Real Number System Review

Draw an Euler Diagram of the Real Number System:

Complex Numbers Euler Diagram:

Imaginary Numbers

- A number whose square is less than zero (negative)
- Imaginary number $\sqrt{-1}$ is called “$i$”
- Other imaginary numbers – write using “$i$” notation: $\sqrt{-16} = _____$ $\sqrt{-8} = _____$
- Adding or subtracting imaginary numbers: add coefficients, just like monomials
  - Add: $5i + 3i = ______$
- Multiplying $i$:
  - $i^0 = _____$ $i^1 = _____$ $i^2 = _____$ $i^3 = _____$
  - $i^4 = _____$ $i^5 = _____$ $i^6 = _____$ $i^7 = _____$
  - $i^8 = _____$ $i^9 = _____$ $i^{10} = _____$ $i^{11} = _____$
  - Is there a pattern? ________________________________
  - Apply the pattern to find: $i^{17} = _____$ $i^{26} = _____$
Complex Numbers

- Take the form \(a + bi\), where \(a\) and \(b\) are real numbers, and \(i\) is the imaginary number \(\sqrt{-1}\)
- \(a\) is the real part of the number, and \(bi\) is the imaginary part of the number
- When \(a = 0\), we have a **pure imaginary number**
- Given \(2 + 5i\), the real part is _______ and the imaginary part is ______
- We can plot complex numbers on the complex plane, where the x-axis is the real part, and the y-axis is the imaginary part.
- Plot: \(2 + 3i\), \(-3 + i\), \(3 - 3i\), \(-4 - 2i\)
- What are the magnitudes of these points (absolute value)? Use Pythagorean Theorem.

Adding and Subtracting Complex Numbers

Combine like terms
- Add real parts, add imaginary parts
- When subtracting, distribute the negative then add
- Simplify:
  \[(8 - 3i) + (2 + 5i)\]
  \[(8 + 7i) + (-12 + 11i)\]
  \[(9 - 6i) - (12 + 2i)\]

Multiplying Complex Numbers

- Multiply imaginary parts, remembering \(i^2\) rules
- Use multiple distribution, just like when we multiply polynomials (FOIL)
  \[5\sqrt{-2} \cdot 4\sqrt{-8}\]
  \[(8 + 5i)(2 - 3i)\]
  \[(-6 + 2i)(2 - 3i)\]
Complex Numbers: Division and Complex Conjugates

- The complex conjugate of \(a + bi\) is \(a - bi\)
- Multiply complex conjugates: \((a + bi)(a - bi)\)
- Use conjugates to rationalize complex denominators of fractions
  - Multiply numerator and denominator by conjugate of denominator
  - Examples:
    \[
    \frac{5 + 2i}{7 + 4i} \quad \frac{8 + i}{2 - i} \quad \frac{3 - 2i}{3 + 2i}
    \]

Finding Complex Solutions

Solve: \(4x^2 + 100 = 0\) \(\quad\) \(3x^2 + 48 = 0\)

Complex Numbers: Field Properties

| Closure | • Any algebraic operations of complex numbers result in a complex number
  • Addition: \((a + bi) + (c + di) = (a + c) + (b + d)i\)
  • Subtraction: \((a + bi) - (c + di) = (a - c) + (b - d)i\)
  • Multiplication: \((a + bi)(c + di) = ac + (ad + bc)i + bd i^2 = (ac - bd) + (ad + bc)i\)
  • Division: \[\frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i\]

| Equality | • Complex numbers are equal IFF their real parts are equal and their imaginary parts are equal
  • \(a + bi = c + di\) IFF \(a = c\) and \(b = d\)

| Commutative Property of Addition | \((a + bi) + (c + di) = (c + di) + (a + bi)\)

| Commutative Property of Multiplication | \((a + bi)(c + di) = (c + di)(a + bi)\)

| Associative Property of Addition | \(((a + bi) + (c + di)) + (e + fi) = (a + bi) + ((c + di) + (e + fi))\)

| Associative Property of Multiplication | \(((a + bi)(c + di))(e + fi) = (a + bi)((c + di)(e + fi))\)

| Distributive Property | \(a(b + ci) = ab + aci\)

| Additive Identity | \((a + bi) + (0 + 0i) = a + bi\)

| Multiplicative Identity | \((a + bi)(1 + 0i) = a + bi\)

| Additive Inverse | \((a + bi) + (-a - bi) = 0\)

| Multiplicative Inverse | \(\frac{1}{a + bi}\)
Algebra 2 AII.3 Complex Numbers NOTES
Mrs. Grieser
Practice
Simplify:
1) \( \sqrt{-144} \)  
2) \( 2 + \sqrt{-25} \)  
3) \( -4 + \sqrt{-49} \)  
4) \( \sqrt{-20} \)

5) \( 6 - \sqrt{-12} \)  
6) \( i^{35} \)  
7) \( i^{62} \)  
8) \( i^{16} \)

9) \( 4i^3 + 7i^9 \)  
10) \( (3i^5)^2 \)  
11) \( i^{16} + i^{10} + i^8 - i^{14} \)  
12) \( i^{12} \cdot 3i^2 \cdot 2i^8 \)

13) \( (5 + 7i) + (-2 + 6i) \)  
14) \( (8 + 3i) - (2 + 4i) \)

15) \( (3 - 2\sqrt{-18}) - (2 + 3\sqrt{-8}) \)  
16) \( (5 - 2i)(3 + 4i) \)

17) \( (9 + 3i)(9 - 3i) \)  
18) \( (4 - 3i)^2 \)

19) \( (5 + \sqrt{-16})(2 - \sqrt{-9}) \)  
20) \( (9 - i\sqrt{2})(9 + i\sqrt{2}) \)

21) \( (10 - 5i) ÷ 5 \)  
22) \( (1 - 2i) ÷ 3i \)

23) \( \frac{2 - 3i}{1 + i} \)  
24) \( \frac{5 + 2i}{3 - i} \)

Solve the equations:
25) \( x^2 + 27 = 0 \)  
26) \( x^2 + 81 = 0 \)

27) \( 4x^2 - 9 = -41 \)  
28) \( x^2 - 2x + 7 = 4 \)

29) Find the additive inverse of \(-2 + 5i\)  
30) Find the multiplicative inverse of \(7 - 5i\)