Geometry Notes G.11 Circles: Angle Relationships Mrs. Grieser

Name: _____ Date: _____ Block: _____

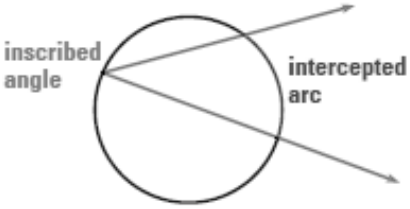
Definitions

Inscribed Angle Angle whose vertex is on a circle and whose sides contain chords of the circle.

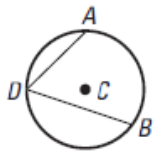
Intercepted Arc The arc that lies in the interior of an inscribed angle and has endpoints on the angle.

Inscribed Polygon Polygon whose vertices lie on a circle.

Circumscribed Circle A circle that contains the vertices of an inscribed polygon.

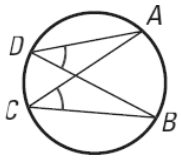


Measure of an Inscribed Angle Theorem
The measure of an inscribed angle is one half the measure of its intercepted arc.



$m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$

Theorem
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

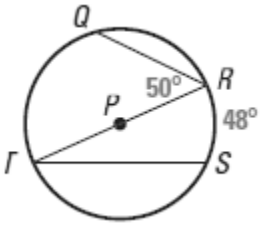


$\angle ADB \cong \angle ACB$

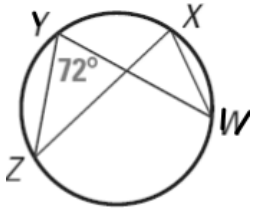
Example:

1)
a) Find $m\angle T$

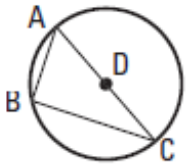
b) Find $m\overset{\frown}{QT}$



2) Find $m\angle WXZ$



Theorem
A right triangle is inscribed in a circle IFF the hypotenuse is the diameter of the circle.

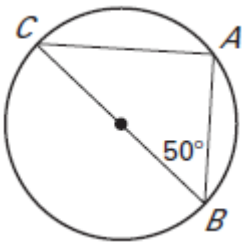


Theorem
A quadrilateral can be inscribed in a circle IFF its opposite angles are supplementary.

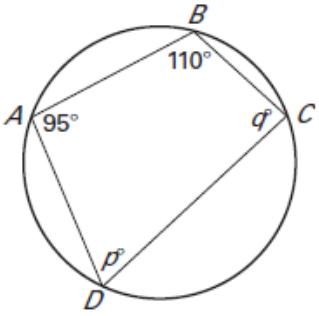
$m\angle ABC = 90°$ if and only if $\overline{AC}$ is a diameter of the circle.

Examples/You Try…

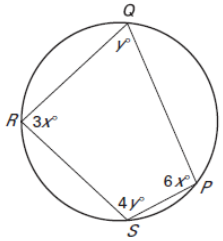
a) Find $m\angle C$



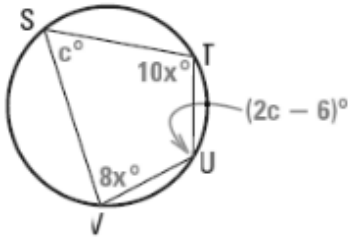
b) Find p and q



c) Find the measure of each angle



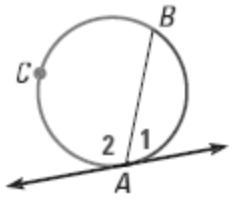
d) Find c and x



Other Angle Relationships

- *Tangent Lines and Circles*

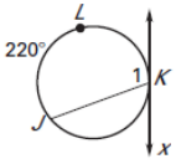
Theorem
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.



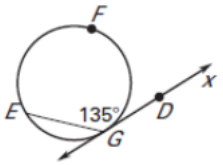
$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

Examples:

a) Find $m\angle 1$

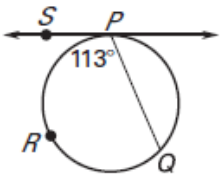


b) Find $m\overset{\frown}{EFG}$

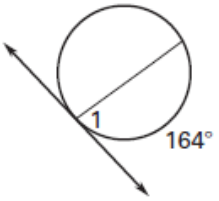


You Try...

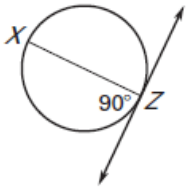
c) Find $m\overset{\frown}{PRQ}$



d) Find $m\angle 1$



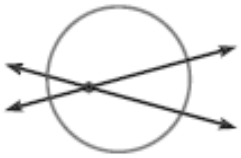
e) Find $m\overset{\frown}{XZ}$



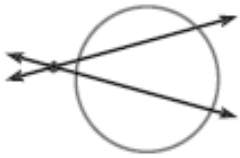
- *Intersecting Lines and Circles*

Three places:

on the circle

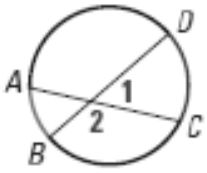


inside the circle



outside the circle

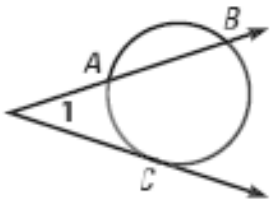
Angles Inside the Circle Theorem
If two chords intersect **inside** a circle, then the measure of each angle is one half the **sum** of the intercepted angles.



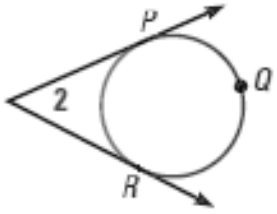
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$,

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

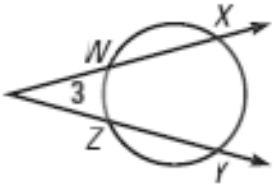
Angles Outside the Circle Theorem
If a tangent and secant, two tangents, or two secants intersect **outside** a circle, then the measure of the angle formed is one half the **difference** of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



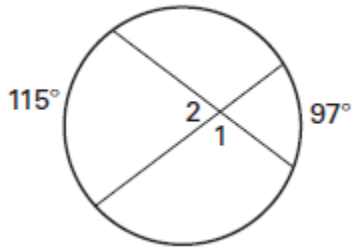
$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$



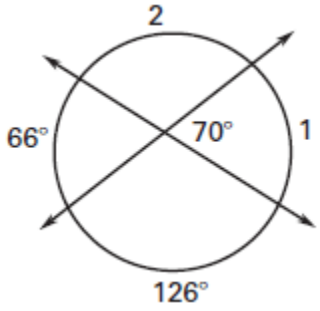
$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Examples:

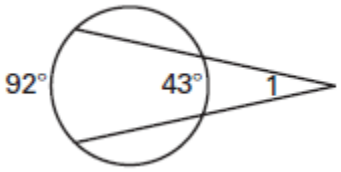
a) Find the missing angles



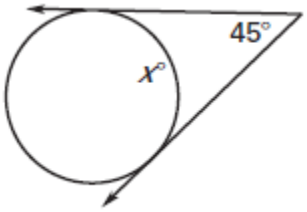
b) Find the missing arcs



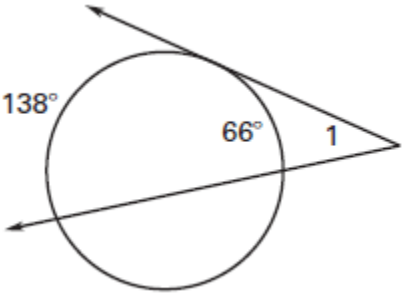
c) Find the missing angle



d) Find x

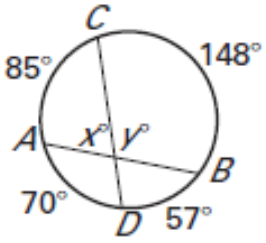


e) Find the missing angle

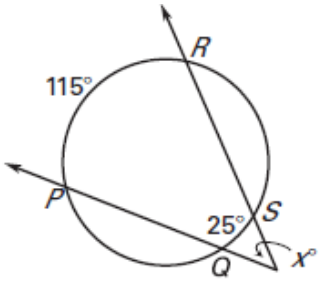


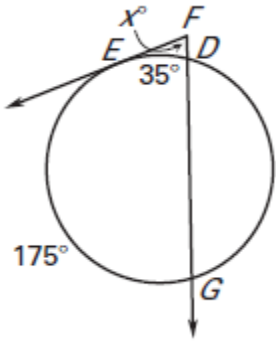
You Try:

a) Find x and y



b) Find x for each figure





c) Find $m\angle 1$