICE TEST ITEM** Point N is moved to a new location, N′. Which white shape shows where the shaded figure would be if it was translated in the same way?

- A  A
- B  B
- C  C
- D  D

**Read the Test Item**

You are asked to determine which figure has been moved according to the same translation as Point N.

**Solve the Test Item**

Point N is translated 4 units left and 1 unit up. Identify the figure that is a translation of the shaded figure 4 units left and 1 unit up.

- Figure A: 2 units left and 2 units up
- Figure B: represents a turn, not a translation
- Figure C: 4 units left and 1 unit up

The answer is C.
1. **Which One Doesn’t Belong?** Identify the transformation that is not the same as the other three. Explain your reasoning.

![Images of four different transformations]

2. **OPEN ENDED** Draw a rectangle on grid paper. Then draw the image of the rectangle after a translation 2 units right and 3 units down.

3. Copy the figure at the right on graph paper. Then draw the image of the figure after a translation 4 units left and 1 unit up.

Graph the figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

4. triangle XYZ with vertices X(−4, −4), Y(−3, −1), and Z(2, −2) translated 3 units right and 4 units up

5. trapezoid EFGH with vertices E(0, 3), F(3, 3), G(4, 1), and H(−2, 1) translated 2 units left and 3 units down

**Practice and Applications**

Copy each figure onto graph paper. Then draw the image of the figure after the indicated translation.

6. 5 units right and 3 units up

7. 3 units right and 4 units down

Graph the figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

8. \(\triangle ABC\) with vertices A(1, 2), B(3, 1), and C(3, 4) translated 2 units left and 1 unit up

9. \(\triangle RST\) with vertices R(−5, −2), S(−2, 3), and T(2, −3) translated 1 unit left and 3 units down

10. rectangle JKLM with vertices J(−3, 2), K(3, 5), L(4, 3), and M(−2, 0) translated by 1 unit right and 4 units down

11. parallelogram ABCD with vertices A(6, 3), B(4, 0), C(6, −2), and D(8, 1) translated 3 units left and 2 units up
12. **ART** Explain why Andy Warhol’s 1962 *Self Portrait*, shown at the right, is an example of an artist’s use of translations.

**MUSIC** For Exercises 13 and 14, use the following information. The sound wave of a tuning fork is given below.

13. Look for a pattern in the sound wave. Then copy the sound wave and indicate where this pattern repeats or is translated.

14. How many translations of the original pattern are shown?

15. **CRITICAL THINKING** Triangle RST has vertices R(4, 2), S(−8, 0), and T(6, 7). When translated, R’ has coordinates (−2, 4). Find the coordinates of S’ and T’.

16. **MULTIPLE CHOICE** Which of the following is a vertex of the figure shown at the right after a translation 4 units down?
   - A. (1, −5)  
   - B. (−6, −2)  
   - C. (1, −3)  
   - D. (−2, −2)

17. **SHORT RESPONSE** What are the coordinates of W(−6, 3) after it is translated 2 units right and 1 unit down?

18. Graph polygon ABCDE with vertices A(−5, −3), B(−2, 1), C(−3, 4), D(0, 2), and E(0, −3). Then graph the image of the figure after a reflection over the y-axis, and write the coordinates of its vertices. (Lesson 6-7)

**LIFE SCIENCE** For Exercises 19 and 20, use the diagram of the diatom at the right. (Lesson 6-6)

19. Does the diatom have line symmetry? If so, trace the figure and draw any lines of symmetry. If not, write none.

20. Does the diatom have rotational symmetry? Write yes or no. If yes, name its angle(s) of rotation.

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Determine whether each figure has rotational symmetry. Write yes or no. If yes, name its angles of rotation. (Lesson 6-6)

21.  
22.  
23.  
24.  

msmath3.net/self_check_quiz/sol
Rotations

A **rotation** is a transformation involving the turning or spinning of a figure around a fixed point called the **center of rotation**.

**Hands-On Mini Lab**

**Step 1** Draw a polygon, placing a dot at one vertex. Place a second dot, the center of rotation, in a nearby corner.

**Step 2** Form an angle of rotation by connecting the first dot, the center of rotation, and a point on the edge of the paper.

**Step 3** Place a second paper over the first and trace the figure, the dots, and the ray passing through the figure.

**Step 4** With your pencil on the center of rotation, turn the top paper until its ray lines up with the ray passing through the edge of the first paper. Tape the papers together.

1. Measure the distances from points on the original figure and corresponding points on the image to the center of rotation. What do you observe?

2. Measure the angles formed by connecting the center of rotation to pairs of corresponding points. What do you observe?

The Mini Lab suggests the following properties of rotations.

**Key Concept**

**Properties of Rotations**

1. Corresponding points are the same distance from \( R \). The angles formed by connecting \( R \) to corresponding points are congruent.

2. The image is congruent to the original figure, and their orientations are **the same**.

**Model**

\[
\text{Model: } Z'X'Y' = ZXY, \quad m\angle XRX' = m\angle YRY'
\]
Rotations in the Coordinate Plane

Graph \( \triangle XYZ \) with vertices \( X(2, 2) \), \( Y(4, 3) \), and \( Z(3, 0) \). Then graph the image of \( \triangle XYZ \) after a rotation 90° counterclockwise about the origin, and write the coordinates of its vertices.

**Step 1** Lightly draw a line connecting point \( X \) to the origin.

**Step 2** Lightly draw \( \overline{OX} \) so that \( m \angle X'OX = 90° \) and \( OX' = OX \).

**Step 3** Repeat steps 1–2 for points \( Y \) and \( Z \). Then erase all lightly drawn lines and connect the vertices to form \( \triangle X'Y'Z' \).

Triangle \( X'Y'Z' \) has vertices \( X'(−2, 2), Y'(−3, 4) \), and \( Z'(0, 3) \).

**Your Turn** Graph \( \triangle ABC \) with vertices \( A(1, −2) \), \( B(4, 1) \), and \( C(3, −4) \). Then graph the image of \( \triangle ABC \) after the indicated rotation about the origin, and write the coordinates of its vertices.

a. 90° counterclockwise  

b. 180° counterclockwise

If a figure touches its center of rotation, then one or more rotations of the figure can be used to create a new figure that has rotational symmetry.

**Use a Rotation**

**FOLK ART** Copy and complete the barn sign shown so that the completed figure has rotational symmetry with 90°, 180°, and 270° as its angles of rotation.

Use the procedure described above and the points indicated to rotate the figure 90°, 180°, and 270° counterclockwise. Use a 90° rotation clockwise to produce the same rotation as a 270° rotation counterclockwise.
1. **OPEN ENDED** Give three examples of rotating objects you see every day.

2. **FIND THE ERROR** Anita and Manuel are graphing \( \triangle MNP \) with vertices \( M(-3, 2), N(-1, -1), \) and \( P(-4, -2) \) and its image after a rotation 90° counterclockwise about the origin. Who is correct? Explain.

   ![Graphs of Anita and Manuel's solutions](image)

   - **Anita**
   - **Manuel**

**Graph the figure with the given vertices. Then graph the image of the figure after the indicated rotation about the origin, and write the coordinates of its vertices.**

3. triangle \( ABC \) with vertices \( A(-2, -4), B(2, -1), \) and \( C(4, -3); \) 90° counterclockwise

4. quadrilateral \( DFGH \) with vertices \( D(-3, 2), F(-1, 0), G(-3, -4), \) and \( H(-4, -2); \) 180°

**Practice and Applications**

Graph the figure with the given vertices. Then graph the image of the figure after the indicated rotation about the origin, and write the coordinates of its vertices.

5. triangle \( VWX \) with vertices \( V(-4, 2), W(-2, 4), \) and \( X(2, 1); \) 180°

6. triangle \( BCD \) with vertices \( B(-5, 3), C(-2, 5), \) and \( D(-3, 2); \) 90° counterclockwise

7. trapezoid \( LMNP \) with vertices \( L(0, 3), M(4, 3), N(1, -3), \) and \( P(-1, 1); \) 90° counterclockwise

8. quadrilateral \( FGHJ \) with vertices \( F(-5, 4), G(-3, 4), H(0, -1), \) and \( J(-5, 2); \) 180°

Determine whether the figure in green is a rotation of the figure in blue about the origin. Write **yes** or **no**. Explain.

9. ![Yes](image)

10. ![Yes](image)

11. ![Yes](image)

12. ![No](image)
13. **FABRIC DESIGN** Copy and complete the handkerchief design at the right so that it has rotational symmetry. Rotate the figure 90°, 180°, and 270° counterclockwise about point C.

14. **CRITICAL THINKING** What are the new coordinates of a point at \((x, y)\) after the point is rotated 90° counterclockwise? 180°?

15. **SHORT RESPONSE** Draw a rectangle. Then draw the image of the rectangle after it has been translated 1.5 inches to the right and then rotated 90° counterclockwise about the bottom left vertex. Label this rectangle I.

16. **MULTIPLE CHOICE** Which illustration shows the figure at the right rotated 180°?

Identify each transformation as a **reflection**, a **translation**, or a **rotation**. (Lessons 6-7, 6-8, and 6-9)

17. 18. 19. 20.

For Exercises 21–25, use the graphic at the right.

21. Describe a translation used in this graphic. (Lesson 6-8)

22. Trace at least two examples of figures or parts of figures in the graphic that appear to have line symmetry. Then draw all lines of symmetry. (Lesson 6-6)

23. Trace a figure or part of a figure used in the graphic that appears to have rotational symmetry. (Lesson 6-6)

24. Trace and then classify the quadrilateral that makes up the top portion of the collar on the shirt. (Lesson 6-4)

25. Trace the two triangles that make up the collar on the shirt. Classify each triangle by its angles and by its sides and then determine whether the two triangles are congruent. (Lessons 6-2 and 6-5)

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**USA TODAY Snapshots®**

Crafts are tops on this gift list

If they were a father, what kids say they would like to receive for Father's Day:

- **51%** Offer to help around the house
- **19%** A night out on the town
- **15%** A night of peace and quiet
- **7%** Something your child made

Source: WGBH in conjunction with Applied Research & Consulting LLC for ZOOM

By Cindy Hall and Suzy Parker, USA TODAY
**Tessellations**

Maurits Cornelis Escher (1898–1972) was a Dutch artist whose work used tessellations. A **tessellation** is a tiling made up of copies of the same shape or shapes that fit together without gaps and without overlapping. The sum of the angle measures where vertices meet in a tessellation must equal 360°. For this reason, equilateral triangles and squares will tessellate a plane.

\[
\begin{align*}
6 \times 60^\circ &= 360^\circ \\
4 \times 90^\circ &= 360^\circ
\end{align*}
\]

**ACTIVITY**

Create a tessellation using a translation.

**STEP 1**

Draw a square on the back of an index card. Then draw a triangle inside the top of the square as shown below.

**STEP 2**

Cut out the square. Then cut out the triangle and translate it from the top to the bottom of the square.

**STEP 3**

Tape the triangle and square together to form a pattern.

**STEP 4**

Trace this pattern onto a sheet of paper as shown to create a tessellation.

\[\text{Your Turn}\]

Make an Escher-like drawing using each pattern.

a.  

b.  

c. 
Lesson 6-9b
Hands-On Lab: Tessellations

1. Design and draw a pattern for an Escher-like drawing.
2. Describe how to use your pattern to create a pattern unit for your tessellation. Then create a tessellation using your pattern.
3. Name another regular polygon other than an equilateral triangle or square that will tessellate a plane. Explain your reasoning.

Determine whether each of the following figures will tessellate a plane. Explain your reasoning.

4. \[
\begin{array}{c}
\text{70°} \\
\text{110°} \\
\text{70°} \\
\end{array}
\]
5. \[
\begin{array}{c}
\text{65°} \\
\text{65°} \\
\text{115°} \\
\text{115°} \\
\text{65°} \\
\end{array}
\]
6. \[
\begin{array}{c}
\text{60°} \\
\text{30°} \\
\end{array}
\]

**Writing Math**

1. Design and draw a pattern for an Escher-like drawing.
2. Describe how to use your pattern to create a pattern unit for your tessellation. Then create a tessellation using your pattern.
3. Name another regular polygon other than an equilateral triangle or square that will tessellate a plane. Explain your reasoning.

Create a tessellation using a rotation.

**Step 1**
Draw an equilateral triangle on the back of an index card. Then draw a right triangle inside the left side of the triangle as shown below.

**Step 2**
Cut out the equilateral triangle. Then cut out the right triangle and rotate it so that the right triangle is on the right side as indicated.

**Step 3**
Tape the right triangle and equilateral triangle together to form a pattern unit.

**Step 4**
Trace this pattern onto a sheet of paper as shown to create a tessellation.

**Your Turn**
Make an Escher-like drawing using each pattern.

d.

![Equilateral Triangle and Right Triangle]

e.

![Equilateral Triangle and Right Triangle]

f.

![Equilateral Triangle and Right Triangle]
**Vocabulary and Concept Check**

State whether each sentence is **true** or **false**. If **false**, replace the underlined word to make a true sentence.

1. A(n) **acute** angle has a measure greater than 90° and less than 180°.
2. The sum of the measures of **supplementary** angles is 180°.
3. Parallel lines intersect at a right angle.
4. In a(n) **scalene** triangle, all three sides are congruent.
5. A(n) **rhombus** is a parallelogram with four congruent sides.
6. An isosceles trapezoid has **rotational** symmetry.
7. The orientations of a figure and its reflected image are **different**.

**Lesson-by-Lesson Exercises and Examples**

**6-1 Line and Angle Relationships** (pp. 256–260)

Find the value of \( x \) in each figure.

8. \[ 125° + x = 180° \]

9. \[ 43° + x = 180° \]

For Exercises 10 and 11, use the figure at the right.

10. Find \( m \angle 8 \) if \( m \angle 4 = 118° \).

11. Find \( m \angle 6 \) if \( m \angle 2 = 135° \).

**Example 1**

If \( m \angle 1 = 105° \), find \( m \angle 3, m \angle 5, \) and \( m \angle 8 \).

Since \( \angle 1 \) and \( \angle 3 \) are vertical angles, \( \angle 1 \equiv \angle 3 \).

So, \( m \angle 3 = 105° \).

Since \( \angle 1 \) and \( \angle 5 \) are corresponding angles, \( \angle 1 \equiv \angle 5 \). Therefore, \( m \angle 5 = 105° \).

Since \( \angle 5 \) and \( \angle 8 \) are supplementary, \( m \angle 8 = 180° - 105° \) or 75°.
19. In quadrilateral $JKLM$, $m\angle J = 123^\circ$, $m\angle K = 90^\circ$, and $m\angle M = 45^\circ$. Find $m\angle L$.

20. Classify the triangle in Exercise 13 by its angles and by its sides.

21. In the figure, $FGHJ \cong YXWZ$. Find each measure.
   - $m\angle X$
   - $WZ$
   - $YX$
   - $m\angle Z$

22. In the figure, $\triangle ABC \cong \triangle RPQ$. Find $PQ$.
   - $PQ$ corresponds to $BC$. Since $BC = 5$ feet, $PQ = 5$ feet.
**Symmetry** (pp. 286–289)

**BOATING** Determine whether each signal flag has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write *none*.

25.  

26.  

27.  

28. Which of the figures above has rotational symmetry? Name the angle(s) of rotation.

**Example 6** Determine whether the logo at the right has rotational symmetry. If it does, name its angles of rotation.

The logo has rotational symmetry. Its angles of rotation are 90°, 180°, and 270°.

---

**Reflections** (pp. 290–294)

Graph parallelogram QRST with vertices Q(2, 5), R(4, 5), S(3, 1), and T(1, 1). Then graph its image after a reflection over the given axis, and write the coordinates of its vertices.

29. x-axis  

30. y-axis

**Example 7** Graph \(\triangle FGH\) with vertices \(F(1, -1), G(3, 1),\) and \(H(2, -3)\) and its image after a reflection over the \(y\)-axis.

---

**Translations** (pp. 296–299)

Graph \(\triangle ABC\) with vertices \(A(2, 2), B(3, 5),\) and \(C(5, 3)\). Then graph its image after the indicated translation, and write the coordinates of its vertices.

31. 6 units down  

32. 2 units left and 4 units down

**Example 8** Graph \(\triangle XYZ\) with vertices \(X(3, 1), Y(1, 0),\) and \(Z(2, 3)\) and its image after a translation 4 units right and 1 unit up.

---

**Rotations** (pp. 300–303)

Graph \(\triangle JKL\) with vertices \(J(-1, 3), K(1, 1),\) and \(L(3, 4)\). Then graph its image after the indicated rotation about the origin, and write the coordinates of its vertices.

33. 90° counterclockwise  

34. 180° counterclockwise

**Example 9** Graph \(\triangle PQR\) with vertices \(P(1, 3), Q(2, 1),\) and \(R(4, 2)\) and its image after a rotation of 90° counterclockwise about the origin.
1. Draw a pair of complementary angles. Label the angles \( \angle 1 \) and \( \angle 2 \).

2. OPEN ENDED Draw an obtuse isosceles triangle.

Skills and Applications

For Exercises 3–5, use the figure at the right.

3. Find \( m\angle 6 \) if \( m\angle 5 = 60^\circ \).
4. Find \( m\angle 8 \) if \( m\angle 1 = 82^\circ \).
5. Name a pair of corresponding angles.

Find each missing measure. Round to the nearest tenth.

6. \[
\begin{align*}
30^\circ & \quad 32 \text{ cm} \\
60^\circ & \quad a
\end{align*}
\]

7. \[
\begin{align*}
c & \quad 45^\circ \\
45^\circ & \quad 6 \text{ in.}
\end{align*}
\]

DESIGN Identify each quadrilateral in the stained glass window using the name that best describes it.

8. A
9. B
10. C

In the figure at the right, \( \triangle MNP \cong \triangle ZYX \). Find each measure.

11. \( ZY \)
12. \( \angle Z \)

MUSIC Determine whether each figure has line symmetry. If it does, trace the figure and draw all lines of symmetry. If not, write none.

13.
14.
15.

16. Which of the figures in Exercises 13–15 has rotational symmetry?

Graph \( \triangle JKL \) with vertices \( J(2, 3), K(-1, 4), \) and \( L(-3, -5) \). Then graph its image, and write the coordinates of its vertices after each transformation.

17. reflection over the \( x \)-axis
18. translation by \((-2, 5)\)
19. rotation \( 180^\circ \)

Standardized Test Practice

20. MULTIPLE CHOICE \( \overline{WY} \) is a diagonal of rectangle \( WXZY \). Which angle is congruent to \( \angle WYZ \)?

\[ \text{A} \quad \angle WXY \quad \text{B} \quad \angle WYX \quad \text{C} \quad \angle ZYW \quad \text{D} \quad \angle XWY \]
1. One week Alexandria ran 500 meters, 600 meters, 800 meters, and 1,100 meters. How many kilometers did she run that week? (Prerequisite Skill, pp. 606–607)

- A 1
- B 2
- C 3
- D 4

2. The graph shows the winning times in seconds of the women’s 4 × 100-meter freestyle relay for several Olympic games.

What is a reasonable prediction for the winning time in 2008? (Lesson 1-1)

- F 212 s
- G 215 s
- H 218 s
- I 221 s

3. Which expression is equivalent to \( xy^2z^{-1} \)? (Lesson 2-2)

- A \( \frac{1}{x \cdot y \cdot y \cdot z} \)
- B \( x + y + y - z \)
- C \( x \cdot y \cdot y - z \)
- D \( \frac{x \cdot y \cdot y}{z} \)

4. Aleta went to the grocery store and paid $19.71 for her purchases. A portion of her receipt is shown below.

About how much did the beef cost per pound? (Lesson 4-1)

- F $2.60
- G $3.34
- H $3.80
- I $4.25

5. Which of the stores represented in the circle graph is preferred by \( \frac{7}{25} \) of the students? (Lesson 5-1)

- A Ultimate Jeans
- B All That!
- C Terrific Trends
- D Formal For You

6. Which of the following could not be the measure of \( \angle M \)? (Lesson 6-4)

- F 35°
- G 50°
- H 45°
- I 116°

7. Which of the following figures is not a rotation of the figure at the right? (Lesson 6-9)

- A
- B
- C
- D
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Ms. Neville has 26 students in her homeroom. All of her students take at least one foreign language. Thirteen students take Spanish, 11 students take French, and 5 students take Japanese. No student takes all three languages. How many students take more than one language? (Lesson 1-1)

9. Write \(\sqrt{8}, \sqrt{6}, 3.2, \text{ and } \frac{1}{3}\) in order from least to greatest. (Lesson 3-3)

10. An area of 2,500 square feet of grass produces enough oxygen for a family of 4. What is the area of grass needed to supply a family of 5 with oxygen? (Lesson 4-4)

11. You buy a sweater on sale for $29.96. You paid 25% less than the original price. What was the original price of the sweater? (Lesson 5-7)

12. If \(a \parallel b\), find the value of \(x\). (Lesson 6-1)

13. Name a quadrilateral with one pair of parallel sides and one pair of sides that are not parallel. (Lesson 6-4)

14. How many lines of symmetry does the figure at the right have? (Lesson 6-6)

15. If \(\triangle JKL \cong \triangle MNP\), name the segment in \(\triangle MNP\) that is congruent to \(Lj\). (Lesson 6-5)

16. If \(\triangle ABC\) is reflected about the \(y\)-axis, what are the coordinates of point \(A'\)? (Lesson 6-7)

17. The graph below shows Arm 1 of the design for a company logo. (Lesson 6-9)

- a. To create Arm 2 of the logo, graph the image of figure \(ABCDE\) after a rotation 90° counterclockwise about the origin. Write the coordinates of the vertices of Arm 2.
- b. To create Arm 3 of the logo, graph the coordinates of figure \(ABCDE\) after a rotation 180° about the origin. Write the coordinates of the vertices of Arm 3.
- c. To create Arm 4 of the logo, graph the coordinates of Arm 2 after a rotation 180° about the origin. Write the coordinates of the vertices of Arm 4.
- d. Does the completed logo have rotational symmetry? If so, name its angle(s) of rotation.