Polynomials TEST STUDY GUIDE

Test covers:

- Polynomial division: long and synthetic.
- Using synthetic division to evaluate a function at a value of x (remainder theorem).
- Factor a function given a factor; finding all zeros of a function given one zero.
- Using the rational zero theorem to find all possible rational zeros of a function.
- Using Descartes’ rule of signs to find all possible combinations of real positive, real negative, or imaginary zeros.
- Using the fundamental theorem of Algebra to determine the total number of zeros of a polynomial function.
- Writing polynomial functions given the zeros of a function.
- Graphing functions, finding intercepts, considering multiplicity (touches or crosses x-axis), and end behaviors, and determining local maxima, minima, and intervals where the function increases and decreases.

Practice Questions:

1) Divide the polynomials, using synthetic division where possible:
   a) \((9x^3 - 18x^2 - x + 2) ÷ (3x + 1)\)
   b) \((3x^3 - 2x^2 + 4x - 3) ÷ (x^2 + 3x + 3)\)
   c) \((x^3 + 3x^2 - 6x + 7) ÷ (x + 4)\)
   d) \((x^3 - 13x - 12) ÷ (x - 4)\)

2) Use synthetic division to evaluate the polynomial functions at the given input value.
   a) \(f(x) = x^4 - 5x^2 + 4x + 12\) at \(x = -4\)
   b) \(g(x) = 2x^4 + 6x^3 - 5x^2 - 60\) at \(x = -1\)

3) Given \(f(x) = x^3 + 4x^2 + x - 6\), determine whether the following binomials are factors. Explain why or why not.
   a) \(x + 3\)  
   b) \(x - 3\)

4) Given polynomial \(f(x)\) and a factor of \(f(x)\), factor \(f(x)\) completely:
   a) \(y = x^3 + 2x^2 - 5x - 6;\) factor \((x + 1)\)
   b) \(f(x) = x^3 - 4x^2 - 9x + 36;\) factor \((x + 3)\)

5) Given \(f(x) = x^3 - 7x^2 + 15x - 9\) and 3 is a zero of \(f\), find all the zeros of \(f\).
6) Find the zeros of \( f(x) = x^3 + 4x^2 + 4x \) given \( f(-2) = 0 \)

7) Use the rational zero theorem to list the possible rational zeros for the following polynomial functions:
   a) \( f(x) = x^3 - 2x^2 - 5x + 10 \)  
   b) \( g(x) = 2x^3 + 3x^2 - 8x + 3 \)

8) Find the rational zeros of the polynomial functions in the previous question.

9) Use Descartes’ rule of signs to describe the possible positive real zeros, negative real zeros, and imaginary zeros of the given polynomial functions. Use the tables to indicate all the possibilities.
   a) \( f(x) = 6x^5 - 4x^4 - 63x^3 + 42x^2 + 147x - 98 \)  
   b) \( f(x) = 2x^6 - 2x^5 + x^4 - 2x^3 - 4x^2 - 3x - 1 \)

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<th>Positive real zeros</th>
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10) Find all zeros of the polynomials (real and imaginary):
   a) \( f(x) = 4x^3 + 16x^2 - 22x - 10 \)  
   b) \( g(x) = x^3 - 2x^2 + 5x - 10 \)

11) Solve the polynomials, finding all solutions (real and imaginary).
   a) \( 2x^4 - 5x^3 - 17x^2 + 41x - 21 = 0 \)  
   b) \( x^3 - 5x^2 + 7x - 35 = 0 \)

12) Write a polynomial of least degree that has rational coefficients, a leading coefficient of 1, and the given the zeros of the polynomial function.
   a) \( 1, 3i \)  
   b) \( \sqrt{3}, 1 - i \)  
   c) \( 3 + \sqrt{2}, \sqrt{5} \)

13) Write a cubic function whose graph passes through the given points, expressing the polynomial in standard form.
   a) \(-2, 0\), \((-1,0\), \((0, -8\), \((2, 0)\)  
   b) \((-2, 0\), \((0, 0\), \((1, 0\), \((2, 1)\)
14) Sketch an example of polynomial functions with the given end behaviors.

a) a negative leading coefficient; odd degree  
   

d) a positive leading coefficient; even degree

15) Graph the functions WITHOUT THE CALCULATOR giving the degree, max turning points, y-intercept, number of zeros, possible rational zeros, zeros (and whether they touch or cross the x-axis), and end behaviors. Plot some points. After graphing, estimate the turning points and indicate whether they are local minima or maxima, then describe the intervals the function increases and decreases. (You may use the calculator for simple calculations when calculating coordinate pair values)

a) \( f(x) = 2x^3 - 4x^2 - 2x + 4 \)

b) \( f(x) = \frac{1}{4}(x^3 - 2x^2 - 7x - 4) \)
STUDY GUIDE ANSWERS

1) a) \(3x^2 - 7x + 2\)  
b) \(3x - 11 + \frac{28x + 30}{x^2 + 3x + 3}\)  
c) \(x^2 - x - 2 + \frac{15}{x + 4}\)  
d) \(x^2 + 4x + 3\)

2) a) 172  
b) -69

3) a) yes, because the remainder is 0  
b) no, because the remainder is not 0

4) a) \(y = (x+1)(x+3)(x-2)\)  
b) \(f(x) = (x+3)(x-3)(x-4)\)

5) 1, 3 (twice)

6) 0, -2 (twice)

7) a) \(\pm 1, \pm 2, \pm 5, \pm 10\)  
b) \(\pm 1, \pm \frac{1}{2}, \pm \frac{3}{2}\)

8) a) \(2, \pm \sqrt{5}\)  
b) 1, -3, \(\frac{1}{2}\)

9) a) 3 or 1 pos. real zeros; 2 or 0 neg. real zeros  
b) 3 or 1 pos real zeros; 3 or 1 neg real zeros

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10) a) \(-5, \pm \frac{\sqrt{3}}{2}\)  
b) 2, \(\pm i\sqrt{5}\)

11) a) \(-3, 1\) (twice), \(\frac{7}{2}\)  
b) 5, \(\pm i\sqrt{7}\)

12) a) \(f(x) = x^3 - x^2 + 9x - 9\)  
b) \(f(x) = x^4 - 2x^3 - x^2 + 6x - 6\)  
c) \(f(x) = x^4 - 6x^3 + 2x^2 + 30x - 35\)

13) a) \(f(x) = 2x^3 + 2x^2 - 8x - 8\)  
b) \(f(x) = \frac{1}{8}(x^3 + x^2 - 2x) = \frac{1}{8}x^3 + \frac{1}{8}x^2 - \frac{1}{4}x\)

14) Answers may vary, but end behaviors are described:
   a) as \(x \to +\infty\), \(f(x) \to -\infty\); as \(x \to -\infty\), \(f(x) \to +\infty\) (end behaviors match \(f(x) = -x^3\))
   b) as \(x \to +\infty\), \(f(x) \to +\infty\); as \(x \to -\infty\), \(f(x) \to -\infty\) (end behaviors match \(f(x) = x^3\))
   c) as \(x \to +\infty\), \(f(x) \to -\infty\); as \(x \to -\infty\), \(f(x) \to +\infty\) (end behaviors match \(f(x) = -x^3\))
   d) as \(x \to +\infty\), \(f(x) \to +\infty\); as \(x \to -\infty\), \(f(x) \to +\infty\) (end behaviors match \(f(x) = x^3\))

15) a) \(3^{rd}\) degree; 2 max turning points, y-int = 4; 3 zeros; possible rational zeros: \(\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}\); factored: \(f(x) = 2(x-2)(x+1)(x-1)\); zeros at 2, -1, 1; crosses x-axis at all zeros; end behaviors: as \(x \to +\infty\), \(f(x) \to +\infty\); as \(x \to -\infty\), \(f(x) \to -\infty\); some points to graph (make a table): (-2, -24), (-1, 0), (-.5, 3.75), (0, 4), (.5, 2.25), (1, 0), (1.5, -1.25)  
   turning points approximately (-.2, 4.5) (local maximum), (1.5, -1.25) (local minimum); function increases on intervals \((-\infty, -2)\) and \((1.5, \infty)\) and decreases on interval \((-2, 1.5)\)
   b) \(3^{rd}\) degree; 2 max turning points; y-int=-1; 3 zeros; possible rational zeros: \(\pm 1, \pm 2, \pm 4\); factored: \(f(x) = \frac{1}{4}(x+1)^2(x-4)\); zeros at -1, 4; touches at -1, crosses at 4; end behaviors: as \(x \to +\infty\), \(f(x) \to +\infty\); as \(x \to -\infty\), \(f(x) \to -\infty\); some points to graph (make a table): (-3, 7), (-2, -1.5), (-1, 0), (0, -1), (1, -3), (2, -4.5), (3, -4), (4, 0), (5, 9)  
   turning points approximately (-1, 0) (local max), (2.3, 4.6) (local min); increases on intervals \((-\infty, -1)\) and \((2.3, \infty)\) and decreases on interval \((-1, 2.3)\)