Algebra 1 Factoring Polynomials Test STUDY GUIDE

Know how to...

- Factoring
  - Always apply Type I factoring (factor out GCF) before factoring any polynomial!!
  - Always multiply your answer back to a polynomial to verify!!
  - Type I Factoring – factor out GCF
    - Factor out the Greatest Common Factor (GCF) of the terms in the polynomial
    - Example: \(4x^4 + 24x^3 = 4x^3(x + 6)\)
  - Type II Factoring – notice special patterns
    - If polynomial follows special product pattern (as described in section 9.3), we can easily factor: \(a^2 - b^2 = (a + b)(a - b)\) or \(a^2 + 2ab + b^2 = (a + b)^2\)
    - Example: \(x^2 - 81 = (x + 9)(x - 9)\)
  - Type III Factoring – Factor \(x^2 + bx + c\)
    - Draw two sets of parentheses, with the variable as the first term in each.
    - Find two factors of \(c\) that add up to \(b\) (when \(c\) is positive) or subtract to \(b\) (when \(c\) is negative)
    - When \(c\) is positive, the signs in the parentheses will be the same (the sign of \(b\))
    - When \(c\) is negative, the signs in the parentheses will be different (the bigger number takes the sign of \(b\))
    - Example: \(x^2 - x - 6 = (x - 3)(x + 2)\)
  - Type IV Factoring – Factor by Grouping
    - Applies when a polynomial has 4 terms
    - Group the first two terms, and the second two terms; factor out the GCF for each set
    - Factor out the common polynomial
    - Example: \(x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6) = x^2(x - 3) + 2(x - 3) = (x - 3)(x^2 + 2)\)
  - Type V Factoring – Factor \(ax^2 + bx + c\)
    - There are several methods for factoring – here is the grouping method:
      - Multiply \(ac\), and find the factors of \(ac\) that add or subtract to \(b\).
      - Re-write the polynomial, splitting up \(b\) into the sum found above.
      - Factor by grouping.
    - Example: Factor \(2x^2 - 7x + 3\)
      - \(a\cdot c = 2\cdot3 = 6\); the factors of 6 that add up to 7 are 1 and 6
      - Re-write: \(2x^2 - 6x - x + 3\)
      - Group and factor: \((2x^2 - 6x) + (-x + 3) = 2x(x - 3) - (x - 3)\)
      - Factor common binomial: \((x - 3)(2x - 1)\)

A polynomial is factored completely when each factor is prime, that is, each factor cannot be factored further. Always check each factor for further factoring (especially look for further Type I (GCF) or Type II (special product) factoring).

- Solving Polynomials
  - Make sure all terms are one side of the equation, and \(0\) is on the other.
  - Factor completely (see below for factoring summary).
  - Use the zero product property (if \(ab = 0\), then \(a = 0\) or \(b = 0\)) to solve.
1) Factor completely:
   a) \(2x + 2y\)  
   b) \(7w^5 - 35w^2\)  
   c) \(25x^2 - 100\)
   d) \(x^2 + 11x + 18\)  
   e) \(n^2 - 6n + 8\)  
   f) \(-y^2 - 2y + 15\)
   g) \(x^2 - 2x - 24\)  
   h) \(8x^2 - 10x + 3\)  
   i) \(x^4 - 1\)
   j) \(4v^2 - 32v + 28\)  
   k) \(-6k^2 + 30k\)  
   l) \(n^2 + 15n + 56\)
   m) \(a^2 - 8a + 16\)  
   n) \(16p^2 - 121\)  
   o) \(7r^3 - 42r^2 + 2r - 12\)

2) Solve (find the roots):
   a) \((2x - 3)(x + 2) = 0\)  
   b) \(x^2 + 3x = 18\)  
   c) \(x^2 - 14x + 45 = 0\)
   d) \(x^2 - 10x + 25 = 0\)  
   e) \(2x^2 - 3x - 35 = 0\)  
   f) \(m^3 - 3m^2 = 4m - 12\)
g) \( 7a^2 + 2a = 0 \)  \hspace{1cm} h) \( 6x^3 - 36x^2 + 30x = 0 \)  \hspace{1cm} i) x (x - 7) = 0

j) \( (8v - 7)(2v + 5) = 0 \)  \hspace{1cm} k) m^2 + 6 = -7m  \hspace{1cm} l) 9n^2 + 5 = -18n

m) \( 7x^2 - 19x = -10 \)  \hspace{1cm} n) 3b^2 = -4b + 15  \hspace{1cm} o) 5p^2 = -6p

3) Find the zeroes of the functions.

a) \( f(x) = 7x^2 + 2x - 5 \)  \hspace{1cm} b) \( g(x) = 5x^3 - 30x^2 + 40x \)  \hspace{1cm} c) \( h(x) = 6x^2 - 5x - 14 \)

d) \( m(x) = 3x^2 - 22x - 16 \)  \hspace{1cm} e) \( n(x) = 5x^2 - 43x + 56 \)  \hspace{1cm} f) \( p(x) = 3x^2 - 7x - 6 \)

4) The length of a rectangle is 7 inches more than 5 times its width. The area of the rectangle is 6 square inches. What is the width?

\[
\begin{array}{c}
\text{x} \\
5x + 7
\end{array}
\]
### STUDY QUESTION ANSWERS

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<thead>
<tr>
<th>1)</th>
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<tbody>
<tr>
<td>a)</td>
<td>$2(x + y)$</td>
<td>b) $7w^2(w^3 - 5)$</td>
<td>c) $25(x + 2)(x - 2)$</td>
</tr>
<tr>
<td>d)</td>
<td>$(x + 9)(x + 2)$</td>
<td>e) $(n - 4)(n - 2)$</td>
<td>f) $-(y - 3)(y + 5)$</td>
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<tr>
<td>g)</td>
<td>$(x - 6)(x + 4)$</td>
<td>h) $(2x - 1)(4x - 3)$</td>
<td>i) $(x^2 + 1)(x + 1)(x - 1)$</td>
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<td>j)</td>
<td>$4(v - 1)(v - 7)$</td>
<td>k) $-6k(k - 5)$</td>
<td>l) $(n + 7)(n + 8)$</td>
</tr>
<tr>
<td>m)</td>
<td>$(a - 4)^2$</td>
<td>n) $(4p + 11)(4p - 11)$</td>
<td>o) $(7r^2 + 2)(r - 6)$</td>
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<tr>
<td>a)</td>
<td>$\left{-2, \frac{3}{2}\right}$</td>
<td>b) ${-6, 3}$</td>
<td>c) ${5, 9}$</td>
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<td>d)</td>
<td>${5}$</td>
<td>e) ${-\frac{7}{2}, 5}$</td>
<td>f) ${-2, 2, 3}$</td>
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<td>g)</td>
<td>${0, -\frac{2}{7}}$</td>
<td>h) ${0, 1, 5}$</td>
<td>i) ${0, 7}$</td>
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<tr>
<td>j)</td>
<td>$\left{\frac{7}{8}, -\frac{5}{2}\right}$</td>
<td>k) ${-1, -6}$</td>
<td>l) $\left{-\frac{1}{3}, -\frac{5}{3}\right}$</td>
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<tr>
<td>m)</td>
<td>$\left{\frac{5}{7}, 2\right}$</td>
<td>n) $\left{\frac{5}{3}, -3\right}$</td>
<td>o) $\left{-\frac{6}{5}, 0\right}$</td>
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<tbody>
<tr>
<td>a)</td>
<td>$x = -1, \frac{5}{7}$</td>
<td>b) $x = 0, 2, 4$</td>
<td>c) $x = -\frac{7}{6}, 2$</td>
</tr>
<tr>
<td>d)</td>
<td>$x = -\frac{2}{3}, 8$</td>
<td>e) $x = 7, \frac{8}{5}$</td>
<td>f) $x = -\frac{2}{3}, 3$</td>
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| 4) | $\frac{3}{5}$ in. |   |   |