Although they may seem unrelated, proportions, algebra, and geometry are closely related. In this unit, you will use proportions and algebra to solve problems involving geometry and percents.
IT’S A MASTERPIECE!

Grab some canvas, some paint, and some paintbrushes. You’re about to create a masterpiece! On this adventure, you’ll learn about the art of painting the human face. Along the way, you’ll research the methods of a master painter and learn about how artists use the Golden Ratio to achieve balance in their works. Don’t forget to bring your math tool kit and a steady hand. This is an adventure you’ll want to frame!

Log on to mmath3.net/webquest to begin your WebQuest.
What do the planets have to do with math?

The circumference of Earth is about 40,000 kilometers. If you know the circumference of the other planets, you can use proportions to make a scale model of our solar system.

You will solve problems involving scale models in Lesson 4-6.
Take this quiz to see whether you are ready to begin Chapter 4. Refer to the lesson or page number in parentheses if you need more review.

### Vocabulary Review

Complete each sentence.

1. A **letter** used to represent an unknown number. (Lesson 1-2)
2. The coordinate system includes a **vertical number line** called the **x-axis**. (Lesson 3-6)
3. An **ordered pair** names any given point on the coordinate plane with its x-coordinate and y-coordinate. (Lesson 3-6)

### Prerequisite Skills

**Simplify each fraction.** (Page 611)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. (\frac{10}{24})</td>
<td>(\frac{5}{12})</td>
</tr>
<tr>
<td>5. (\frac{88}{104})</td>
<td>(\frac{11}{13})</td>
</tr>
<tr>
<td>6. (\frac{36}{81})</td>
<td>(\frac{4}{9})</td>
</tr>
<tr>
<td>7. (\frac{49}{91})</td>
<td>(\frac{7}{13})</td>
</tr>
</tbody>
</table>

**Evaluate each expression.** (Lesson 1-2)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. (\frac{6 - 2}{5 + 5})</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>9. (\frac{7 - 4}{8 - 4})</td>
<td>(\frac{3}{4})</td>
</tr>
<tr>
<td>10. (\frac{3 - 1}{1 + 9})</td>
<td>(\frac{1}{5})</td>
</tr>
<tr>
<td>11. (\frac{5 + 7}{8 - 6})</td>
<td>(\frac{12}{2})</td>
</tr>
</tbody>
</table>

**Subtract.** (Lesson 1-5)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>12. 16 - 7</td>
<td>9</td>
</tr>
<tr>
<td>13. 5 - 12</td>
<td>-7</td>
</tr>
<tr>
<td>14. -8 - 10</td>
<td>-18</td>
</tr>
<tr>
<td>15. 4 - (-3)</td>
<td>7</td>
</tr>
<tr>
<td>16. -11 - 2</td>
<td>-13</td>
</tr>
<tr>
<td>17. -8 - (-9)</td>
<td>1</td>
</tr>
</tbody>
</table>

**Solve each equation.** (Lesson 1-9)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>18. 5 \cdot 6 = x \cdot 2</td>
<td>(x = 15)</td>
</tr>
<tr>
<td>19. (c \cdot 1.5 = 3 \cdot 7)</td>
<td>(c = 7)</td>
</tr>
<tr>
<td>20. 12 \cdot z = 9 \cdot 4</td>
<td>(z = \frac{3}{4})</td>
</tr>
<tr>
<td>21. 7 \cdot 2 = 8 \cdot g</td>
<td>(g = \frac{7}{4})</td>
</tr>
<tr>
<td>22. 3 \cdot 11 = 4 \cdot y</td>
<td>(y = \frac{33}{4})</td>
</tr>
<tr>
<td>23. (b \cdot 6 = 7 \cdot 9)</td>
<td>(b = \frac{63}{6})</td>
</tr>
</tbody>
</table>
Express ratios as fractions in simplest form and determine unit rates.

**NEW Vocabulary**

- ratio
- rate
- unit rate

**Math Symbols**

\[ \approx \] approximately equal to

**TRAIL MIX** The diagram shows a batch of trail mix that is made using 3 scoops of raisins and 6 scoops of peanuts.

1. Which combination of ingredients below would you use to make a smaller amount of the same recipe? Explain.

   - Combination #1: raisins, peanuts
   - Combination #2: raisins, peanuts

2. In order to make the same recipe of trail mix, how many scoops of peanuts should you use for every scoop of raisins?

A **ratio** is a comparison of two numbers by division. If a batch of trail mix contains 3 scoops of raisins and 6 scoops of peanuts, then the ratio comparing the raisins to the peanuts can be written as follows.

\[
\frac{3}{6} \quad \text{or} \quad 3:6 \quad \text{or} \quad \frac{3}{6}
\]

Since a ratio can be written as a fraction, it can be simplified.

**EXAMPLES**

**Write Ratios in Simplest Form**

1. **Express 8 Siamese cats out of 28 cats** in simplest form.

   \[
   \frac{8}{28} = \frac{2}{7}
   \]

   Divide the numerator and denominator by the greatest common factor, 4.

   The ratio of Siamese cats to cats is \( \frac{2}{7} \) or 2 out of 7.

2. **Express 10 ounces to 1 pound** in simplest form.

   \[
   \frac{10 \text{ ounces}}{1 \text{ pound}} = \frac{5 \text{ ounces}}{8 \text{ ounces}}
   \]

   Convert 1 pound to 16 ounces. Divide the numerator and the denominator by 2.

   The ratio in simplest form is \( \frac{5}{8} \) or 5:8.

**Your Turn**

Express each ratio in simplest form.

a. 16 pepperoni pizzas out of 24 pizzas
b. 12 minutes to 2 hours
A **rate** is a special kind of ratio. It is a comparison of two quantities with different types of units. Here are two examples of rates.

\[
\text{\$5 for 2 pounds}
\]

\[
130 \text{ miles in } 2 \text{ hours}
\]

When a rate is simplified so it has a denominator of 1, it is called a **unit rate**. An example of a unit rate is \$6.50 per hour, which means \$6.50 per 1 hour.

### Find a Unit Rate

**TRAVEL** On a trip from Nashville, Tennessee, to Birmingham, Alabama, Darrell drove 187 miles in 3 hours. What was Darrell’s average speed in miles per hour?

Write the rate that expresses the comparison of miles to hours. Then find the average speed by finding the unit rate.

\[
\frac{187 \text{ miles}}{3 \text{ hours}} \approx \frac{62 \text{ miles}}{1 \text{ hour}} \quad \text{Divide the numerator and denominator by 3 to get a denominator of 1.}
\]

Darrell drove an average speed of about 62 miles per hour.

### Compare Unit Rates

**CIVICS** For the 2000 census, the population of Texas was about 20,900,000, and the population of Virginia was about 7,000,000. There were 30 members of the U.S. House of Representatives from Texas and 11 from Virginia. In which state did a member represent more people?

For each state, write a rate that compares the state’s population to its number of representatives. Then find the unit rates.

- **Texas**
  \[
  \frac{20,900,000 \text{ people}}{30 \text{ representatives}} \approx \frac{700,000 \text{ people}}{1 \text{ representative}}
  \]

- **Virginia**
  \[
  \frac{7,000,000 \text{ people}}{11 \text{ representatives}} \approx \frac{640,000 \text{ people}}{1 \text{ representative}}
  \]

Therefore, in Texas, a member of the U.S. House of Representatives represented more people than in Virginia.
1. OPEN ENDED Write a ratio about the marbles in the jar. Simplify your ratio, if possible. Then explain the meaning of your ratio.

2. Explain how to write a rate as a unit rate.

Express each ratio in simplest form.

3. 12 missed days in 180 school days
4. 12 wins to 18 losses
5. 24 pints:1 quart
6. 8 inches out of 4 feet

Express each rate as a unit rate.

7. $50 for 4 days work
8. 3 feet of snow in 5 hours

9. SHOPPING You can buy 4 Granny Smith apples at Ben’s Mart for $0.95. SaveMost sells the same quality apples 6 for $1.49. Which store has the better buy? Explain your reasoning.

Express each ratio in simplest form.

10. 33 brown eggs to 18 white eggs
11. 56 boys to 64 girls
12. 14 chosen out of 70 who applied
13. 28 out of 100 doctors
14. 400 centimeters to 1 meter
15. 6 feet : 9 yards
16. 2 cups to 1 gallon
17. 153 points in 18 games

Express each rate as a unit rate.

18. $22 for 5 dozen donuts
19. $73.45 in 13 hours
20. 1,473 people entered the park in 3 hours
21. 11,025 tickets sold at 9 theaters
22. 100 meters in 12.2 seconds
23. 21.5 pounds in 12 weeks

SHOPPING For Exercises 24–27, decide which is the better buy. Explain.

24. a 17-ounce box of cereal for $4.89 or a 21-ounce box for $5.69
25. 6 cans of green beans for $1 or 10 cans for $1.95
26. 1 pound 4 ounces of meat for $4.99 or 2 pounds 6 ounces for $9.75
27. a 2-liter bottle of soda for $1.39 or a 12-pack of 12-ounce cans for $3.49

Use ratios to convert the following rates.

28. 60 mi/h = ___ ft/s
29. 180 gal/h = ___ oz/min

30. CARS Gas mileage is the average number of miles you can drive a car per gallon of gasoline. A test of a new car resulted in 2,250 miles being driven using 125 gallons of gas. Find the car’s gas mileage.
SPORTS For Exercises 31 and 32, use the graph at the right.

31. Write a ratio comparing the amount of money Jeff Gordon earned in the Winston Cup Series in 2001 to his number of wins that year.

32. **MULTI STEP** On average, who earned more money per win in their sport in 2001, Jeff Gordon or Tiger Woods? Explain.

33. **ART** At an auction in New York City, a 2.55-square inch portrait of George Washington sold for $1.2 million. About how much did the buyer pay per square inch for the portrait?

34. **WRITE A PROBLEM** Write about a real-life situation that can be represented by the ratio 2:5.

35. **CRITICAL THINKING** Luisa and Rachel have some trading cards. The ratio of their cards is 3:1. If Luisa gives Rachel 2 cards, the ratio will be 2:1. How many cards does Luisa have?

36. **MULTIPLE CHOICE** Which of the following cannot be written as a ratio?
- A: two pages for every one page
- B: three more chips than she has he reads
- C: half as many CDs as he has
- D: twice as many pencils as she has

37. **SHORT RESPONSE** Three people leave at the same time from town A to town B. Sarah averaged 45 miles per hour for the first third of the distance, 55 miles per hour for the second third, and 75 miles per hour for the last third. Darnell averaged 55 miles per hour for the first half of the trip and 70 miles per hour for the second half. Megan drove at a steady speed of 60 miles per hour the entire trip. Who arrived first?

Graph each pair of ordered pairs. Then find the distance between the points. Round to the nearest tenth. **(Lesson 3-6)**

38. (1, 4), (6, −3) 39. (−1, 5), (3, −2) 40. (−5, −2), (−1, 0) 41. (−2, −3), (3, 1)

42. **GYMNASTICS** A gymnast is making a tumbling pass along the diagonal of a square floor exercise mat measuring 40 feet on each side. Find the measure of the diagonal. **(Lesson 3-5)**

---

**PREREQUISITE SKILL** Evaluate each expression. **(Lesson 1-5)**

43. \( \frac{45 - 33}{10 - 8} \) 44. \( \frac{85 - 67}{2001 - 1995} \) 45. \( \frac{29 - 44}{55 - 50} \) 46. \( \frac{18 - 19}{25 - 30} \)
Rate of Change

HOBBIES Alicia likes to collect teddy bears. The graph shows the number of teddy bears in her collection between 1997 and 2002.

1. By how many bears did Alicia’s collection increase between 1997 and 1999? Between 1999 and 2002?

2. Between which years did Alicia’s collection increase the fastest?

A rate of change is a rate that describes how one quantity changes in relation to another. In the example above, the rate of change in Alicia’s teddy bear collection from 1997 to 1999 is shown below:

\[
\frac{(22 - 8) \text{ bears}}{(1999 - 1997) \text{ years}} = \frac{14 \text{ bears}}{2 \text{ years}} = 7 \text{ bears per year}
\]

EXAMPLE Find a Rate of Change

HEIGHTS The table at the right shows Ramón’s height in inches between the ages of 8 and 13. Find the rate of change in his height between ages 8 and 11.

\[
\frac{\text{change in height}}{\text{change in age}} = \frac{(58 - 51) \text{ inches}}{(11 - 8) \text{ years}} = \frac{7 \text{ inches}}{3 \text{ years}} \approx \frac{2.3 \text{ inches}}{1 \text{ year}}
\]

Ramón grew from 51 to 58 inches tall from age 8 to age 11.

Subtract to find the change in heights and ages.

Express this rate as a unit rate.

Ramón grew an average of about 2.3 inches per year.

Your Turn

a. Find the rate of change in his height between ages 11 and 13.

Mental Math

You can also find a unit rate by dividing the numerator by the denominator.
A graph of the data in Example 1 is shown at the right. The data points are connected by segments. On a graph, a rate of change measures how fast a segment goes up when the graph is read from left to right.

A formula for rate of change using data coordinates is given below.

**Key Concept**

<table>
<thead>
<tr>
<th>Words</th>
<th>To find the rate of change, divide the difference in the ( y )-coordinates by the difference in the ( x )-coordinates.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbols</td>
<td>The rate of change between ((x_1, y_1)) and ((x_2, y_2)) is ( \frac{y_2 - y_1}{x_2 - x_1} ).</td>
</tr>
</tbody>
</table>

Rates of change can be positive or negative. This corresponds to an increase or decrease in the \( y \)-value between the two data points.

**EXAMPLE**

**Find a Negative Rate of Change**

**MUSIC** The graph shows cassette sales from 1994 to 2000. Find the rate of change between 1996 and 2000, and describe how this rate is shown on the graph.

Use the formula for the rate of change.

Let \((x_1, y_1) = (1996, 19.3)\) and \((x_2, y_2) = (2000, 4.9)\).

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4.9 - 19.3}{2000 - 1996} = \frac{-14.4}{4} = -3.6
\]

Write the formula for rate of change.

Simplify.

Express this rate as a unit rate.

The rate of change is \(-3.6\) million dollars in sales per year. The rate is negative because between 1996 and 2000, the cassette sales decreased. This is shown on the graph by a line slanting downward from left to right.

**Your Turn**

b. In the graph above, find the rate of change between 1994 and 1996.

c. Describe how this rate is shown on the graph.
When a quantity does not change over a period of time, it is said to have a zero rate of change.

### Zero Rates of Change

**MAIL** The graph shows the cost in cents of mailing a 1-ounce first-class letter. Find a time period in which the cost of a first-class stamp did not change.

Between 1992 and 1994, the cost of a first-class stamp did not change. It remained 29¢. This is shown on the graph by a horizontal line segment.

**MAIL** Find the rate of change from 1992 to 1994.

Let \((x_1, y_1) = (1992, 29)\) and \((x_2, y_2) = (1994, 29)\).

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{29 - 29}{1994 - 1992}
\]

Write the formula for rate of change.

\[
= \frac{0}{2} \text{ or } 0
\]

Simplify.

The rate of change in the cost of a first-class stamp between 1992 and 1994 is 0 cents per year.

**Your Turn**

d. Find another time period in which the cost of a first-class stamp did not change. Explain your reasoning.

The table below summarizes the relationship between rates of change and their graphs.
1. **OPEN ENDED** Describe a situation involving a zero rate of change.

2. **NUMBER SENSE** Does the height of a candle as it burns over time show a positive, negative, or zero rate of change? Explain your reasoning.

**GUIDED PRACTICE** For Exercises 3–6, use the table at the right. It shows the outside air temperature at different times during one day.

3. Find the rate of temperature change in degrees per hour from 6 A.M. to 8 A.M. and from 4 P.M. and 8 P.M.
4. Between which of these two time periods was the rate of change in temperature greater?
5. Make a graph of this data.
6. During which time period(s) was the rate of change in temperature positive? negative? 0° per hour? How can you tell this from your graph?

**TEMPERATURE**

<table>
<thead>
<tr>
<th>Time</th>
<th>Temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 A.M.</td>
<td>33</td>
</tr>
<tr>
<td>8 A.M.</td>
<td>45</td>
</tr>
<tr>
<td>12 P.M.</td>
<td>57</td>
</tr>
<tr>
<td>3 P.M.</td>
<td>57</td>
</tr>
<tr>
<td>4 P.M.</td>
<td>59</td>
</tr>
<tr>
<td>8 P.M.</td>
<td>34</td>
</tr>
</tbody>
</table>

**Practice and Applications**

**ADVERTISING** For Exercises 7–10, use the following information. Tanisha’s job is to neatly fold flyers for the school play. She started folding at 12:55 P.M. The table below shows her progress.

<table>
<thead>
<tr>
<th>Time</th>
<th>12:55</th>
<th>1:00</th>
<th>1:20</th>
<th>1:25</th>
<th>1:30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flyers Folded</td>
<td>0</td>
<td>21</td>
<td>102</td>
<td>102</td>
<td>125</td>
</tr>
</tbody>
</table>

7. Find the rate of change in flyers per minute between 1:00 and 1:20.
8. Find her rate of change between 1:25 and 1:30.
9. During which time period did her folding rate increase the fastest?
10. Find the rate of change from 1:20 to 1:25 and interpret its meaning.

**BIRDS** For Exercises 11–14, use the information below and at the right. The graph shows the approximate number of American Bald Eagle pairs from 1963 to 1998.

11. Find the rate of change in the number of eagle pairs from 1974 to 1984.
12. Find the rate of change in the number of eagle pairs from 1984 to 1994.
13. During which of these two time periods did the eagle population grow faster?
14. Find the rate of change in the population from 1994 to 1998. Then interpret its meaning.

**Bald Eagle Population Growth**

Source: birding.about.com
FAST FOOD  For Exercises 15 and 16, use the graph at the right.
15. During which time period was the rate of change in sales greatest? Explain.
16. Find the rate of change during that period.

CANDY  For Exercises 17 and 18, use the following information.
According to the National Confectioners Association, candy sales during the winter holidays in 1995 totaled $1,342 billion. By 2001, this figure had risen to $1,474 billion.
17. Find the rate of change in candy sales during the winter holidays from 1995 to 2001.
18. If this rate of change were to continue, what would the total candy sales during the winter holidays be in 2005?

Data Update  What were candy sales during the winter holidays last year? Visit msmath3.net/data_update to learn more.

CRITICAL THINKING  The rate of change between point A and point B on the graph is 3 meters per day. Find the value of y.

SHORT RESPONSE  Nine days ago, the area covered by mold on a piece of bread was 3 square inches. Today the mold covers 9 square inches. Find the rate of change in the mold’s area.

MULTIPLE CHOICE  The graph shows the altitude of a falcon over time. Between which two points on the graph was the bird’s rate of change in height negative?
A  A and B  B  B and C  C  C and D  D  D and E

Express each ratio in simplest form.  (Lesson 4-1)
22. 42 red cars to 12 black cars
23. 1,500 pounds to 2 tons

GEOMETRY  A triangle has vertices A(−2, −5), B(−2, 8), and C(1, 4). Find the perimeter of triangle ABC.  (Lesson 3-6)

PREREQUISITE SKILL  Evaluate each expression.  (Lesson 1-2)
25. $\frac{8 - 5}{3 - 1}$
26. $\frac{3 - 7}{4 - (-4)}$
27. $\frac{-5 - (-2)}{-1 - 8}$
28. $\frac{2 - (-4)}{-2 - (-3)}$
**What You’ll Learn**

Find rates of change using a spreadsheet.

**Lesson 4-2b**

**Spreadsheet Investigation**

**A Follow-Up of Lesson 4-2**

**Constant Rates of Change**

You can calculate rates of change using a spreadsheet.

**ACTIVITY**

Andrew earns $18 per hour mowing lawns. Calculate the rate of change in the amount he earns between each consecutive pair of times. Then interpret your results.

Set up a spreadsheet like the one shown below.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
</tr>
<tr>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>72</td>
</tr>
</tbody>
</table>

The rate of change between each consecutive pair of data is the same, or constant—$18 per hour.

**EXERCISES**

1. Graph the data given in the activity above. Then describe the figure formed when the points on the graph are connected.

**PARKING** For Exercises 2–4, use the information in the table. It shows the charges for parking at a football stadium.

2. Use a spreadsheet to find the rate of change in the amount charged between each consecutive pair of times.

3. Interpret your results from Exercise 2.

4. Graph the data. Then describe the figure formed when the points on the graph are connected.
Slope

**EXERCISE** As part of Cameron’s fitness program, he tries to run every day. He knows that after he has warmed up, he can maintain a constant running speed of 8 feet per second. This is shown in the table and in the graph.

1. Pick several pairs of points from those plotted and find the rate of change between them. Write each rate in simplest form.
2. What is true of these rates?

In the graph above, the rate of change between any two points on a line is always the same. This constant rate of change is called the slope of the line. **Slope** is the ratio of the rise, or vertical change, to the run, or horizontal change.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} \quad \text{← vertical change between any two points}
\]

\[
\text{slope} = \frac{\text{rise}}{\text{run}} \quad \text{← horizontal change between the same two points}
\]

**EXAMPLE** Find Slope Using a Graph

1. **Find the slope of the line.**

Choose two points on the line. The vertical change is 2 units while the horizontal change is 3 units.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} \quad \text{Definition of slope}
\]

\[
= \frac{2}{3} \quad \text{rise} = 2, \text{run} = 3
\]

The slope of the line is \(\frac{2}{3}\).

**Your Turn** Find the slope of each line.

a. [Graph of a line with a positive slope]

b. [Graph of a line with a negative slope]

c. [Graph of a line with a horizontal slope]
Since slope is a rate of change, it can be positive (slanting upward), negative (slanting downward), or zero (horizontal).

**EXAMPLE**

**Find Slope Using a Table**

The points given in the table lie on a line. Find the slope of the line. Then graph the line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
</tr>
</tbody>
</table>

The slope is calculated as follows:

$$slope = \frac{\text{rise}}{\text{run}}$$

$$= \frac{-3}{2}$$

or $$\frac{-3}{2}$$

**Your Turn**

The points given in each table lie on a line. Find the slope of the line. Then graph the line.

d. | $x$ | $y$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

e. | $x$ | $y$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-4</td>
</tr>
</tbody>
</table>

Since slope is a rate of change, it can have real-life meaning.

**LIBRARIES**

The library fine increases $2 for every 5 overdue books. Written as a unit rate, $\frac{52}{5}$ is $\frac{56}{1}$.

The fine is $0.40 per overdue book per day.

[Image of a graph showing library fines]
1. **OPEN ENDED** Graph a line whose slope is 2 and another whose slope is 3. Which line is steeper?

2. **FIND THE ERROR** Juan and Martina are finding the slope of the line graphed at the right. Who is correct? Explain.

   - **Juan**
     slope = $\frac{-2}{5}$
   - **Martina**
     slope = $\frac{2}{5}$

---

**Exercise 2**

Find the slope of each line.

3. 4.

5. The points given in the table at the right lie on a line. Find the slope of the line. Then graph the line.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

---

**Practice and Applications**

Find the slope of each line.

6. 7. 8.

9. 10. 11.

The points given in each table lie on a line. Find the slope of the line. Then graph the line.

Find the slope of each line and interpret its meaning as a rate of change.

15. **Ace Pizza Delivery**
   - Graph showing the relationship between the number of pizzas and the cost.

16. **Amount Owed on CD Player**
   - Graph showing the relationship between the number of payments and the balance.

17. **Scuba-Diving Pressure**
   - Graph showing the relationship between depth and pressure.

**SAVINGS** For Exercises 18 and 19, use the following information.

Pedro and Jenna are each saving money to buy the latest video game system. Their savings account balances over 7 weeks are shown in the graph at the right.

18. Find the slope of each person’s line.
19. Who is saving more money each week? Explain.

**CRITICAL THINKING** According to federal guidelines, wheelchair ramps for access to public buildings are allowed a maximum of one inch of rise for every foot of run. Would a ramp with a slope of \( \frac{1}{10} \) comply with this guideline? Explain your reasoning. (Hint: Convert feet to inches.)

21. **GRID IN** Find the slope of the roof shown.

22. **MULTIPLE CHOICE** The first major ski slope at a resort rises 8 feet vertically for every 48-foot run. The second rises 12 feet vertically for every 72-foot run. Which statement is true?
   - A. The first slope is steeper than the second.
   - B. The second slope is steeper than the first.
   - C. Both slopes have the same steepness.
   - D. This cannot be determined from the information given.

23. **POOL MAINTENANCE** After 15 minutes of filling a pool, the water level is at 2 feet. Twenty minutes later the water level is at 5 feet. Find rate of change in the water level between the first 15 minutes and the last 20 minutes in inches per minute. (Lesson 4-2)

24. Express $25 for 10 disks as a unit rate. (Lesson 4-1)

**PREREQUISITE SKILL** Solve each equation. Check your solution. (Lesson 1-9)

25. \( 5 \cdot x = 6 \cdot 10 \)
26. \( 8 \cdot 3 = 4 \cdot y \)
27. \( 2 \cdot d = 3 \cdot 5 \)
28. \( 2.1 \cdot 7 = 3 \cdot a \)

msmath3.net/self_check_quiz/sol

Lesson 4-3 Slope 169
Solving Proportions

**NUTRITION** Part of the nutrition label from a granola bar is shown at the right.

1. Write a ratio that compares the number of Calories from fat to the total number of Calories. Write the ratio as a fraction in simplest form.

2. Suppose you plan to eat two such granola bars. Write a ratio comparing the number of Calories from fat to the total number of Calories.

3. Is the ratio of Calories the same for two granola bars as it is for one granola bar? Why or why not?

In the example above, the ratio \(\frac{20}{110}\) simplifies to \(\frac{2}{11}\). The equation \(\frac{20}{110} = \frac{2}{11}\) indicates that the two ratios are equivalent. This is an example of a **proportion**.

### Key Concept

#### Proportion

**Words**
A proportion is an equation stating that two ratios are equivalent.

**Symbols**

\[
\begin{align*}
\frac{a}{b} &= \frac{c}{d} \quad & (a, b, c, d \neq 0) \\
6 &= 3 \\ 8 &= 4
\end{align*}
\]

In a proportion, the two **cross products** are equal.

\[
\begin{align*}
6 \times 3 &= 24 \\
8 \times 4 &= 24
\end{align*}
\]

**The cross products are equal.**

### Key Concept

#### Property of Proportions

**Words**
The cross products of a proportion are equal.

**Symbols**

If \(\frac{a}{b} = \frac{c}{d}\), then \(ad = bc\).

You can use cross products to determine whether a pair of ratios forms a proportion. If the cross products of two ratios are equal, then the ratios form a proportion. If the cross products are not equal, the ratios do not form a proportion.
Identify a Proportion

Determine whether the ratios $\frac{6}{9}$ and $\frac{8}{12}$ form a proportion.

Find the cross products.

$6 \cdot 8 \rightarrow 9 \cdot 12$  
$48 \rightarrow 108$

Since the cross products are equal, the ratios form a proportion.

Your Turn

Determine whether the ratios form a proportion.

a. $\frac{2}{5}, \frac{4}{10}$  

You can also use cross products to solve proportions in which one of the terms is not known.

Solve a Proportion

Solve $\frac{x}{4} = \frac{9}{10}$.

Write the equation.

$\frac{x}{4} = \frac{9}{10}$

Find the cross products.

$x \cdot 10 = 4 \cdot 9$

Multiply.

$10x = 36$

Divide each side by 10.

$\frac{10x}{10} = \frac{36}{10}$

Simplify.

$x = 3.6$

The solution is 3.6. Check the solution by substituting the value of $x$ into the original proportion and checking the cross products.

Your Turn

Solve each proportion.

d. $\frac{7}{d} = \frac{2}{3}$  
e. $\frac{2}{34} = \frac{5}{y}$  
f. $\frac{7}{3} = \frac{n}{2.1}$

Proportions can be used to make predictions.

Use a Proportion to Solve a Problem

LIFE SCIENCE A microscope slide shows 37 red blood cells out of 60 blood cells. How many red blood cells would be expected in a sample of the same blood that has 925 blood cells?

Write a proportion. Let $r$ represent the number of red blood cells.

$\frac{37}{60} = \frac{r}{925}$

Find the cross products.

$37 \cdot 925 = 60 \cdot r$  

Multiply.

$34,225 = 60r$  

Divide each side by 60.

$\frac{34,225}{60} = \frac{60r}{60}$  

Simplify.

$r = 570.4$

You would expect to find 570 or 571 red blood cells out of 925 blood cells.
1. **OPEN ENDED** List four different ratios that form a proportion with \(\frac{12}{40}\).

2. **NUMBER SENSE** What would be a good estimate of the value of \(n\) in the equation \(\frac{3}{5} = \frac{n}{11}\)? Explain your reasoning.

**GUIDED PRACTICE**

Determine whether each pair of ratios form a proportion.

3. \(\frac{8}{5} : \frac{40}{25}\)
4. \(\frac{3}{5} : \frac{8}{2}\)
5. \(\frac{6}{16} : \frac{9}{24}\)

Solve each proportion.

6. \(\frac{a}{13} = \frac{7}{1}\)
7. \(\frac{41}{x} = \frac{5}{2}\)
8. \(\frac{3}{9} = \frac{n}{36}\)

Write a proportion that could be used to solve for each variable.
Then solve.

9. 18 heart beats in 15 seconds
   \(b\) times in 60 seconds
10. 483 miles on 14 gallons of gas
    600 miles on \(g\) gallons of gas

**Practice and Applications**

Determine whether each pair of ratios form a proportion.

11. \(\frac{8}{7} : \frac{10}{9}\)
12. \(\frac{12}{6} : \frac{14}{7}\)
13. \(\frac{16}{12} : \frac{12}{9}\)
14. \(\frac{3}{5} : \frac{55}{200}\)
15. \(\frac{42}{56} : \frac{3}{4}\)
16. \(\frac{5}{18} : \frac{18}{65}\)
17. \(\frac{0.4}{5} : \frac{0.6}{7.5}\)
18. \(\frac{1.5}{0.5} : \frac{2.1}{7}\)

Solve each proportion.

19. \(\frac{k}{7} = \frac{32}{56}\)
20. \(\frac{44}{p} = \frac{11}{5}\)
21. \(\frac{45}{y} = \frac{3}{8}\)
22. \(\frac{x}{13} = \frac{18}{39}\)
23. \(\frac{6}{25} = \frac{d}{30}\)
24. \(\frac{48}{9} = \frac{72}{n}\)
25. \(\frac{15}{2.1} = \frac{12}{c}\)
26. \(\frac{2.5}{6} = \frac{h}{9}\)
27. \(\frac{3.5}{8} = \frac{a}{3.2}\)
28. \(\frac{2}{w} = \frac{0.4}{0.7}\)
29. \(\frac{2}{3} = \frac{18}{x+5}\)
30. \(\frac{m-4}{10} = \frac{7}{5}\)

Write a proportion that could be used to solve for each variable.
Then solve.

31. 6 Earth-pounds equals 1 moon-pound
76 Earth-pounds equals \(p\) moon-pounds
32. 2 pages typed in 13 minutes
25 pages typed in \(m\) minutes
33. 3 pounds of seed for 2,000 square feet
\(x\) pounds of seed for 3,500 square feet
34. \(n\) cups flour used with \(\frac{3}{4}\) cup sugar
1 \(\frac{1}{2}\) cups flour used with \(\frac{1}{2}\) cup sugar

35. **LIFE SCIENCE** About 4 out of every 5 people are right-handed. If there are 30 students in a class, how many would you expect to be right-handed?
PEOPLE For Exercises 36 and 37, use the following information. Although people vary in size and shape, in general, people do not vary in proportion. The head height to overall height ratio for an adult is given in the diagram at the right.

36. About how tall is an adult with a head height of 9.6 inches?
37. Find the average head height of an adult that is 64 inches tall.

38. RECYCLING The amount of paper recycled is directly proportional to the number of trees that recycling saves. If recycling 2,000 pounds of paper saves 17 trees, how many trees are saved when 5,000 pounds of paper are recycled?

MEASUREMENT For Exercises 39–42, refer to the table. Write and solve a proportion to find each quantity.

39. 12 inches = □ centimeters 40. 20 miles = □ kilometers
41. 2 liters = □ gallons 42. 45 kilograms = □ pounds

CRITICAL THINKING Classify the following pairs of statements as having a proportional or nonproportional relationship. Explain.

43. You jump 63 inches and your friend jumps 42 inches. You jump 1.5 times the distance your friend jumps.
44. You jump 63 inches and your friend jumps 42 inches. You jump 21 more inches than your friend jumps.

45. MULTIPLE CHOICE At Northside Middle School, 30 students were surveyed about their favorite type of music. The results are graphed at the right. If there are 440 students at the middle school, predict how many prefer country music.

46. SHORT RESPONSE Yutaka can run 3.5 miles in 40 minutes. About how many minutes would it take him to run 8 miles at this same rate?

47. The points given in the table lie on a line. Find the slope of the line. Then graph the line. (Lesson 4-3)

48. GARDENING Three years ago, an oak tree in Emily’s back yard was 4 feet 5 inches tall. Today it is 6 feet 3 inches tall. How fast did the tree grow in inches per year? (Lesson 4-2)

BASIC SKILL Name the sides of each figure.

49. triangle ABC 50. rectangle DEFG 51. square LMNP
1. Explain the meaning of a rate of change of $-2^\circ$ per hour. (Lesson 4-2)

2. Describe how to find the slope of a line given two points on the line. (Lesson 4-3)

Express each ratio in simplest form. (Lesson 4-1)

3. 32 out of 100 dentists
4. 12 chosen out of 60
5. 300 points in 20 games
6. Express $420 for 15 tickets as a unit rate. (Lesson 4-1)

**TEMPERATURE** For Exercises 7 and 8, use the table at the right. (Lesson 4-2)

7. Find the rate of the temperature change in degrees per hour from 1 P.M. to 3 P.M. and from 5 P.M. to 6 P.M.
8. Was the rate of change between 12 P.M. and 3 P.M. positive, negative, or zero?

Find the slope of each line. (Lesson 4-3)

9. 
10. 
11. 

Solve each proportion. (Lesson 4-4)

12. \[ \frac{33}{r} = \frac{11}{2} \]
13. \[ \frac{x}{36} = \frac{15}{24} \]
14. \[ \frac{5}{9} = \frac{4.5}{a} \]

**GRID IN** A typical 30-minute TV program in the United States has about 8 minutes of commercials. At that rate, how many commercial minutes are shown during a 2-hour TV movie? (Lesson 4-4)

**MULTIPLE CHOICE** There are 2 cubs for every 3 adults in a certain lion pride. If the pride has 8 cubs, how many adults are there? (Lesson 4-4)
The Game Zone: Identifying Proportions

Criss Cross

GET READY!

Players: two to four
Materials: paper; scissors; 24 index cards

GET SET!

- Each player should copy the game board shown onto a piece of paper.
- Cut each index card in half, making 48 cards.
- Copy the numbers below, one number onto each card.

1 1 1 1 2 2 2 3 3 3 4 4
4 5 5 5 6 6 6 7 7 7 8 8
8 9 9 9 10 10 11 11 12 12 13 13
14 14 15 15 16 16 18 18 20 22 24 25

- Deal 8 cards to each player. Place the rest facedown in a pile.

GO!

- The player to the dealer’s right begins by trying to form a proportion using his or her cards. If a proportion is formed, the player says, “Criss cross!” and displays the cards on his or her game board.
- If the cross products of the proportion are equal, the player forming the proportion is awarded 4 points and those cards are placed in a discard pile. If not that player loses his or her turn.
- If a player cannot form a proportion, he or she draws a card from the first pile. If the player cannot use the card, play continues to the right.
- When there are no more cards in the original pile, shuffle the cards in the discard pile and use them.
- Who Wins? The first player to reach 20 points wins the game.
### Problem-Solving Strategy

#### A Preview of Lesson 4-5

**Virginia SOL Standard 8.3** The student will solve practical problems involving rational numbers, percents, ratios, and proportions. Problems will be of varying complexities and will involve real-life data, such as finding a discount and discount prices and balancing a checkbook.

### What You’ll Learn

Solve problems by using the draw a diagram strategy.

### Draw a Diagram

Cleaning tanks for the city aquarium sure is hard work, and filling them back up seems to take forever. It’s been 3 minutes and this 120-gallon tank is only at the 10-gallon mark!

I wonder how much longer it will take? Let’s **draw a diagram** to help us picture what’s happening.

### Explore

The tank holds 120 gallons of water. After 3 minutes, the tank has 10 gallons of water in it. How many more minutes will it take to fill the tank?

### Plan

Let’s draw a diagram showing the water level after every 3 minutes.

### Solve

The tank will be filled after twelve 3-minute time periods. This is a total of $12 \times 3$ or 36 minutes.

### Examine

The tank is filling at a rate of 10 gallons every 3 minutes, which is about 3 gallons per minute. So a 120-gallon tank will take about $120 \div 3$ or 40 minutes to fill. Our answer of 36 minutes seems reasonable.

### Analyze the Strategy

1. **Tell** how drawing a diagram helps solve this problem.
2. **Describe** another method the students could have used to find the number of 3-minute time periods it would take to fill the tank.
3. **Write** a problem that can be solved by drawing a diagram. Then draw a diagram and solve the problem.
Solve. Use the draw a diagram strategy.

4. **AQUARIUM** Angelina fills another 120-gallon tank at the same time Kyle is filling the first 120-gallon tank. After 3 minutes, her tank has 12 gallons in it. How much longer will it take Kyle to fill his tank than Angelina?

5. **LOGGING** It takes 20 minutes to cut a log into 5 equally-sized pieces. How long will it take to cut a similar log into 3 equally-sized pieces?

Solve. Use any strategy.

6. **STORE DISPLAY** A stock clerk is stacking oranges in the shape of a square-based pyramid, as shown at the right. If the pyramid is to have 5 layers, how many oranges will he need?

**FOOD** For Exercises 7 and 8, use the following information.
Of the 30 students in a life skills class, 19 like to cook main dishes, 15 prefer baking desserts, and 7 like to do both.

7. How many like to cook main dishes, but not bake desserts?
8. How many do not like either baking desserts or making main dishes?

9. **MOVIES** A section of a theater is arranged so that each row has the same number of seats. You are seated in the 5th row from the front and the 3rd row from the back. If your seat is 6th from the left and 2nd from the right, how many seats are in this section of the theater?

10. **MONEY** Mi-Ling has only nickels in her pocket. Julián has only quarters in his and Aisha has only dimes in hers. Hannah approached all three for a donation for the school fund-raiser. What is the least each person could donate so that each one gives the same amount?

11. **TOURISM** An amusement park in Texas features giant statues of comic strip characters. If you multiply one character’s height by 4 and add 1 foot, you will find the height of its statue. If the statue is 65 feet tall, how tall is the character?

**TECHNOLOGY** For Exercises 12 and 13, use the diagram below and the following information.
Seven closed shapes are used to make the digits 0 to 9 on a digital clock. (The number 1 is made using the line segments on the right side of the figure.)

12. In forming these digits, which line segment is used most often?
13. Which line segment is used the least?

14. **SPORTS** The width of a tennis court is ten more than one-third its length. If the court is 78 feet long, what is its perimeter?

15. **STANDARDIZED TEST PRACTICE**
Three-inch square tiles that are 2 inches high are being packaged into boxes like the one at the right. If the tiles must be laid flat, how many will fit in one box?

   - A 140  
   - B 150  
   - C 450  
   - D 900
4–5

**Similar Polygons**

*Virginia SOL Standard 8.17 The student will create and solve problems, using proportions, formulas, and functions.*

**What You'll Learn**
Identify similar polygons and find missing measures of similar polygons.

**NEW Vocabulary**
- polygon
- similar
- corresponding parts
- congruent
- scale factor

**MATH Symbols**
- $\angle$ angle
- $AB$ segment $AB$
- $\sim$ is similar to
- $\equiv$ is congruent to
- $AB$ measure of $AB$

**Handson Mini Lab**

**Work with a partner.**
Follow the steps below to discover how the triangles at the right are related.

1. **Copy both triangles onto tracing paper.**

2. **Measure and record the sides of each triangle.**

3. **Cut out both triangles.**

- Compare the angles of the triangles by matching them up. Identify the angle pairs that have equal measure.

- Express the ratios $\frac{DE}{LK}$, $\frac{EF}{JK}$, and $\frac{DF}{IJ}$ as decimals to the nearest tenth.

- What do you notice about the ratios of the matching sides of matching triangles?

A simple closed figure in a plane formed by three or more line segments is called a **polygon**. Polygons that have the same shape are called **similar** polygons. In the figure below, polygon $ABCD$ is similar to polygon $WXYZ$. This is written as polygon $ABCD \sim$ polygon $WXYZ$.

The parts of similar figures that “match” are called **corresponding parts**.

---

**Corresponding Angles**
- $\angle A \sim \angle W$, $\angle B \sim \angle X$,
- $\angle C \sim \angle Y$, $\angle D \sim \angle Z$

**Corresponding Sides**
- $AB \sim WX$, $BC \sim XY$,
- $CD \sim YZ$, $DA \sim ZW$
The similar triangles in the Mini Lab suggest that the following properties are true for similar polygons.

**Key Concept**

**Similar Polygons**

**Words** If two polygons are similar, then
- their corresponding angles are congruent, or have the same measure, and
- their corresponding sides are proportional.

**Models**

\[ \triangle ABC \sim \triangle XYZ \]

**Symbols**

\[ \angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z \text{ and } \frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ} \]

**EXAMPLE**

**Identify Similar Polygons**

1. Determine whether rectangle \( HJKL \) is similar to rectangle \( MNPQ \). Explain your reasoning.

   First, check to see if corresponding angles are congruent.

   Since the two polygons are rectangles, all of their angles are right angles. Therefore, all corresponding angles are congruent.

   Next, check to see if corresponding sides are proportional.

   \[
   \frac{HJ}{MN} = \frac{7}{10}, \quad \frac{JK}{NP} = \frac{3}{6} \quad \text{or} \quad \frac{1}{2}, \quad \frac{KH}{PM} = \frac{7}{10}, \quad \frac{PQ}{KL} = \frac{3}{6} \quad \text{or} \quad \frac{1}{2}.
   \]

   Since \( \frac{7}{10} \) and \( \frac{1}{2} \) are not equivalent ratios, rectangle \( HJKL \) is not similar to rectangle \( MNPQ \).

2. **Your Turn**

   a. Determine whether these polygons are similar. Explain your reasoning.

   The ratio of the lengths of two corresponding sides of two similar polygons is called the **scale factor**. The squares below are similar.

   - The scale factor from square \( ABCD \) to square \( EFGH \) is \( \frac{3}{2} \) or \( 1.5 \).
   - The scale factor from square \( EFGH \) to square \( ABCD \) is \( \frac{3}{2} \) or \( 1.5 \).

---

**Congruence**

Arrows are used to show congruent angles.

**Common Error**

Do not assume that two polygons are similar just because their corresponding angles are congruent. Their corresponding sides must also be proportional.
Find Missing Measures

Given that polygon $ABCD \sim$ polygon $WXYZ$, write a proportion to find the measure of $XY$. Then solve.

The scale factor from polygon $ABCD$ to polygon $WXYZ$ is $\frac{CD}{YZ}$, which is $\frac{10}{15}$ or $\frac{2}{3}$. Write a proportion with this scale factor. Let $m$ represent the measure of $XY$.

$BC$ corresponds to $XY$. The scale factor is $\frac{2}{3}$.

\[
\frac{BC}{XY} = \frac{2}{3} \quad \text{BC corresponds to XY. The scale factor is } \frac{2}{3}.
\]


\[
\frac{12}{m} = \frac{2}{3} \quad BC = 12 \text{ and } XY = m
\]


\[
12 \cdot 3 = m \cdot 2 \quad \text{Find the cross products.}
\]

\[
\frac{36}{2} = \frac{2m}{2} \quad \text{Multiply. Then divide each side by 2.}
\]

\[
18 = m \quad \text{Simplify.}
\]

Your Turn

Write a proportion to find the measure of each side above. Then solve.

b. $WZ$

c. $AB$

Scale Factor and Perimeter

MULTIPLE-CHOICE TEST ITEM

Triangle $LMN \sim \triangle PQR$. Each side of $\triangle LMN$ is $1\frac{1}{3}$ times longer than the corresponding sides of $\triangle PQR$.

If the perimeter of $\triangle LMN$ is 64 centimeters, what is the perimeter of $\triangle PQR$?

A. $5\frac{1}{3}$ cm  B. 16 cm  C. 48 cm  D. 61 cm

Read the Test Item

Since each side of $\triangle LMN$ is $1\frac{1}{3}$ times longer than the corresponding sides of $\triangle PQR$, the scale factor from $\triangle LMN$ to $\triangle PQR$ is $1\frac{1}{3}$ or $\frac{4}{3}$.

Solve the Test Item

Let $x$ represent the perimeter of $\triangle PQR$. The ratio of the perimeters is equal to the ratio of the sides.

\[
\text{ratio of perimeters } \rightarrow \frac{64}{x} = \frac{4}{3} \quad \leftarrow \text{ratio of sides}
\]

\[
64 \cdot 3 = x \cdot 4 \quad \text{Find the cross products.}
\]

\[
\frac{192}{4} = \frac{4x}{4} \quad \text{Multiply. Then divide each side by 4.}
\]

\[
48 = x \quad \text{Simplify.}
\]

The answer is C.
1. Explain how you can determine whether two polygons are similar.

2. OPEN ENDED Draw and label a pair of similar rectangles. Then draw a third rectangle that is not similar to the other two.

Determine whether each pair of polygons is similar. Explain your reasoning.

3.  

4.  

5. In the figure at the right, \( \triangle FGH \sim \triangle KIJ \). Write a proportion to find each missing measure. Then solve.

Determine whether each pair of polygons is similar. Explain your reasoning.

6.  

7.  

8.  

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.

9.  

10.  

11.  

12.  

13.  

Extra Practice

See pages 625, 651.
14. **YEARBOOK** In order to fit 3 pictures across a page, the yearbook staff must reduce their portrait proofs using a scale factor of 8 to 5. Find the dimensions of the pictures as they will appear in the yearbook.

**MOVIES** For Exercises 15 and 16, use the following information. Film labeled 35-millimeter is film that is 35 millimeters wide.

15. When a frame of 35-millimeter movie film is projected onto a movie screen, the image from the film is 9 meters high and 6.75 meters wide. Find the height of the film.

16. If the image from this same film is projected so that it appears 8 meters high, what is the width of the projected image?

17. **GEOMETRY** Find the ratio of the area of rectangle $A$ to the area of rectangle $B$ for each of the following scale factors of corresponding sides. What can you conclude?
   a. $\frac{1}{2}$  
   b. $\frac{1}{3}$  
   c. $\frac{1}{4}$  
   d. $\frac{1}{5}$

**CRITICAL THINKING** Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

18. Two rectangles are similar.  
19. Two squares are similar.

20. **MULTIPLE CHOICE** Which triangle is similar to $\triangle ABC$?

21. **SHORT RESPONSE** Polygon $JKLM \sim$ polygon $QRST$. If $JK = 2$ inches and $QR = 2\frac{1}{2}$ inches, find the measure of $ST$ if $LM = 3$ inches.

22. **BAKING** A recipe calls for 4 cups of flour for 64 cookies. How much flour is needed for 96 cookies? (Lesson 4-4)

23. Graph each pair of points. Then find the slope of the line that passes through each pair of points. (Lesson 4-3)
   23. $(-3, 9), (1, -5)$  
   24. $(2, 4), (-6, 7)$  
   25. $(3, -8), (-1, -8)$

**PREREQUISITE SKILL** Write a proportion and solve for $x$. (Lesson 4-4)

26. 3 cm is to 5 ft as $x$ cm is to 9 ft  
27. 4 in. is to 5 mi as 5 in. is to $x$ mi
Find the value of the golden ratio.

**Materials**
- grid paper
- scissors
- calculator
- tape measure

Lesson 4-5b
Hands-On Lab: The Golden Rectangle

A Follow-Up of Lesson 4-5

The Golden Rectangle

**INVESTIGATE** Work in groups of three.

**STEP 1**
Cut a rectangle out of grid paper that measures 34 units long by 21 units wide. Using your calculator, find the ratio of the length to the width. Express it as a decimal to the nearest hundredth. Record your data in a table like the one below.

| length | 34 | 21 | ? | ? | ? |
| width  | 21 | 13 | ? | ? | ? |

**STEP 2**
Cut this rectangle into two parts, in which one part is the largest possible square and the other part is a rectangle. Record the rectangle’s length and width. Write the ratio of length to width. Express it as a decimal to the nearest hundredth and record in the table.

**STEP 3**
Repeat the procedure described in Step 2 until the remaining rectangle measures 3 units by 5 units.

**Writing Math**

1. **Describe** the pattern in the ratios you recorded.
2. If the rectangles you cut out are described as golden rectangles, **make a conjecture** as to what the value of the golden ratio is.
3. **Write** a definition of golden rectangle. Use the word ratio in your definition. Then describe the shape of a golden rectangle.
4. **Determine** whether all golden rectangles are similar. Explain your reasoning.
5. **RESEARCH** There are many examples of the golden rectangle in architecture. One is shown at the right. Use the Internet or another resource to find three places where the golden rectangle is used in architecture.

Taj Mahal, India
Solve problems involving scale drawings.

**NEW Vocabulary**
- scale drawing
- scale model
- scale

**FLOOR PLANS** The blueprint for a bedroom is given below.

1. How many units wide is the room?
2. The actual width of the room is 18 feet. Write a ratio comparing the drawing width to the actual width.
3. Simplify the ratio you found and compare it to the scale shown at the bottom of the drawing.

A **scale drawing** or a **scale model** is used to represent an object that is too large or too small to be drawn or built at actual size. Examples are blueprints, maps, models of vehicles, and models of animal anatomy.

The **scale** is determined by the ratio of a given length on a drawing or model to its corresponding actual length. Consider the scales below.

- 1 inch represents an actual distance of 4 feet.
- 1 inch represents an actual distance of 30 units.
- 1:30

Distances on a scale drawing are proportional to distances in real-life.

**EXAMPLE**

**Find a Missing Measurement**

**RECREATION** The distance from the roller coaster to the food court on the map is 3.5 centimeters. Find the actual distance to the food court.

Let \( x \) represent the actual distance to the food court. Write and solve a proportion.

\[
\frac{\text{map distance}}{\text{actual distance}} = \frac{1 \text{ cm}}{10 \text{ m}} = \frac{3.5 \text{ cm}}{x \text{ m}}
\]

\[
1 \cdot x = 10 \cdot 3.5
\]

\[
x = 35
\]

The actual distance to the food court is 35 meters.
To find the scale factor for scale drawings and models, write the ratio given by the scale in simplest form.

**Find the Scale Factor**

Find the scale factor for the map in Example 1.

\[
\frac{1 \text{ cm}}{10 \text{ m}} = \frac{1 \text{ cm}}{1,000 \text{ cm}} \quad \text{Convert 10 meters to centimeters.}
\]

The scale factor is \(\frac{1}{1,000}\) or 1:1,000. This means that each distance on the map is \(\frac{1}{1,000}\) the actual distance.

**Find the Scale**

**MODEL TRAINS** A passenger car of a model train is 6 inches long. If the actual car is 80 feet long, what is the scale of the model?

Write a ratio comparing the length of the model to the actual length of the train. Using \(x\) to represent the actual length of the train, write and solve a proportion to find the scale of the model.

\[
\frac{\text{model length}}{\text{actual length}} = \frac{6 \text{ in}}{80 \text{ ft}} \quad \text{or} \quad x \text{ ft}
\]

Find the cross products.

\[
6x = 80 \cdot 1
\]

Multiply. Then divide each side by 6.

\[
x = 13\frac{1}{3}
\]

So, the scale is 1 inch = 13\(\frac{1}{3}\) feet.

To construct a scale drawing of an object, find an appropriate scale.

**Construct a Scale Model**

**SOCIAL STUDIES** Each column of the Lincoln Memorial is 44 feet tall. Michaela wants the columns of her model to be no more than 12 inches tall. Choose an appropriate scale and use it to determine how tall she should make the model of Lincoln’s 19-foot statue.

Try a scale of 1 inch = 4 feet.

\[
\frac{1 \text{ in}}{4 \text{ ft}} = \frac{x \text{ in}}{44 \text{ ft}} \quad \text{Find the cross products.}
\]

Multiply. Then divide each side by 4.

\[
x = 11
\]

The columns are 11 inches tall.

Use this scale to find the height of the statue.

\[
\frac{1 \text{ in}}{4 \text{ ft}} = \frac{y \text{ in}}{19 \text{ ft}} \quad \text{or} \quad 1 \cdot 19 = 4 \cdot y
\]

\[
y = 4\frac{3}{4}
\]

The statue is 4\(\frac{3}{4}\) inches tall.
1. OPEN ENDED Choose an appropriate scale for a scale drawing of a bedroom 10 feet wide by 12 feet long. Identify the scale factor.

2. FIND THE ERROR On a map, 1 inch represents 4 feet. Jacob and Luna are finding the scale factor of the map. Who is correct? Explain.

<table>
<thead>
<tr>
<th>Jacob</th>
<th>Luna</th>
</tr>
</thead>
<tbody>
<tr>
<td>scale factor: 1:4</td>
<td>scale factor: 1:48</td>
</tr>
</tbody>
</table>

On a map of the United States, the scale is 1 inch = 120 miles. Find the actual distance for each map distance.

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>Map Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>3. South Bend, Indiana</td>
<td>Enid, Oklahoma</td>
<td>6 inches</td>
</tr>
<tr>
<td>4. Atlanta, Georgia</td>
<td>Memphis, Tennessee</td>
<td>( \frac{3}{4} ) inches</td>
</tr>
</tbody>
</table>

MONUMENTS For Exercises 5 and 6, use the following information.
At 555 feet tall, the Washington Monument is the highest all-masonry tower.

5. A scale model of the monument is 9.25 inches high. What is the model’s scale?

6. What is the scale factor?

The scale on a set of architectural drawings for a house is 0.5 inch = 3 feet. Find the actual length of each room.

<table>
<thead>
<tr>
<th>Room</th>
<th>Drawing Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>7. Bed Room 2</td>
<td>2 inches</td>
</tr>
<tr>
<td>8. Living Room</td>
<td>3 inches</td>
</tr>
<tr>
<td>9. Kitchen</td>
<td>1.4 inches</td>
</tr>
<tr>
<td>10. Dining Room</td>
<td>2.1 inches</td>
</tr>
<tr>
<td>11. Master Bedroom</td>
<td>2 ( \frac{1}{4} ) inches</td>
</tr>
<tr>
<td>12. Bath</td>
<td>1 ( \frac{1}{8} ) inches</td>
</tr>
</tbody>
</table>

13. Refer to Exercises 7–12. What is the scale factor of these drawings?

14. MULTI STEP On the drawings for Exercises 7–12, the area of the living room is 15 square inches. What is the actual area of the living room?

15. LIFE SCIENCE In the picture of a paramecium at the right, the length of the single celled organism is 4 centimeters. If the paramecium’s actual size is 0.006 millimeter, what is the scale of the drawing?

16. MOVIES One of the models of the gorilla used in the filming of a 1933 movie was only 18 inches tall. In the movie, the gorilla was seen as 24 feet high. What was the scale used?
17. **SPIDERS** A tarantula’s body length is 5 centimeters. Choose an appropriate scale for a model of the spider that is to be just over 6 meters long. Use it to determine how long the tarantula’s 9-centimeter legs should be.

**SPACE** For Exercises 18 and 19, use the information in the table.

18. You decide to use a basketball to represent Earth in a scale model of Earth and the moon. A basketball’s circumference is about 30 inches. What is the scale of your model?

19. Which of the following should you use to represent the moon in your model? (The number in parentheses is the object’s circumference.) Explain your reasoning.

   a. a soccer ball (28 in.)
   b. a tennis ball (8.25 in.)
   c. a golf ball (5.25 in.)
   d. a marble (4 in.)

20. **CONSTRUCT A SCALE DRAWING** Choose a large rectangular space such as the floor or wall of a room. Find its dimensions and choose an appropriate scale for a scale drawing of the space. Then construct a scale drawing and write a problem that uses your drawing.

21. **NUMBER SENSE** One model of a building is built on a 1:75 scale. Another model of the same building is built on a 1:100 scale. Which model is larger? Explain your reasoning.

22. **CRITICAL THINKING** Describe how you could find the scale of a map that did not have a scale printed on it.

23. **MULTIPLE CHOICE** Using which scale would a scale model of a statue appear \( \frac{1}{12} \) the size of the actual statue?

   A. 4 in. = 8 ft  
   B. 3 in. = 36 ft  
   C. 3 in. = 4 ft  
   D. 4 in. = 4 ft

24. **SHORT RESPONSE** The distance between San Antonio and Houston is \( 6\frac{3}{4} \) inches on a map with a scale of \( \frac{1}{2} \) inch = 15 miles. About how long would it take to drive this distance going 60 miles per hour?

25. Determine whether the polygons at the right are similar. Explain your reasoning. (Lesson 4-5)

**SOL Practice**

26. \( \frac{120}{b} = \frac{24}{60} \)

27. \( \frac{0.6}{5} = \frac{1.5}{n} \)

28. \( \frac{10}{6} = \frac{p}{26} \)

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** In the figure, \( \triangle ABC \sim \triangle DEC \). (Lesson 4-5)

29. Identify the corresponding angles in the figure.

30. Identify the corresponding sides in the figure.
Indirect Measurement

Virginia SOL Standard 8.17 The student will create and solve problems, using proportions, formulas, and functions.

am I ever going to use this?

COMICS The caveman is trying to measure the distance to the Sun.

1. How is the caveman measuring the distance to the Sun?

Distances or lengths that are difficult to measure directly can sometimes be found using the properties of similar polygons and proportions. This kind of measurement is called indirect measurement.

One type of indirect measurement is called shadow reckoning. Two objects and their shadows form two sides of similar triangles from which a proportion can be written.

\[ \frac{\text{stick's shadow}}{\text{tree's shadow}} = \frac{\text{stick's height}}{\text{tree's height}} \]

EXAMPLE

Use Shadow Reckoning

FLAGS One of the tallest flagpoles in the U.S. is in Winsted, Minnesota. At the same time of day that Karen’s shadow was about 0.8 meter, the flagpole’s shadow was about 33.6 meters. If Karen is 1.5 meters tall, how tall is Winsted’s flagpole?

Mental Math
Karen’s height is about 2 times her shadow’s length. So the flagpole’s height is about 2 times its shadow’s length.

Karen’s shadow \( \rightarrow \) 0.8 = \( \frac{1.5}{h} \) ← Karen’s height
flagpole’s shadow \( \rightarrow \) 33.6 = \( \frac{50.4}{x} \) ← flagpole’s height

\[ 0.8h = 33.6 \cdot 1.5 \]
\[ 0.8h = 50.4 \]
\[ 0.8x = 50.4 \]
\[ 0.8 \]
\[ x = 63 \]

Find the cross products.
Multiply.
Divide each side by 0.8
Use a calculator.

The flagpole is 63 meters tall.

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Johnny Hart/Creators Syndicate, Inc.
You can also use similar triangles that do not involve shadows to find missing measurements.

**Use Indirect Measurement**

**SURVEYING** The two triangles shown in the figure are similar. Find the distance \( d \) across Coyote Ravine.

In the figure, \( \triangle STV \sim \triangle XWV \).
So, \( ST \) corresponds to \( XW \), and \( TV \) corresponds to \( WV \).

\[
\frac{ST}{XW} = \frac{TV}{WV}
\]
Write a proportion.

\[
\frac{350}{d} = \frac{400}{180}
\]
Find the cross products.

\[
350 \cdot 180 = d \cdot 400
\]
Multiply. Then divide each side by 400.

\[
\frac{63000}{400} = \frac{400d}{400}
\]

\[
157.5 = d
\]
Use a calculator.

The distance across the ravine is 157.5 meters.

1. **Draw and label** similar triangles to illustrate the following problem. Then write an appropriate proportion.

   A building’s shadow is 14 feet long, and a street sign’s shadow is 5 feet long. If the street sign is 6 feet tall, how tall is the building?

2. **OPEN ENDED** Write a problem that requires shadow reckoning. Explain how to solve the problem.

3. **ARCHITECTURE** How tall is the pyramid?

4. **BRIDGES** How far is it across the river?

5. A building casts a 18.5-foot shadow. How tall is the building if a 10-foot tall sculpture nearby casts a 7-foot shadow? Draw a diagram of the situation. Then write a proportion and solve the problem.
In Exercises 6–9, the triangles are similar. Write a proportion and solve the problem.

6. **REPAIRS** How tall is the telephone pole?

7. **LIGHTHOUSE** How tall is the house?

8. **ZOO** How far are the elephants from the aquarium?

9. **SURVEYING** How far is it across Mallard Pond? \( (\text{Hint: } \triangle ABC \sim \triangle ADE) \)

For Exercises 10–15, draw a diagram of the situation. Then write a proportion and solve the problem.

10. **NATIONAL MONUMENT** Devil’s Tower in Wyoming was the United States’ first national monument. At the same time this natural rock formation casts a 181-foot shadow, a nearby 224-foot tree casts a 32-foot shadow. How tall is the monument?

11. **FAIR** Reaching 212 feet tall, the Texas Star at Fair Park in Dallas, Texas, is the tallest Ferris wheel in the United States. A man standing near this Ferris wheel casts a 3-foot shadow. At the same time, the Ferris wheel’s shadow is 106 feet long. How tall is the man?

12. **TOWER** The Stratosphere Tower in Las Vegas is the tallest free-standing observation tower in the United States. If the tower casts a 22.5-foot shadow, about how tall is a nearby flagpole that casts a 3-foot shadow? Use the information at the right.

13. **LAKES** From the shoreline, the ground slopes down under the water at a constant incline. If the water is 3 feet deep when it is 5 feet from the shore, about how deep will it be when it is 62.5 feet from the shore?

14. **LANDMARKS** The Gateway to the West Arch in St. Louis casts a shadow that is 236 foot 3 inches. At the same time, a 5 foot 4 inch tall tourist casts a 2-foot shadow. How tall is the arch?
15. **SPACE SCIENCE** You cut a square hole \( \frac{1}{4} \) inch wide in a piece of cardboard. With the cardboard 30 inches from your face, the moon fits exactly into the square hole. The moon is about 240,000 miles from Earth. Estimate the moon’s diameter. Draw a diagram of the situation. Then write a proportion and solve the problem.

**CRITICAL THINKING** For Exercises 16–18, use the following information.

Another method of indirect measurement involves the use of a mirror as shown in the diagram at the right. The two triangles in the diagram are similar.

16. Write a statement of similarity between the two triangles.

17. Write a proportion that could be used to solve for the height \( h \) of the light pole.

18. What information would you need to know in order to solve this proportion?

---

19. **MULTIPLE CHOICE** A child \( 4 \frac{1}{2} \) feet tall casts a 6-foot shadow. A nearby statue casts a 12-foot shadow. How tall is the statue?

- A) 8 \( \frac{1}{4} \) ft
- B) 9 ft
- C) 13 \( \frac{1}{2} \) ft
- D) 24 ft

20. **GRID IN** A guy wire attached to the top of a telephone pole goes to the ground 9 feet from its base. When Jorge stands under the guy wire so that his head touches the wire, he is 2 feet 3 inches from where the wire goes into the ground. If Jorge is 5 feet tall, how tall in feet is the telephone pole?

On a city map, the scale is 1 centimeter = 2.5 miles. Find the actual distance for each map distance. (Lesson 4-6)

21. 4 cm  
22. 10 cm  
23. 13 cm  
24. 8.5 cm

25. The triangles at the right are similar. Write a proportion to find the missing measure. Then solve. (Lesson 4-5)

Solve each equation. Check your solution. (Lesson 2-7)

26. \( \frac{2}{3}x + 4 = -6 \)  
27. \( a - 2\frac{3}{5} = -6\frac{7}{10} \)  
28. \( -2.3 = \frac{k}{-8} \)  
29. \( -4\frac{1}{2}x = 6 \)

Express each number in scientific notation. (Lesson 2-9)

30. 0.0000236  
31. 4,300,000  
32. 504,000  
33. 0.0000002

---

**GETTING READY FOR THE NEXT LESSON**

**PREREQUISITE SKILL** Graph each pair of ordered pairs. Then find the distance between the points. (Lesson 3-6)

34. (3, 4), (3, 8)  
35. (−2, −1), (6, −1)  
36. (1, 4), (5, 1)  
37. (−1, −2), (4, 10)
**Trigonometry**

**INVESTIGATE** Work in groups of three.

Trigonometry is the study of the properties of triangles. The word trigonometry means triangle measure. A trigonometric ratio is the ratio of the lengths of two sides of a right triangle.

In this Lab you will discover and apply the most common trigonometric ratios: sine, cosine, and tangent.

In any right triangle, the side opposite an angle is the side that is not part of the angle. In the triangle shown,

- side \(a\) is opposite \(\angle A\),
- side \(b\) is opposite \(\angle B\), and
- side \(c\) is opposite \(\angle C\).

The side that is not opposite an angle and not the hypotenuse is called the adjacent side. In \(\triangle ABC\),

- side \(b\) is adjacent to \(\angle A\), and
- side \(a\) is adjacent to \(\angle B\).

Each person in the group should complete steps 1–6.

**STEP 1** Copy the table shown.

<table>
<thead>
<tr>
<th></th>
<th>30° angle</th>
<th>60° angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm) of opposite leg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm) of adjacent leg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm) of hypotenuse</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cosine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tangent</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**STEP 2** Draw a right triangle \(XYZ\) so that \(m\angle X = 30^\circ\), \(m\angle Y = 60^\circ\), and \(m\angle Z = 90^\circ\).

**STEP 3** Find the length to the nearest millimeter of the leg opposite the angle that measures 30°. Record the length.

**STEP 4** Find the length of the leg adjacent to the 30° angle. Record the length.

**STEP 5** Find the length of the hypotenuse. Record the length.
Use the measurements and a calculator to find each of the following ratios to the nearest hundredth. Notice that each of these ratios has a special name.

\[
\begin{align*}
\text{sine} & = \frac{\text{opposite}}{\text{hypotenuse}} & \text{cosine} & = \frac{\text{adjacent}}{\text{hypotenuse}} & \text{tangent} & = \frac{\text{opposite}}{\text{adjacent}}
\end{align*}
\]

Compare your ratios with the others in your group.

Repeat the procedure for the 60º angle. Record the results.

Work with your group.
1. Make a conjecture about the ratio of the sides of any 30º-60º-90º triangle.
2. Repeat the activity with a triangle whose angles measure 45º, 45º, and 90º.
3. Make a conjecture about the ratio of the sides of any 45º-45º-90º triangle.

Use triangle \(ABC\) to find each of the following ratios to the nearest hundredth.

4. cosine of \(\angle A\)
5. sine of \(\angle A\)
6. tangent of \(\angle A\)

You can use a scientific calculator to find the sine [SIN], cosine [COS], or tangent [TAN] ratio for an angle with a given degree measure. Be sure your calculator is in degree mode. Find each value to the nearest thousandth.

7. \(\sin 46º\)
8. \(\cos 63º\)
9. \(\tan 82º\)

10. **SHADOWS** An angle of elevation is formed by a horizontal line and a line of sight above it. A flagpole casts a shadow 35 meters long when the angle of elevation of the Sun is 50º. How tall is the flagpole? (Hint: Use the tangent ratio.)

11. Describe a triangle whose sine and cosine ratios are equal.
Graph dilations on a coordinate plane.

**NEW Vocabulary**

**dilation**

Everyday Meaning of dilation: the act of enlarging or expanding, as in dilating the pupils of your eyes

---

**Link to READING**

Virginia SOL Standard 8.17 The student will create and solve problems, using proportions, formulas, and functions.

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**Graph dilations on a coordinate plane.**

---

**Dilations**

Everyday Meaning of dilation: the act of enlarging or expanding, as in dilating the pupils of your eyes

---

**Work with a partner.**

Plot \(A(0, 0), B(1, 4),\) and \(C(4, 3)\) on a coordinate plane. Then draw \(\triangle ABC.\)

1. Multiply each coordinate by 2 to find the coordinates of points \(A', B',\) and \(C'.\)
2. On the same coordinate plane, graph points \(A', B',\) and \(C'.\) Then draw \(\triangle A'B'C'.\)
3. Determine whether \(\triangle ABC \sim \triangle A'B'C'.\) Explain your reasoning.

---

In mathematics, the image produced by enlarging or reducing a figure is called a dilation. In the Mini Lab, \(\triangle A'B'C'\) has the same shape as \(\triangle ABC,\) so the two figures are similar. Recall that similar figures are related by a scale factor.

---

**Graph a Dilation**

Graph \(\triangle JKL\) with vertices \(J(3, 8), K(10, 6),\) and \(L(8, 2).\) Then graph its image \(\triangle J'K'L'\) after a dilation with a scale factor of \(\frac{1}{2}.\)

To find the vertices of the dilation, multiply each coordinate in the ordered pairs by \(\frac{1}{2}.\) Then graph both images on the same axes.

\[
\begin{align*}
J(3, 8) &\rightarrow \left(3 \cdot \frac{1}{2}, 8 \cdot \frac{1}{2}\right) \rightarrow J'(\frac{3}{2}, 4) \\
K(10, 6) &\rightarrow \left(10 \cdot \frac{1}{2}, 6 \cdot \frac{1}{2}\right) \rightarrow K'(5, 3) \\
L(8, 2) &\rightarrow \left(8 \cdot \frac{1}{2}, 2 \cdot \frac{1}{2}\right) \rightarrow L'(4, 1)
\end{align*}
\]

**Check** Draw lines through the origin and each of the vertices of the original figure. The vertices of the dilation should lie on those same lines.

**Your Turn** Find the coordinates of \(\triangle JKL\) after a dilation with each scale factor.

a. scale factor: 2
b. scale factor: \(\frac{1}{3}\)
Notice that the dilation of $\triangle ABC$ in the Mini Lab is an *enlargement* of the original figure. The dilation of $\triangle JKL$ in Example 1 is a *reduction* of the original figure.

### Example

**Find and Classify a Scale Factor**

Segment $V'W'$ is a dilation of segment $VW$. Find the scale factor of the dilation, and classify it as an enlargement or as a reduction.

Write a ratio of the $x$- or $y$-coordinate of one vertex of the dilation to the $x$- or $y$-coordinate of the corresponding vertex of the original figure. Use the $y$-coordinates of $V(-2, 2)$ and $V'(-5, 5)$.

$$\frac{y\text{-coordinate of point } V'}{y\text{-coordinate of point } V} = \frac{5}{2}$$

The scale factor is $\frac{5}{2}$. Since the image is larger than the original figure, the dilation is an enlargement.

**Your Turn** Segment $A'B'$ is a dilation of segment $AB$. The endpoints of each segment are given. Find the scale factor of the dilation, and classify it as an *enlargement* or as a *reduction*.

c. $A(4, -8), B(12, -4)$

d. $A(-5, -7), B(-3, 2)$

### Example

**Use a Scale Factor**

*EYES* Carleta’s optometrist uses medicine to dilate her pupils by a factor of $\frac{5}{3}$.

The diagram shows the diameter of Carleta’s pupil before dilation. Find the new diameter once her pupil is dilated.

Write a proportion using the scale factor.

$$\frac{\text{dilated eye}}{\text{normal eye}} = \frac{x}{5} = \frac{\frac{5}{3}}{3}$$

$$x \cdot 3 = 5 \cdot 5$$

Find the cross products.

$$\frac{3x}{3} = \frac{25}{3}$$

Multiply. Then divide each side by 3.

$$x \approx 8.3$$

Simplify.

Her pupil will be about 8.3 millimeters in diameter once dilated.
1. **OPEN ENDED** Draw a triangle on the coordinate plane. Then graph its image after a dilation with a scale factor of 3.

2. **Which One Doesn’t Belong?** Identify the pair of points that does not represent a dilation with a factor of 2. Explain your reasoning.

   - \(P(3, -1), P'(5, 1)\)
   - \(Q(4, 2), Q'(8, 4)\)
   - \(R(-5, 3), R'(-10, 6)\)
   - \(S(1, -7), S'(2, -14)\)

3. Triangle \(ABC\) has vertices \(A(-4, 12), B(-2, -4),\) and \(C(8, 6)\). Find the coordinates of \(\triangle ABC\) after a dilation with a scale factor of \(\frac{1}{4}\). Then graph \(\triangle ABC\) and its dilation.

4. In the figure at the right, the green rectangle is a dilation of the blue rectangle. Find the scale factor and classify the dilation as an enlargement or as a reduction.

5. Segment \(C'D'\) with endpoints \(C'(-3, 12)\) and \(D'(6, -9)\) is a dilation of segment \(CD\). If segment \(CD\) has endpoints \(C(-2, 8)\) and \(D(4, -6)\), find the scale factor of the dilation. Then classify the dilation as an enlargement or as a reduction.

---

**Practice and Applications**

Find the coordinates of the vertices of polygon \(H'J'K'L'\) after polygon \(HJKL\) is dilated using the given scale factor. Then graph polygon \(HJKL\) and its dilation.

6. \(H(-1, 3), J(3, 2), K(2, -3), L(-2, -2);\) scale factor 2
7. \(H(0, 2), J(3, 1), K(0, -4), L(-2, -3);\) scale factor 3
8. \(H(-6, 2), J(4, 4), K(7, -2), L(-2, -4);\) scale factor \(\frac{1}{2}\)
9. \(H(-8, 4), J(6, 4), K(6, -4), L(-8, -4);\) scale factor \(\frac{3}{4}\)

10. Write a general rule for finding the new coordinates of any ordered pair \((x, y)\) after a dilation with a scale factor of \(k\).

   Segment \(P'Q'\) is a dilation of segment \(PQ\). The endpoints of each segment are given. Find the scale factor of the dilation, and classify it as an enlargement or as a reduction.

   11. \(P(0, -10)\) and \(Q(5, -15)\)
   12. \(P(-1, 2)\) and \(Q(3, -3)\)
   13. \(P(-3, -9)\) and \(Q(6, -3)\)
   14. \(P(-5, 6)\) and \(Q(4, 3)\)

   13. \(P'(-4, -12)\) and \(Q'(8, -4)\)
   14. \(P'(-2.5, 3)\) and \(Q'(2, 1.5)\)

For Exercises 15 and 16, graph each figure on dot paper.

15. a square and its image after a dilation with a scale factor of 4
16. a right triangle and its image after a dilation with a scale factor of 0.5.
In each figure, the green figure is a dilation of the blue figure. Find the scale factor of each dilation and classify as an *enlargement* or as a *reduction*.

\[ \text{DESIGN} \] For Exercises 21 and 22, use the following information.
Simone designed a logo for her school. The logo, which is 5 inches wide and 8 inches long, will be enlarged and used on a school sweatshirt. On the sweatshirt, the logo will be 12\( \frac{1}{2} \) inches wide.

21. What is the scale factor for this enlargement?
22. How long will the logo be on the sweatshirt?

\[ \text{ART} \] For Exercises 23 and 24, use the painting at the right and the following information.
Painters use dilations to create the illusion of distance and depth. To create this illusion, the artist establishes a *vanishing point* on the horizon line. Objects are drawn using intersecting lines that lead to the vanishing point.

23. Find the vanishing point in this painting.
24. **RESEARCH** Use the Internet or other reference to find examples of other paintings that use dilations. Identify the vanishing point in each painting.

\[ \text{CRITICAL THINKING} \] Describe the image of a figure after a dilation with a scale factor of \(-2\).

\[ \text{MULTIPLE CHOICE} \] Square A is a dilation of square B. What is the scale factor of the dilation?

\[ A \quad \frac{1}{7} \quad B \quad \frac{3}{5} \quad C \quad \frac{5}{3} \quad D \quad 7 \]

\[ \text{MULTIPLE CHOICE} \] A photo is 8 inches wide by 10 inches long. You want to make a reduced color copy of the photo that is 5 inches wide for your scrapbook. What scale factor should you choose on the copy machine?

\[ A \quad \frac{1}{2} \text{ or } 50\% \quad B \quad \frac{5}{8} \text{ or } 62.5\% \quad C \quad \frac{8}{5} \text{ or } 160\% \quad D \quad 2 \text{ or } 200\% \]

\[ \text{ARCHITECTURE} \] The Empire State Building casts a shadow 156.25 feet long. At the same time, a nearby building that is 84 feet high casts a shadow 10.5 feet long. How tall is the Empire State Building? (Lesson 4-7)

\[ \text{HOBBIES} \] A model sports car is 10 inches long. If the actual car is 14 feet, find the scale of the model. (Lesson 4-6)
Vocabulary and Concept Check

congruent (p. 179)  rate (p. 157)  scale drawing (p. 184)
corresponding parts (p. 178) rate of change (p. 160) scale factor (p. 179)
cross products (p. 170) ratio (p. 156) scale model (p. 184)
dilation (p. 194) rise (p. 166) similar (p. 178)
indirect measurement (p. 188) run (p. 166) slope (p. 166)
polygon (p. 178) scale (p. 184) unit rate (p. 157)
proportion (p. 170)

Choose the letter of the term that best matches each statement or phrase.
1. polygons that have the same shape  a. slope
2. a rate with a denominator of one  b. rate of change
3. the constant rate of change between two points on a line  c. dilation
4. a comparison of two numbers by division  d. proportion
5. two equivalent ratios  e. unit rate
6. ratio of a length on a drawing to its actual length  f. similar
7. describes how one quantity changes in relation  g. ratio
to another  h. scale

Lesson-by-Lesson Exercises and Examples

4-1 Ratios and Rates (pp. 156–159)

Express each ratio in simplest form.
9. 7 chaperones for 56 students
10. 12 peaches: 8 pears
11. 5 inches out of 5 feet

Example 1  Express the ratio 10 milliliters to 8 liters in simplest form.

\[ \frac{10 \text{ milliliters}}{8 \text{ liters}} = \frac{10 \text{ milliliters}}{8,000 \text{ milliliters}} = \frac{1}{800} \]

4-2 Rate of Change (pp. 160–164)

12. MONEY  The table below shows Victor’s weekly allowance for different ages.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ per week</td>
<td>0.25</td>
<td>1.00</td>
<td>2.00</td>
<td>2.00</td>
<td>3.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Find the rate of change in his allowance between ages 12 and 15.

Example 2  At 5 A.M., it was 54°F. At 11 A.M., it was 78°F. Find the rate of temperature change in degrees per hour.

\[ \frac{\text{change in temperature}}{\text{change in hours}} = \frac{(78 - 54)\degree}{(11 - 5) \text{ hours}} = \frac{24\degree}{6 \text{ hours}} = \frac{4\degree}{1 \text{ hour}} \]
**Slope** (pp. 166–169)

Find the slope of each line graphed at the right.

13. \( AB \)
14. \( CD \)

15. The points in the table lie on a line. Find the slope of the line. Then graph the line.

<table>
<thead>
<tr>
<th>( x )</th>
<th>6</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>5</td>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Solving Proportions** (pp. 170–173)

Solve each proportion.

16. \( \frac{3}{r} = \frac{6}{8} \)
17. \( \frac{7}{4} = \frac{n}{2} \)
18. \( \frac{k}{5} = \frac{72}{8} \)
19. \( \frac{8}{3.8} = \frac{6}{x} \)

20. **ANIMALS** A turtle can move 5 inches in 4 minutes. How far will it travel in 10 minutes?

**Similar Polygons** (pp. 178–182)

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.

21.

22.

23. **PARTY PLANNING** For your birthday party, you make a map to your house on a 3-inch wide by 5-inch long index card. How long will your map be if you use a copier to enlarge it so that it is 8 inches wide?

**Example 3** Find the slope of the line.

The vertical change from point \( J \) to point \( K \) is -5 units while the horizontal change is 4 units.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-5}{4} \quad \text{or} \quad \frac{-5}{4} \quad \text{rise} = -5, \text{run} = 4
\]

**Example 4** Solve \( \frac{9}{x} = \frac{4}{18} \).

\[
\frac{9}{x} = \frac{4}{18}
\]

Write the equation.

\[
9 \cdot 18 = x \cdot 4
\]

Find the cross products.

\[
162 = 4x
\]

Multiply.

\[
\frac{162}{4} = \frac{4x}{4}
\]

Divide each side by 10.

\[
40.5 = x
\]

Simplify.

**Example 5** Rectangle \( GHJK \) is similar to rectangle \( PQRS \). Find the value of \( x \).

\[
\text{The scale factor from } GHJK \text{ to } PQSR \text{ is } \frac{GK}{PR} \text{ which is } \frac{3}{9} \text{ or } \frac{1}{3}.
\]

\[
\frac{GH}{PQ} = \frac{1}{3}
\]

Write a proportion.

\[
\frac{4.5}{x} = \frac{1}{3}
\]

\( GH = 4.5 \) and \( PQ = y \)

\[
13.5 = x
\]

Find the cross products. Simplify.
4-6  **Scale Drawings and Models** (pp. 184–187)

The scale on a map is 2 inches = 5 miles. Find the actual distance for each map distance.
24. 12 in. 25. 9 in. 26. 2.5 in.

27. **HOBBIES** Mia’s sister’s dollhouse is a replica of their townhouse. The outside dimensions of the dollhouse are 25 inches by 35 inches. If the actual outside dimensions of the townhouse are 25 feet by 35 feet, what is the scale of the dollhouse?

**Example 6** The scale on a model is 3 centimeters = 45 meters. Find the actual length for a model distance of 5 centimeters.

\[
\frac{\text{model length}}{\text{actual length}} = \frac{3 \text{ cm}}{45 \text{ m}} = \frac{5 \text{ cm}}{x \text{ m}} \quad \leftarrow \text{model length}
\]

\[
3 \cdot x = 45 \cdot 5
\]

\[
x = 75
\]

The actual length is 75 meters.

4-7  **Indirect Measurement** (pp. 188–191)

Write a proportion. Then determine the missing measure.

28. **MAIL** A mailbox casts an 18-inch shadow. A tree casts a 234-inch shadow. If the mailbox is 4 feet tall, how tall is the tree?

29. **WATER** From the shoreline, the ground slopes down under the water at a constant incline. If the water is 5\(\frac{1}{2}\) feet deep when it is 2\(\frac{1}{4}\) feet from the shore, about how deep will it be when it is 6 feet from the shore?

**Example 7** A house casts a shadow that is 5 meters long. A tree casts a shadow that is 2.5 meters long. If the house is 20 meters tall, how tall is the tree?

\[
\frac{\text{house's shadow}}{\text{tree's shadow}} = \frac{5 \text{ m}}{2.5 \text{ m}} = \frac{20}{x} \quad \leftarrow \text{house's height}
\]

\[
5 \cdot x = 20 \cdot 2.5
\]

\[
x = 10
\]

The tree is 10 meters tall.

4-8  **Dilations** (pp. 194–197)

Segment \(C'D'\) is a dilation of segment \(CD\). The endpoints of each segment are given. Find the scale factor of the dilation, and classify it as an enlargement or as a reduction.

30. \(C(-2, 5), D(1, 4); C'(−8, 20), D'(4, 16)\)

31. \(C(-5, 10), D(0, 5); C'(-2, 4), D'(0, 2)\)

**Example 8** Segment \(XY\) has endpoints \(X(-4, 1)\) and \(Y(8, −2)\). Find the coordinates of its image for a dilation with a scale factor of \(\frac{3}{4}\).

\(X(-4, 1) \rightarrow \left(-4 \cdot \frac{3}{4}, 1 \cdot \frac{3}{4}\right) \rightarrow X'(-3, \frac{3}{4})\)

\(Y(8, −2) \rightarrow \left(8 \cdot \frac{3}{4}, −2 \cdot \frac{3}{4}\right) \rightarrow Y'(6,−1\frac{1}{2})\)
1. **OPEN ENDED** List four different ratios that form a proportion with \( \frac{8}{12} \).

2. **Describe** a reasonable scale for a scale drawing of your classroom.

### Skills and Applications

3. Express 15 inches to 1 foot in simplest form.

4. Express $1,105 for 26 jerseys as a unit rate.

**BUSINESS** For Exercises 5 and 6, use the table at the right.

5. Find the rate of change in new customers per hour between 4 P.M. and 5 P.M.

6. Find the rate of change in new customers per hour between 12 P.M. and 2 P.M. Then interpret its meaning.

Find the slope of each line graphed at the right.

7. \( \frac{AB}{CD} \)

8. \( \frac{CD}{AB} \)

Solve each proportion.

9. \( \frac{5}{3} = \frac{20}{y} \)

10. \( \frac{x}{2} = \frac{5}{8} \)

Each pair of polygons is similar. Write a proportion to find each missing measure. Then solve.

11. \( \frac{5}{2} \)

12. \( \frac{10}{x} \)

13. **GEOMETRY** Graph triangle \( FGH \) with vertices \( F(-4, -2), G(-1, 2), \) and \( H(3, 0) \). Then graph its image after a dilation with a scale factor of \( \frac{3}{2} \).

14. On a map, 1 inch = 7.5 miles. How many miles does 2.5 inches represent?

### Standardized Test Practice

15. **GRID IN** If it costs an average of $102 to feed a family of three for one week, on average, how much will it cost in dollars to feed a family of five for one week?

16. **MULTIPLE CHOICE** A 36-foot flagpole casts a 9-foot shadow at the same time a building casts a 15-foot shadow. How tall is the building?

   - A 21.6 ft
   - B 60 ft
   - C 135 ft
   - D 375 ft
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the numbers below is not a prime number?  (Prerequisite Skill, p. 609)
   - A 23  B 49  C 59  D 61

2. One floor of a house is divided into two apartments as shown below.

<table>
<thead>
<tr>
<th>Apartment A</th>
<th>Apartment B</th>
</tr>
</thead>
<tbody>
<tr>
<td>21 ft</td>
<td>15 ft</td>
</tr>
<tr>
<td>14 ft</td>
<td></td>
</tr>
</tbody>
</table>

   How much larger is the area of apartment A than the area of apartment B?  (Lesson 1-1)
   - F 90 ft²  G 100 ft²
   - H 105 ft²  I 115 ft²

3. Which of the following numbers could replace the variable $n$ to make the inequality true?  (Lesson 2-2)
   \[
   \frac{4}{9} < n < 0.72
   \]
   - A $\frac{1}{3}$  B $\frac{6}{8}$  C $\frac{3}{2}$  D $\frac{4}{6}$

4. Which of the following could not be the side lengths of a right triangle?  (Lesson 3-4)
   - F 2, 3, 5  G 6, 10, 8
   - H 8, 15, 17  I 13, 5, 12

5. Last week, Caleb traveled from home to his grandmother’s house. The graph below shows the relationship between his travel time and the distance he traveled.

   ![Distance Traveled Over Time Graph]

   Which best describes his trip?  (Lesson 4-2)
   - A He drove on a high-speed highway, then slowly on a dirt road, and finished his trip on a high-speed highway.
   - B He drove slowly on a dirt road, stopped for lunch, and then got on a high-speed highway for the rest of his trip.
   - C He drove slowly on a dirt road, then on a high-speed highway, and finished his trip on a dirt road.
   - D He started on a high-speed highway, stopped for lunch, and then got on a dirt road for the rest of his trip.

6. You are making a scale model of the car shown below. If your model is to be $\frac{1}{25}$ of car’s actual size, which proportion could be used to find the measure $\ell$ of the model’s length?  (Lesson 4-6)
   - F $\frac{\ell}{14} = \frac{25}{1}$  G $\frac{14}{\ell} = \frac{1}{25}$
   - H $\frac{\ell}{14} = \frac{1}{25}$  I $\frac{14}{25} = \frac{1}{\ell}$

**Question 3** It will save you time to memorize the decimal equivalents or approximations of some common fractions.

   - $\frac{3}{4} = 0.75$  $\frac{1}{3} \approx 0.33$  $\frac{2}{3} = 0.66$  $\frac{3}{2} = 1.5$
7. The temperature at 9:00 A.M. was \(-20^\circ F\). If the temperature rose 15° from 9:00 A.M. to 12:00 noon, what was the temperature at noon? (Lesson 1-4)

8. Suppose you made fruit punch for a party using \(3\frac{1}{2}\) cups of apple juice, 2 cups of orange juice, and \(2\frac{1}{2}\) cups of cranberry juice. How many quarts of juice did you make? (Lesson 2-5)

9. Estimate the value of \(\sqrt{47}\) to the nearest whole number. (Lesson 3-2)

10. A swan laid 5 eggs. Only 4 of the eggs hatched, and only 3 of these swans grew to become adults. Write the ratio of swans that grew to adulthood to the number of eggs that hatched as a fraction. (Lesson 4-1)

11. Find the slope of the line graphed at the right. (Lesson 4-3)

12. A truck used 6.3 gallons of gasoline to travel 107 miles. How many gallons of gasoline would it need to travel an additional 250 miles? (Lesson 4-4)

13. Triangle \(FGH\) is similar to triangle \(JKL\). The perimeter of triangle \(FGH\) is 30 centimeters.

   \[
   \begin{align*}
   F & \quad G \\
   & \quad H \\
   12 \text{ cm} & \quad 9 \text{ cm}
   \end{align*}
   \]

   What is the perimeter of triangle \(JKL\) in centimeters? (Lesson 4-5)

14. The distance between Jasper and Cartersville on a map is 3.8 centimeters. If the actual distance between these two cities is 209 miles, what is the scale for this map? (Lesson 4-6)

15. A 6-foot tall man casts a shadow that is 8 feet long. At the same time, a nearby crane casts a 20-foot long shadow. How tall is the crane? (Lesson 4-7)

16. The table below shows how much Susan earns for different amounts of time she works at a fast food restaurant. (Lesson 4-3)

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wages ($)</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
</tr>
</tbody>
</table>

   a. Graph the data from the table and connect the points with a line.
   b. Find the slope of the line.
   c. What is Susan’s rate of pay?
   d. If Susan continues to be paid at this rate, how much money will she make for working 10 hours?

17. Triangle \(ABC\) has vertices \(A(-6, 3), B(3, 6),\) and \(C(6, -9)\). (Lesson 4-8)

   a. Find the coordinates of the vertices of \(\triangle A'B'C'\) after \(\triangle ABC\) is dilated using a scale factor of \(\frac{2}{3}\).
   b. Graph \(\triangle ABC\) and its dilation.
   c. Name a scale factor that would result in \(\triangle ABC\) being enlarged.
   d. Find the coordinates of the vertices of \(\triangle A'B'C'\) after this enlargement.