Radical Review

- A perfect square is _________________________________.
- The first 12 perfect squares are: _________________________________.
- A square root of a number is _________________________________.
- Find: \( \sqrt{36} \) \( \sqrt{49} \) \( \sqrt{144} \) \( \sqrt{x^2} \) \( \sqrt{4x^4} \)
- Estimating square roots: find the closest perfect square
  o Estimate to the nearest whole number: \( \sqrt{35} \) \( \sqrt{55} \) \( \sqrt{99} \)
- Other roots:
  o A perfect cube is _________________________________.
  o The first 10 perfect cubes are: _________________________________.
  o A cube root of a number is _________________________________.
- Find: \( \sqrt[3]{8} \) \( \sqrt[3]{27} \) \( \sqrt[3]{125} \) \( \sqrt[3]{x^5} \) \( \sqrt[3]{8x^6} \)
- \( \sqrt[3]{y} \) means the \( x \)th root of \( y \). The \( \sqrt{ } \) symbol is called a **radical**. The number inside is the **radicand**.
- By default, take the square root. Example: \( \sqrt{4} = 2 \)

Simplifying Radicals

A radical expression is in simplest form if:

- No perfect squares other than 1 are in the radicand
- No fractions are in the radicand
- No radicals appear in the denominator of a fraction

We use the Product Property of Radicals and Quotient Property of Radicals to simplify:

**Product Property of Radicals**
The square root of a product equals the product of the square roots of the factors:
\( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \)

**Quotient Property of Radicals**
The square root of a quotient equals the quotient of the square roots of the numerator and denominator:
\( \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \)

- Write the radicand as the **product of perfect squares**, and then take the roots of those perfect squares.
  a) Simplify \( \sqrt{32} \)
  \( \sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4 \sqrt{2} \)
  b) Simplify \( \sqrt{9x^3} \)
  \( \sqrt{9x^3} = \sqrt{9} \cdot x \cdot x = \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x} = 3x \sqrt{x} \)

You try: Simplify the radical expressions...

a) Simplify \( \sqrt{24} \)

b) Simplify \( \sqrt{25x^2} \)
Examples for Quotient Property of Radicals:

a) Simplify \( \frac{\sqrt{13}}{\sqrt{100}} = \frac{\sqrt{13}}{10} \)

b) Simplify \( \frac{1}{\sqrt{y^2}} = \frac{1}{y} \)

**Multiplying Radicals**

We can use the product property of radicals to multiply radicals.

Examples:

a) \( \sqrt{6} \cdot \sqrt{6} = \sqrt{36} = 6 \) (makes sense!)

b) \( \sqrt{3x} \cdot 4\sqrt{x} = 4 \sqrt{3x^2} = 4x\sqrt{3} \)

c) \( \sqrt{7x^2} \cdot 3\sqrt{x} = 3\sqrt{7x^2}y = 3xy\sqrt{7} \)

**Note:** When we solve \( \sqrt{x^2} \), we always take the positive value of \( x \).

**Rationalizing Denominators**

The process of eliminating a radical in a denominator is called rationalizing the denominator.

Examples: Rationalize the denominator...

a) \( \frac{5}{\sqrt{7}} = \frac{5\sqrt{7}}{7} \)

b) \( \frac{4}{\sqrt{2x}} = \frac{4\sqrt{2x}}{2x} = \frac{2\sqrt{2x}}{x} \)

You try: Simplify the radical expressions...

a) \( \sqrt{20} \)

b) \( \sqrt{72} \)

c) \( \sqrt{32x^5} \)

d) \( \sqrt{96x^2} \)

e) \( \frac{9x}{\sqrt{16}} \)

f) \( \frac{4}{\sqrt{3}} \)

g) \( \frac{1}{\sqrt{x}} \)

h) \( \sqrt{\frac{2x^2}{5}} \)

i) \( \sqrt{\frac{8}{3n^2}} \)

j) \( \sqrt[3]{27x^3} \)

k) \( \sqrt[3]{16x^4} \)

l) \( \sqrt[3]{\frac{64x^6y^9}{8}} \)