Introduction to Slope

Name: _______________________________ Date: _____________ Block: ______

Slope of a line:

- Ratio of the vertical change (the “rise”) to the horizontal change (the “run”) between any two points in a line.
- The letter 'm' is used to denote slope.
- The slope will calculate to the same value for any 2 points on a line.

Finding a line’s slope:

- Identify two points:
  - “Rise” is the difference in the ordinates (y-values)
  - “Run” is the difference in the abscissas (x-values)

To calculate slope for two points \((x_1, y_1)\) and \((x_2, y_2)\):

\[
slope = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}
\]

- The Greek letter delta (\(\Delta\)) denotes the difference between two values and may be used as a short cut: e.g. \(\Delta y\) is shorthand for \(y_2 - y_1\).

Examples - find the slopes given the following sets of points:

a) \((-4, 2)\) and \((2, 6)\)

\[
x_1 = -4, y_1 = 2 \quad x_2 = 2, y_2 = 6
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{2 - (-4)} = \frac{4}{6} = \frac{2}{3}
\]

b) \((3, 5)\) and \((6, -1)\)

\[
x_1 = 3, y_1 = 5 \quad x_2 = 6, y_2 = -1
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{6 - 3} = \frac{-6}{3} = -2
\]

c) \((-2, 4)\) and \((4, 4)\)

\[
x_1 = -2, y_1 = 4 \quad x_2 = 4, y_2 = 4
\]

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{4 - (-2)} = \frac{0}{6} = 0
\]

Things to know about slope:

- **Positive slopes:** range increases as \(x\) increases - line rises from left to right
- **Negative slopes:** range decreases as \(x\) increases - line falls from left to right
- **Zero (0) slope:** horizontal lines have a slope of 0 because the difference of their \(y\)-values is always 0
- **Undefined slope:** vertical lines have an undefined slope because the difference of the \(x\)-values is always 0 (so we can’t divide!)

Find the Slope of a Line

- Identify any two points on the line
- Use the slope formula
Rate of Change

- A rate of change compares a change in one quantity to a change in another quantity.
- Slope is a rate of change comparing the change in vertical to horizontal.
- We can compare and graph real-world data in the same way as slope.

Example:
The table to the right shows the distance a person walks for exercise. Find the rate of change in distance with respect to time.

Rate of change = \frac{\text{change in distance}}{\text{change in time}} = \frac{4.5 - 1.5}{90 - 30} = \frac{3}{60} = \frac{1}{20} = 0.05 \text{ mi/min}

Example: Analyzing real-world data

A community theatre performed a play each Saturday evening for 10 consecutive weeks. The graph shows the attendance for the performances in weeks 1, 4, 6, and 10. Describe the rates of change in attendance with respect to time.

Weeks 1-4:
\[ m = \frac{232 - 124}{4 - 1} = \frac{108}{3} = 36 \text{ people/week} \]

Weeks 4-6:
\[ m = \frac{204 - 232}{6 - 4} = \frac{-28}{2} = -14 \text{ people/week} \]

Weeks 6-10:
\[ m = \frac{72 - 204}{10 - 6} = \frac{-132}{4} = -33 \text{ people/week} \]

Interpretation: During the first four weeks, we see an increase in attendance of 36 people per week. During weeks 4-6 we see a decrease in attendance of 14 people per week. During the final weeks we see an even greater decrease in attendance.

Example: Analyzing a graph.
The graph to the right shows a student commuting from home to school both by walking and taking the bus. Use the graph to describe the student’s commute in words.

- During the first part of the student’s commute, we see an increase in distance, but it is not very steep in relation to time. The student must have been walking.
- Then the rate of change stops – there is no increase in distance as time increases. The student is not moving, so perhaps is waiting for the bus.
- Then we see a sharp increase in distance in relation to time, so the student is probably riding the bus at that point.

You try: For a and b, find the slope of the line that passes through the given points.

a) (-4, -1) and (5, 9)  b) (-2, 5) and (-7, 8)  c) Find the rate of change in calories with respect to time: