Normal Distribution

- Density curve of a distribution that graphs as a bell-shaped curve
- Properties
  - bell-shaped, symmetric curve
  - defined by the mean and standard deviation of a data set
  - arithmetic mean = median = mode
  - arithmetic mean is located on axis of symmetry
  - 50% of data is on each side of the mean
  - percents of data fit within standard deviations as shown in diagram (68-95-99.7 rule):

![Normal Distribution Diagram]

- The area under the curve represents the probability of the data
- 100% of the data fits under the curve
- Therefore, the area under every normal curve is ________

Standard Normal Curve

- Defined as $\mu = 0$ and $\sigma = 1$
- Graphing with calculator:
  1) Press y=
  2) Press $<2^{nd}><$VARS$><1>$ (DISTR) to get normalpdf
  3) Enter normalpdf(X, 0, 1)
  4) Adjust window
     - x-min -4, x-max 4
     - y-min 0, y-max: .5
  5) Press Graph
Questions:

1) What happens if $\mu$ is not 0?
   a) In Y2=, add another normalpdf statement, but change 0 (\(\mu\) value) to 5
   b) Change window to x-max 9 before graphing
   c) What happened? ______________________________________________________
   d) Why? ___________________________________________________________________

2) What happens if $\sigma$ is not 1?
   a) In Y3=, add another normalpdf statement, but change 0 (\(\mu\) value) back to 0, and 1 (\(\sigma\) value) to 3
   b) Change window to x-min -7 before graphing
   c) What happened? ______________________________________________________
   d) Why? ___________________________________________________________________

Your notes:

Explain how to use the graphing calculator to graph a normal curve. Include parameters to normalpdf...

How do you set the window values?

Other:

Probabilities and the Normal Distribution

We can use the known area percentages of the normal curve to determine probabilities of a range of values.
**Example 1:**
Given a normally distributed data set of 500 observations measuring tree heights in a forest, what is the approximate number of observations that fall within two standard deviations from the mean?

- What percentage of data falls within $2\sigma$ of the mean? __________
- Based on the given total observations, how many are within $2\sigma$? ________________________

**Example 2:**
A normally distributed data set containing the number of ball bearings produced during a specified interval of time has a mean of 150 and a standard deviation of 10. What percentage of the observed values fall between 140 and 160?

- 140 and 160 are how many standard deviations from the mean? (Use z-score!) ______________
- Use the normal distribution percentages to answer the question. ______________

**Using the Calculator**

**Example 2** can also be solved by using the calculator. The `normalcdf` (cumulative distribution function) will give you a percentage between two intervals on a normal curve. The calling parameters:

\[
\text{normalcdf}(lb, ub, \mu, \sigma), \text{ where } lb = \text{lower bound}, \ ub = \text{upper bound}
\]

- Find `normalcdf` on the calculator by pressing `<2^{nd}> <VARS><2>
- To do the above problem, enter `normalcdf(140, 160, 150, 10)` and pressing enter. You should get the same percentage as when you used the normal curve diagram. _________

Another way to use the calculator is to graph the curve and then shade the area we are interested in.

- Graph the curve as described earlier (use `normalpdf(X, 150, 10)` in Y1= ; can you figure out the window?)
- Press `<2^{nd}><VARS><DRAW><1>` (ShadeNorm)
- Enter parameters: ShadeNorm(140, 160, 150, 10)
  - (parameters are lower bound, upper bound, mean, standard deviation)
- What does the graph show? Sketch it here:
Z-Tables

- Yet another way to solve probability problems is to use standard normal probabilities tables, also known as z-tables.
- Popular in the pre-calculator age, but still useful!
- The area percentages (in decimal form) from $-\infty$ to the z-score you are interested in is shown.
- Also known as percentiles.
- Find the probability if the z-score is 1.0
  - Find the row with 1.0, then the column with .00. What is the probability?______
- Use the z-tables to find the probabilities for the following z-scores:
  - 1.3 ___________  -2.15___________  3.28___________  -0.06_________

Let’s re-do example 2 using z-tables. We want the percentage between 140 and 160 if the mean is 150 and the standard deviation is 10.
Find the z-scores of 140 _____________ and 160 ___________.
Find the corresponding values in the z-tables: 140: ___________ 160:_________
How would you use these table values to find the correct result? What is the result?

Practice:

1) The lifetime of 20,000 flashlight batteries are normally distributed, with a mean of $\mu = 370$ days and a standard deviation of $\sigma = 30$ days.
   a) What percentage of the batteries is expected to last more than 340 days?
   b) How many batteries can be expected to last less than 325 days?

2) The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours. Find the probability that a randomly selected battery lasts longer than 42 hours.

3) The amount of time that Carlos plays video games in any given week is normally distributed. If Carlos plays video games an average of 15 hours per week, with a standard deviation of 3 hours, what is the probability of Carlos playing video games between 15 and 18 hours a week?

4) A Calculus exam is given to 500 students. The scores have a normal distribution with a mean of 78 and a standard deviation of 5. What percent of the students have scores between 82 and 90?
Inverse Norm

What if we are given a probability (percentile), and want to find the value that gives the probability?

The inverse norm function (invNorm), given a probability, mean, and standard deviation, will return the value that gives that probability.

Example:
Given a normal distribution of values for which the mean is 70 and the standard deviation is 4.5. Find:
   a) the probability that a value is between 65 and 80, inclusive.
   b) the probability that a value is greater than or equal to 75.
   c) the probability that a value is less than 62.
   d) the 90th percentile for this distribution.

- We should be able to find a), b), and c) using methods from previous discussions.
- Question d), however, is giving us the probability, and asking for the x-value that gives the probability.
- Use the calculator: \( \text{<2nd><VARS><3> (invnorm)} \)
- Type: invnorm(.90, 70, 4.5)
- Percentile: _______________

Summary of calculator functions:

<table>
<thead>
<tr>
<th>Function</th>
<th>Navigation</th>
<th>Parameters</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>normalpdf</td>
<td>&lt;2nd&gt;&lt;VARS&gt;&lt;1&gt;</td>
<td>x-value, ( \mu, \sigma )</td>
<td>Use to graph curves by using X as first parameter in Y= graphs. Set window before graphing by making x-min and x-max related to ( \mu \pm 3\sigma ) and y-max = ( 1/2\sigma )</td>
</tr>
<tr>
<td>ShadeNorm</td>
<td>&lt;2nd&gt;&lt;VARS&gt;&lt;DRAW&gt;&lt;1&gt;</td>
<td>lower bound, upperbound, ( \mu, \sigma )</td>
<td>After using normalpdf to graph a curve, use ShadeNorm to shade a region of the curve and get the area under the curve</td>
</tr>
<tr>
<td>ClrDraw</td>
<td>&lt;2nd&gt;&lt;PRGM&gt;&lt;1&gt;</td>
<td>none</td>
<td>Clear region shadings from ShadeNorm</td>
</tr>
<tr>
<td>normalcdf</td>
<td>&lt;2nd&gt;&lt;VARS&gt;&lt;2&gt;</td>
<td>lower bound, upperbound, ( \mu, \sigma )</td>
<td>Find the probability for a region of a normal curve</td>
</tr>
<tr>
<td>invNorm</td>
<td>&lt;2nd&gt;&lt;VARS&gt;&lt;3&gt;</td>
<td>probability in decimal form, ( \mu, \sigma )</td>
<td>Find the x-value given a probability (percentile) in decimal form, mean, and standard deviation</td>
</tr>
</tbody>
</table>
You Try…

1) The amount of mustard dispensed from a machine at *The Hotdog Emporium* is normally distributed with a mean of 0.9 ounce and a standard deviation of 0.1 ounce. If the machine is used 500 times, *approximately* how many times will it be expected to dispense 1 or more ounces of mustard?

2) Professor Halen has 184 students in his college mathematics lecture class. The scores on the midterm exam are normally distributed with a mean of 72.3 and a standard deviation of 8.9. How many students in the class can be expected to receive a score between 82 and 90? Express answer to the *nearest student*.

3) A machine is used to fill soda bottles. The amount of soda dispensed into each bottle varies slightly. Suppose the amount of soda dispensed into the bottles is normally distributed. If at least 99% of the bottles must have between 585 and 595 milliliters of soda, find the greatest standard deviation, to the *nearest hundredth*, that can be allowed.

4) Battery lifetime is normally distributed for large samples. The mean lifetime is 500 days and the standard deviation is 61 days. To the *nearest percent*, what percent of batteries have lifetimes longer than 561 days?

5) A shoe manufacturer collected data regarding men’s shoe sizes and found that the distribution of sizes exactly fits the normal curve. If the mean shoe size is 11 and the standard deviation is 1.5, find:
   a) the probability that a man's shoe size is greater than or equal to 11.
   b) the probability that a man's shoe size is greater than or equal to 12.5.