Pythagorean Theorem (converse true, too!)
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.
\[ a^2 + b^2 = c^2 \] (c is the hypotenuse) IFF the triangle is a right triangle

Examples: Find the missing side...

a) 

b) 

c) 

Other examples:

d) Word Problem
A 16 ft ladder rests against the side of a house, and the base of the ladder is 4 ft away from the house. Approximately how high above the ground is the top of the ladder?

e) Find the area of an isosceles triangle with side lengths 10, 13, and 13 meters.
Recall: \[ A = \frac{1}{2}bh \]
Find altitude (height):
Find area:

Pythagorean Triples

Pythagorean Triples are sets of three positive integers that could be the lengths of a right triangle, that is, they satisfy \[ a^2 + b^2 = c^2 \].

Example: 3, 4, 5 is a Pythagorean triple because \[ 3^2 + 4^2 = 5^2 \] (9 + 16 = 25)

- Primitive triples are those with sides that are coprime (no common factors other than 1)
- Find other Pythagorean triples by finding multiples of primitive triples (apply scale factors)
- Example: since 3, 4, 5 is a Pythagorean triple, so is 6, 8, 10 (multiply each member by 2)

<table>
<thead>
<tr>
<th>Scale Factor</th>
<th>The Primitive Triples</th>
</tr>
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<tbody>
<tr>
<td>1:2</td>
<td>3, 4, 5</td>
</tr>
<tr>
<td></td>
<td>5, 12, 13</td>
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<tr>
<td></td>
<td>8, 15, 17</td>
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<td></td>
<td>7, 24, 25</td>
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<tr>
<td>1:3</td>
<td>6, 8, 10</td>
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<tr>
<td></td>
<td>9, 12, 15</td>
</tr>
<tr>
<td>1:4</td>
<td>12, 16, 20</td>
</tr>
<tr>
<td>1:5</td>
<td>15, 20, 25</td>
</tr>
</tbody>
</table>

Memorize the primitive triples, notice multiples, and save time solving right triangle problems!
Using triples example: Find the third side of a right triangle with legs 20 and 48.

- Find greatest common factor: 
- Compare with primitives - which primitive triple matches?
- Apply factor to third side of the triangle:

Go back to previous examples. Would identifying triples have made solving easier?

You Try: Complete the Pythagorean Triples
a) 27, ____, 45 b) ____ , 96, 100 c) 65, ____ , 169 d) 49, 168, ____

Finding the Midpoint and Distance (Length) of a Line

Terms:

<table>
<thead>
<tr>
<th>Terms</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>midpoint</td>
<td>The midpoint of a segment divides the segment into two congruent parts.</td>
</tr>
<tr>
<td>segment</td>
<td>A segment bisector is a point, ray, line, or line segment that intersects</td>
</tr>
<tr>
<td>bisector</td>
<td>the segment at its midpoint (a segment bisector bisects a segment).</td>
</tr>
</tbody>
</table>

Example: Point M is the midpoint of \( VW \). Find the length of \( VM \).

What do we know about \( VM \) and \( MW \)?

Solve:

We can find the midpoint of a segment in the coordinate plane using the midpoint formula.

**Midpoint Formula**

Find the midpoint of a segment in the coordinate plane by finding the average (mean) of the abscissas (x-coordinates) and ordinates (y-coordinates).

Given points \( A(x_1, y_1) \) and \( B(x_2, y_2) \), the midpoint \( M \) has coordinates:

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

Examples:

1) Find the midpoints of \( RS \) if the coordinates are \( R(1, -3) \) and \( S(4,2) \)

2) The midpoint of \( JK \) is \( M(2, 1) \). One endpoint is \( J(1, 4) \). Find the coordinates of the other endpoint \( K \).
We can also find the **distance** (length) of a line segment in the coordinate plane.

### Distance Formula

Given points \(A(x_1, y_1)\) and \(B(x_2, y_2)\) are points on the coordinate plane, then the distance between \(A\) and \(B\) is:

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

The distance formula is based on the Pythagorean Theorem \((a^2 + b^2 = c^2\), where \(c\) is the hypotenuse and \(a\) and \(b\) are legs of a right triangle).

#### Example: Find the length of \(RS\).

\((x_1, y_1) = \ldots\)  
\((x_2, y_2) = \ldots\)  
\((x_2 - x_1) = \ldots\)  
\((y_2 - y_1) = \ldots\)  
Length = \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \ldots\)

**You try:**

a) Find \(AM\) given \(M\) bisects \(AC\)

\[x + 5\]

\[A\]

\[M\]

\[2x\]

\[C\]

b) Find the midpoint given the coordinates of endpoints: \(C(3, 5)\) and \(D(7, 5)\)

c) Find the midpoint given the coordinates of endpoints: \(J(-7, -5)\) and \(K(-3, 7)\)

d) Find the endpoint \(S\) given endpoint \(R\) and midpoint \(M:\)

\(R(5, 1), M(1, 4)\)

e) Find the length of the segment with endpoints \(P(1, 2)\) and \(Q(5, 4)\).

f) Find the length of the segment with endpoints \(S(-1, 2)\) and \(T(3, -2)\).