Circles Review and Study Guide

- **Things to Know** Use your notes, homework, checkpoint, and other materials as well as flashcards at quizlet.com (http://quizlet.com/4776937/chapter-10-circles-flashcards-flash-cards/).
  - Vocabulary:
    - circle, radius, chord, diameter, secant, tangent, concentric circles, common tangent, central angle, minor arc, major arc, semicircle, inscribed angle, intercepted arc, inscribed polygon, circumscribed circle, sector
  - Theorems:
    - In a plane, a line is tangent to a circle IFF the line is perpendicular to a radius of the circle at its endpoint on the circle (tangents to circles are perpendicular to radii)
    - Tangent segments from a common external point are congruent.
    - In the same or congruent circles, two minor arcs are congruent IFF their corresponding chords are congruent.
    - If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.
    - If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.
    - In the same circle or congruent circles, two chords are congruent IFF they are equidistant from the center.
    - The measure of an inscribed angle is one half the measure of its intercepted arc.
    - If two inscribed angles of a circle intercept the same arc, then the angles are congruent.
    - A right triangle is inscribed in a circle IFF the hypotenuse is the diameter of the circle.
    - A quadrilateral can be inscribed in a circle IFF its opposite angles are supplementary.
    - If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.
    - If two chords intersect inside a circle, then the measure of each angle is one half the sum of the intercepted angles.
    - If a tangent and secant, two tangents, or two secants intersect outside a circle, then the measure of the angle formed is one half the difference of the intercepted arcs.
    - If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.
    - If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.
    - If a secant and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.
  - Arc length formula: \( S = \frac{C}{360} \cdot 2\pi \), where \( C \) is the central angle of the arc in degrees
Sector area formula: \( A = \frac{C}{360} \cdot \pi r^2 \), where \( C \) is the central angle of the arc in degrees.

The equation for a standard circle is \( (x-h)^2 + (y-k)^2 = r^2 \), where \((h, k)\) is the center of a circle and \( r \) is the radius.

The quadratic formula for \( ax^2 + bx + c \) is: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) (finds the zeros).

DO NOT LIMIT YOUR STUDYING TO PROBLEMS IN THIS STUDY GUIDE!

Example problems:

1) How many common tangents do the given circles have?
   a) \( \bigcirc \)
   b) \( \bigcirc \)
   c) \( \bigcirc \)

2) \( \overline{XY} \) and \( \overline{XZ} \) are tangent to circle \( W \). Find the value of \( a \):

3) Find the missing variable (lines shown are lines of tangency).
   a) \( Q \)
   b) \( S \)
   c) \( S \)

4) \( \overline{AC} \) and \( \overline{BD} \) are diameters of \( \odot F \). Identify the given arc as a major arc, minor arc, or semicircle. Then find the measure of the arc.
   a) \( \overline{mAB} \)
   b) \( \overline{mBC} \)
   c) \( \overline{mABC} \)
   d) \( \overline{mAE} \)
   e) \( \overline{mCDE} \)
   f) \( \overline{mBDC} \)
5) Find the measure of the given arc.

a) \( \widehat{DF} \)

b) \( \widehat{JML} \)

c) \( \widehat{DE} \)

d) \( \widehat{WVY} \)

6) Find the value(s) of the variable(s).

a) \( \angle ACB \)

b) \( \angle DEF \)

c) \( \angle PQR \)

d) \( \angle U VW \)

e) \( \angle JKL \)

f) \( \angle PQM \)

g) \( \angle JLM \)

h) \( \angle PMK \)

i) \( \angle DFE \)

j) \( \angle 5x + 2 \)

k) \( \angle 2x + 10 \)

l) \( \angle 4x + 5 \)

m) \( \angle 3x + 7 \)

n) \( \angle 14x + 16 \)

o) \( \angle 24x + 3 \)

p) \( \angle 8x + 23 \)
7) In the given circle shown at right, \( m\widehat{PQ} : m\widehat{QR} : m\widehat{PR} = 2:6:10 \). Find the value of \( x \), and the measure of each arc.

8) Find the circumference of the circles:
   a) \( 7 \text{ cm} \)
   b) \( 20 \text{ ft} \)
   c) \( 21.6 \text{ ft} \)

9) Find the length of the radius of the circles given their circumferences:
   a) \( C = 48 \text{ in} \)
   b) \( C = 94 \text{ in} \)
   c) \( C = 58 \text{ ft} \)

10) Find the length of \( AB \).
    a) \( 20 \text{ cm} \)
    b) \( 15.4 \text{ cm} \)
    c) \( 30.8 \text{ m} \)

11) In circle D at right, \( \angle ADC \equiv \angle BDC \). Find the indicated measure.
    a) \( m\widehat{ACB} \)
    b) \( m\widehat{CB} \)
    c) length of \( \widehat{ACB} \)
    d) length of \( \widehat{CB} \)
    e) \( m\widehat{ABC} \)
    f) length of \( \widehat{BAC} \)
12) Find the indicated measure of each circle.

a) radius length

b) radius length

c) circumference

13) Find the area of the shaded region.

a)

b)

c)

14) Find the indicated measure of each circle.

a) Area of the circle

b) diameter

c) perimeter of the shaded region

d) Area of the shaded region (area of the pentagon is approximately 342.5 in$^2$)

15) Find the center and radius of a circle that has the standard equation $(x + 5)^2 + (y - 3)^2 = 81$
16) Write the standard equation of the circle with the given center and radius.
   a) Center (0, 0); radius 5   b) Center (1, 3); radius 10   c) Center (-5, 6); radius $\sqrt{5}$

17) Graph the equation. Use a compass if you wish.
   a) $(x - 2)^2 + (y + 1)^2 = 1$
   b) $x^2 + (y + 3)^2 = 6.25$

18) Find the center and radius of the circles given by the equations below:
   a) $x^2 + y^2 + 4x + 6y - 3 = 0$
   b) $x^2 + y^2 - 8x - 10y = 23$

19) What is the equation of the circle?

20) A wagon wheel has 14 spokes. What is the measure of the angle between any two spokes? Round to the nearest tenth.

21) A tree is 16 feet from a circular garden. The distance from the tree to a point of tangency on the garden is 24 meters. What is the radius of the garden? (Hint: Draw a picture!)
### Answer Key:

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<td>1)</td>
<td>a) 0 b) 4 c) 3</td>
<td>2) 2</td>
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<td>3)</td>
<td>a) 10 b) 3 c) ( \frac{8}{3} )</td>
<td>4) a) minor arc, 70° b) minor arc, 110° c) semicircle, 180° d) minor arc, 60° e) minor arc, 120° f) major arc, 250°</td>
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<td>5)</td>
<td>a) 125° b) 120° c) 80.5° d) 265.4°</td>
<td>6) a) ( x=50 ) b) ( x=80 ) c) ( x=20, y=10 ) d) ( x=121 ) e) ( x=50 ) f) ( x=123 ) g) ( x=33.1 ) h) ( x=106.5, y=73.5 ) i) ( x=12 ) j) ( x=12 ) k) ( x=12.5 ) l) ( x=12 ) m) ( x = \frac{35}{3} ) n) ( x=12 ) o) ( x=9 ) p) ( x = 4\sqrt{2} \approx 5.7 ) q) ( x \approx 1.9 ) r) ( x \approx 5.6 ) s) ( x=3 ) t) ( x=4 )</td>
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<td>7)</td>
<td>20°</td>
<td>8) a) ( 14\pi \approx 43.98 \text{ cm} ) b) ( 20\pi \approx 62.83 \text{ ft} ) c) ( 21.6\pi \approx 67.86 \text{ ft} )</td>
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<td>9)</td>
<td>a) ( \frac{24}{\pi} \approx 7.64 \text{ in} ) b) ( \frac{47}{\pi} \approx 14.96 \text{ in} ) c) ( \frac{29}{\pi} \approx 9.23 \text{ ft} )</td>
<td>10) a) ( \frac{15\pi}{2} \approx 23.56 \text{ cm} ) b) ( \frac{100.1\pi}{9} \approx 34.94 \text{ cm} ) c) ( \frac{515.9\pi}{45} \approx 36.02 \text{ m} )</td>
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<td>11)</td>
<td>a) 320° b) 160° c) 22.34 in d) 11.17 in e) 200° f) 13.96 in</td>
<td>12) a) 36 in b) 6.16 in c) 50.82 cm</td>
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<td>13)</td>
<td>a) ( 64\pi \approx 201.06 \text{ ft}^2 ) b) ( \frac{50\pi}{3} \approx 52.36 \text{ in}^2 ) c) ( \frac{27\pi}{4} \approx 21.21 \text{ in}^2 )</td>
<td>14) a) ( 62.86 \text{ m}^2 ) b) ( 11.50 \text{ ft} ) c) ( 24.57 \text{ units} ) d) ( 109.89 \text{ in}^2 ) (note: answers may vary slightly depending on rounding - but they should be close!)</td>
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<td>15)</td>
<td>center ((-5, 3); r=9)</td>
<td>16) a) ( x^2 + y^2 = 25 ) b) ( (x-1)^2 + (y-3)^2 = 100 ) c) ( (x+5)^2 + (y-6)^2 = 5 )</td>
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<td>17) a)</td>
<td></td>
<td>18) a) center ((-2, -3); r=4) b) center ((4, 5); r=8)</td>
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<td>19)</td>
<td>((x-5)^2 + (y+5)^2 = 16)</td>
<td>20) a) ( 25.7^\circ ) b) 10 ft</td>
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