Algebra 1 Laws of Exponents/Polynomials Test STUDY GUIDE

Know how to...

- Evaluate expressions with exponents using the laws of exponents:
  - \( a^m \cdot a^n = a^{m+n} \): Add exponents when multiplying powers with the same base.
    Example: \( x^3 \cdot x^4 = x^7 \)
  - \( \frac{a^m}{a^n} = a^{m-n} \): Subtract exponents when dividing powers with the same base.
    Example: \( \frac{x^9}{x^4} = x^5 \)
  - \((ab)^m = a^m b^m\): Multiply exponents when raising a power to a power.
    Example: \((x^4)^3 = x^{12}\)

- Evaluate an expression with a number raised to a 0 power... it is 1.
  - Example: \( x^0 = 1 \)

- Evaluate expressions with negative exponents: \( a^{-n} = \frac{1}{a^n} \) or \( a^n = \frac{1}{a^{-n}} \), \( a \neq 0 \)
  - Examples: \( 2^{-2} = \frac{1}{2^2}; \ x^5 = \frac{1}{x^{-5}}; \ x^2 y^{-3} z^4 = \frac{x^2 z^4}{y^3} \)

- Evaluate expressions using scientific notation: \( c \times 10^n \), where \( 1 \leq c < 10 \) and \( n \) is an integer
  - Examples: \( 1,200 = 1.2 \times 10^3; \ 0.0034 = 3.4 \times 10^{-3} \)

- Multiply/divide numbers in scientific notation by using the laws of exponents; you may need to put the number back into scientific notation!
  - Multiply or divide coefficients; multiply or divide exponents
  - Examples: \((4.1 \times 10^4)(2.3 \times 10^{-2}) = 9.43 \times 10^2; \ \frac{5.0 \times 10^8}{2.5 \times 10^5} = 2.0 \times 10^3\)

- Add and Subtract Polynomials
  - A monomial is a number, variable, or product of a number and one or more variables with a whole number coefficient. Since variables in a denominator represent negative coefficients in the numerator, monomials do not have variables in the denominator.
  - Example: \( 2x^2, 14x^2y^5 \) are monomials; \( \frac{4}{x}, \sqrt{x} \) are not
  - The degree of a monomial is the sum of the exponents of the variables in the monomial.
    - Example: the degree of \( 4x^2y \) is 3
    - The degree of a constant (such as 5) is 0
  - A polynomial is a monomial or the sum of monomials.
  - By convention, we write the terms of a polynomial in degree order, from greatest to least (standard form).
  - The degree of a polynomial is the greatest degree of its terms, that is, it takes the degree of the monomial with the largest degree in the polynomial.
Example: the degree of $5x^3y^4 + 6x^2 + 4xy + 6$ is 7
  - Add polynomials by combining like terms.
  - Subtract polynomials by distributing the negative sign, and then adding.

- Multiplying Polynomials
  - Multiply a monomial by a polynomial by distributing the monomial over all the terms in the polynomial.
  - Multiply polynomials by performing multiple distribution for each term in the first polynomial over every term in the second monomial.
  - The FOIL acronym (“First, Outside, Inside, Last”) is a way to remember how to multiply two binomials.

- Special Polynomial Products
  - Square of a polynomial pattern: $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$.
  - Sum and difference pattern: $(a + b)(a - b) = a^2 - b^2$

- Solving Polynomials
  - Make sure all terms are one side of the equation, and 0 is on the other.
  - Factor completely (see below for factoring summary).
  - Use the zero product property (if $ab = 0$, then $a = 0$ or $b = 0$) to solve.

- Factoring
  - Always apply Type I factoring (factor out GCF) before factoring any polynomial!!
  - Always multiply your answer back to a polynomial to verify!!
  - Type I Factoring – factor out GCF
    - Factor out the Greatest Common Factor (GCF) of the terms in the polynomial
    - Example: $4x^4 + 24x^3 = 4x^3(x + 6)$
  - Type II Factoring – notice special patterns
    - If polynomial follows special product pattern (as described in section 9.3), we can easily factor: $a^2 - b^2 = (a + b)(a - b)$ or $a^2 + 2ab + b^2 = (a + b)^2$
    - Example: $x^2 - 81 = (x + 9)(x - 9)$
  - Type III Factoring – Factor $x^2 + bx + c$
    - Draw two sets of parentheses, with the variable as the first term in each.
    - Find two factors of $c$ that add up to $b$ (when $c$ is positive) or subtract to $b$ (when $c$ is negative)
    - When $c$ is positive, the signs in the parentheses will be the same (the sign of $b$)
    - When $c$ is negative, the signs in the parentheses will be different (the bigger number takes the sign of $b$)
    - Example: $x^2 - x - 6 = (x - 3)(x + 2)$
  - Type IV Factoring – Factor by Grouping
    - Applies when a polynomial has 4 terms
    - Group the first two terms, and the second two terms; factor out the GCF for each set
    - Factor out the common polynomial
    - Example: $x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)$
  - Type V Factoring – Factor $ax^2 + bx + c$
There are several methods for factoring – here is the grouping method:

- Multiply ac, and find the factors of ac that add or subtract to b.
- Re-write the polynomial, splitting up b into the sum found above.
- Factor by grouping.

Example: Factor \(2x^2 - 7x + 3\)

- \(a \cdot c = 2 \cdot 3 = 6\); the factors of 6 that add up to 7 are 1 and 6
- Re-write: \(2x^2 - 6x - x + 3\)
- Group and factor: \((2x^2 - 6x) + (-x + 3) = 2x(x - 3) - (x - 3)\)
- Factor common binomial: \((x - 3)(2x - 1)\)

A polynomial is factored completely when each factor is prime, that is, each factor cannot be factored further. Always check each factor for further factoring (especially look for further Type I (GCF) or Type II (special product) factoring).

Study Questions

1) Evaluate the expressions, writing answers using positive exponents:

a) \(8^3 \cdot 8^{11}\)

b) \((13^3)^{10}\)

c) \((4 \cdot 12)^6\)

d) \(3^2 \cdot 3^4 \cdot 3^7\)

e) \(\frac{6^5}{6^2}\)

f) \(\frac{4^2 \times 4^5 \times 4^4}{4^3}\)

g) \(x^5 \cdot x^{12}\)

h) \(\frac{x^{11}}{x^4}\)

i) \(\frac{1}{x^4} \cdot x^{25}\)

j) \((-3x^2y^5)^2\)

k) \((-3y^5)^3 \cdot 2y^2\)

l) \(\left(\frac{b}{c}\right)^7\)

m) \(\left(-\frac{3^2}{w^8}\right)^4\)

n) \(8^6 \cdot \frac{1}{8^{13}}\)

o) \(\left(\frac{1}{5}\right)^7 \cdot 5^{17}\)

p) \(\left(\frac{m^7}{2n^{10}}\right)^6\)

q) \(\left(\frac{x^4y^{10}z^{-7}}{6x^{14}}\right)^0\)

r) \(2^{-3}\)

s) \(\frac{x^{-5}}{x^{-4}}\)

t) \((4x)^4 \cdot 4^{-3}\)

u) \(0^{-10}\)

v) \(\frac{1}{(5y)^{-3}}\)

w) \((7x^5y^{-4})^{-6}\)

x) \(\left(-\frac{x^4}{3}\right)^3\)

2) Re-write the numbers in scientific notation.

a) 48,100

b) 6,235,000

c) 0.05

d) 0.001429

3) Re-write the numbers in standard form.

a) \(4.06 \times 10^5\)

b) \(3.142 \times 10^3\)

c) \(4.5 \times 10^{-5}\)

d) \(6.7 \times 10^{-1}\)
4) Multiply or divide. Express results in scientific notation.

a) \((4.9 \times 10^4)(3.8 \times 10^6)\)  
b) \((7.3 \times 10^6)(4.2 \times 10^{-9})\)  
c) \(\frac{6.2 \times 10^6}{3.1 \times 10^5}\)  
d) \(\frac{2.272 \times 10^{-8}}{7.1 \times 10^{-5}}\)

5) Tell whether the following are monomials; if yes, what is the degree; if not, why not?

a) \(4x^2y^3z^4\)  
b) \(\frac{5x^{-1}}{y^2}\)  
c) \(5\sqrt{x}\)

6) What is the degree of the polynomial? \(5x^4y^2 + 4xy + 3x + 9\)

7) Add or subtract the polynomials:

a) \((5a^2 - 3) + (8a^2 - 1)\)  
b) \((4m^2 - m + 2) + (-3m^2 + 10m + 7)\)  
c) \((9b^3 - 13b^2 + b) - (-13b^2 - 5b + 14)\)

8) Multiply the polynomials:

a) \(x(2x^2 - 3x + 9)\)  
b) \(-5b^3(4b^5 - 2b^3 + b - 11)\)  
c) \((b - 2)(b^2 - b + 1)\)

d) \((y + 6)(y - 5)\)  
e) \((2x + 4)(2x - 4)\)  
f) \((2x + 3)^2\)

g) \((5x - 8)(2x - 5)\)  
h) \((7w + 5)(11w - 3)\)  
i) \((7a - 2)(3a - 4)\)

9) Divide the polynomials:

a) \(\frac{6x\sqrt{30x^3 - 12x^3 + 6x^2}}{3x - 7\sqrt{3x^2 - x - 14}}\)  
b) \(\frac{3x - 5\sqrt{6x^2 - 13x + 11}}{3x - 5}\)
10) Factor completely:
   a) \(2x + 2y\)  
   b) \(7w^5 - 35w^2\)  
   c) \(25x^2 - 100\)
   d) \(x^2 + 11x + 18\)  
   e) \(n^2 - 6n + 8\)  
   f) \(-y^2 - 2y + 15\)
   g) \(x^2 - 2x - 24\)  
   h) \(8x^2 - 10x + 3\)  
   i) \(x^4 - 1\)

11) Solve (find the roots):
   a) \((2x - 3)(x + 2) = 0\)  
   b) \(x^2 + 3x = 18\)  
   c) \(x^2 - 14x + 45 = 0\)
   d) \(x^2 - 10x + 25 = 0\)  
   e) \(2x^2 - 3x - 35 = 0\)  
   f) \(m^3 - 3m^2 = 4m - 12\)
   g) \(7a^2 + 2a = 0\)  
   h) \(6x^3 - 36x^2 + 30x = 0\)

12) Find the zeros of the functions.
   a) \(f(x) = 7x^2 + 2x - 5\)  
   b) \(f(x) = 5x^3 - 30x^2 + 40x\)  
   c) \(f(x) = 6x^2 - 5x - 14\)

13) The length of a rectangle is 7 inches more than 5 times its width. The area of the rectangle is 6 square inches. What is the width?
### STUDY QUESTION ANSWERS

1) a) 8\(^{14}\) b) 13\(^{30}\) c) 4\(^6\)\(\cdot\)12\(^6\) d) 3\(^{14}\)  e) 6\(^3\) f) 4\(^8\)  g) x\(^{17}\)  h) x\(^7\) i) x\(^{21}\)  
   j) 9x\(^4\)y\(^{10}\) k) -54y\(^{17}\)  l) \(\frac{b^7}{c^7}\) m) \(\frac{3^8}{w^{42}}\) n) \(\frac{1}{8^7}\)  o) 5\(^{10}\)  p) \(\frac{m^{42}}{64n^{60}}\)  
   q) 1  r) \(\frac{1}{8}\)  s) \(\frac{1}{x}\)  t) 4x\(^4\)  u) undefined  v) 125y\(^3\)  w) \(\frac{y^{24}}{7^6x^{30}}\)  x) \(-\frac{x^{12}}{27}\)  

2) a) 4.81 \times 10^4  b) 6.235 \times 10^6  c) 5.0 \times 10^{-2}  d) 1.429 \times 10^{-3} 

3) a) 406,000  b) 3,142  c) 0.000045  d) 0.67 

4) a) 1.862 \times 10^{11}  b) 3.066 \times 10^{-2}  c) 2.0 \times 10^3  d) 3.2 \times 10^{-4} 

5) a) yes; degree is 9  b) no; contains exponents that are not whole numbers  c) no; can’t have variables under the radical sign 

6) degree is 6 

7) a) 13a\(^2\) - 4  b) m\(^2\) + 9m + 9  c) 9b\(^3\) + 6b - 14 

8) a) 2x\(^3\) - 3x\(^2\) + 9x  b) -20b\(^8\) + 10b\(^6\) - 5b\(^4\) + 55b\(^3\)  c) b\(^3\) - 3b\(^2\) + 3b - 2  d) y\(^2\) + y - 30  e) 4x\(^2\) - 16  f) 4x\(^2\) + 12x + 9  g) 10x\(^2\) - 41x + 40  h) 77w\(^2\) + 34w - 15  i) 21a\(^2\) - 34a + 8 

9) a) 5x\(^3\) - 2x\(^2\) + x  b) x + 2  c) 2x - 1 + \(\frac{6}{3x+5}\) 

10) a) 2(x + y)  b) 7w\(^2\)(w\(^3\) - 5)  c) 25(x + 2)(x - 2)  d) (x + 9)(x + 2)  e) (n - 4)(n - 2)  f) -(y - 3)(y + 5)  g) (x - 6)(x + 4)  h) (2x - 1)(4x - 3)  i) (x\(^2\) + 1)(x + 1)(x - 1) 

11) a) \{-2, \frac{3}{2}\}  b) \{-6, 3\}  c) \{5, 9\}  d) \{5\}  e) \{-\frac{7}{2}, 5\}  f) \{-2, 2, 3\}  
   g) \{\frac{2}{7}, 0\}  h) \{0, 1, 5\} 

12) a) x = -1, \(\frac{5}{7}\)  b) x = 0, 2, 4  c) x = \(-\frac{7}{6}\), 2 

13) \(\frac{3}{5}\) in.