Chapter 1

Essentials of Geometry

1.1 Points, Lines, and Planes
1.2 Segments and Congruence
1.3 Midpoint and Distance Formulas
1.4 Measure and Classify Angles
1.5 Angle Pair Relationships
### 1.1-1.2 Geometry Essentials

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<th>Figure</th>
<th>Info</th>
<th>Naming</th>
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<tbody>
<tr>
<td>Point</td>
<td>No ________</td>
<td>Use single ________ letter.</td>
<td>A • G</td>
</tr>
<tr>
<td>Line</td>
<td>Goes forever in ______ _____________</td>
<td>Use ___ pts on line, ______ arrow line on top</td>
<td>A —&gt; B</td>
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<tr>
<td>Segment</td>
<td>_______ of line __ endpts</td>
<td>Use both ________ ______arrow line on top</td>
<td>A —&gt; B</td>
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<td>Ray</td>
<td>Goes forever in ______ _____________</td>
<td>Use ______ first &amp; another pt on ray ______ arrow to right</td>
<td>A —&gt; B</td>
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<td>Opposite Rays</td>
<td>Rays with ________ ______</td>
<td>Go in exactly ______ ______</td>
<td>A —&gt; B C</td>
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<tr>
<td>Plane</td>
<td>______ surface</td>
<td>Use 3 ________ pts on plane.</td>
<td>A • G Q</td>
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<td>Congruent Segments</td>
<td>Segments with ______ ______</td>
<td></td>
<td>A —&gt; B C</td>
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**Intersection:**
- The intersection of two lines is __________.
- The intersection of a line and a plane is __________.
- The intersection of two planes is __________.

**Use the diagram to decide whether the given statement is true or false.**

1. Points $H, I,$ and $G$ are collinear.
2. Points $H, I,$ and $J$ are coplanar.
3. $EG$ and $FG$ are opposite rays.
4. All points on $GI$ and $GF$ are coplanar.
5. The intersection of $EF$ and plane $JKH$ is $HI$.
6. The intersection of $EF, HI,$ and $JG$ is point $G$.
7. The intersection of plane $EGH$ and plane $JGI$ is point $G$.
8. The intersection of plane $EFI$ and plane $JKG$ is $HG$. 
Segment Addition Postulate: If _______ is between _______ and _______, then
_________ + ___________ = ___________.

Remember: “The__________ is equal to the____________________.”

Examples:

1. Use the diagram to find $BC$.

   1. \[ \begin{array}{c|c|c} & A & B & C \\ \hline & 5 & 15 \\ \end{array} \]
   2. \[ \begin{array}{c|c|c} & A & B & C \\ \hline & 4 & 16 \\ \end{array} \]
   3. \[ \begin{array}{c|c|c} & A & B & C \\ \hline & 18 & 30 \\ \end{array} \]

Example 1: In the diagram, points $P$, $Q$, $R$, and $S$ are collinear, $PS = 46$, $PR = 18$, and $PQ = QR$. Find the indicated length.
   a. $PQ$
   b. $QR$
   c. $QS$
   d. $RS$

Example 2: FINDING LENGTHS In the diagram, points $V$, $W$, $X$, $Y$, and $Z$ are collinear, $VZ = 52$, $XZ = 20$, and $WX = XY = YZ$. Find the indicated length.
   a. $WX$
   b. $VW$
   c. $WY$
   d. $VX$
   e. $WZ$
   f. $VY$

Examples: Together
   a. Find $VW$.

   \[ \begin{array}{c|c|c|c|c} & T & 3x & V & x + 8 & W \\ \hline & 32 \\ \end{array} \]

On Your Own
   b. Find $YZ$.

   \[ \begin{array}{c|c|c|c|c} & X & 2x + 3 & Y & 3x - 1 & Z \\ \hline & 27 \\ \end{array} \]

Example 5: Point $S$ is between $R$ and $T$ on $\overline{RT}$. Use the given information to write an equation in terms of $x$. Solve the equation. Then, find $RS$ and $ST$.

   a. $RS = 2x + 10$
   b. $RS = 3x - 16$
   \[ ST = x - 4 \]
   \[ ST = 4x - 8 \]
   \[ RT = 21 \]
   \[ RT = 2x + 36 \]
1.3 Midpoint and Distance

Midpoint: ____________________________________________

Segment bisector: _______________________________________

Ex: Point M is the midpoint of \( \overline{VW} \). Find the length of \( VM \).

\[ VM = \frac{4x - 1 + 3x + 3}{2} = \frac{7x + 2}{2} \]

PRACTICE: In the diagram, M is the midpoint of the segment. Find the indicated length.

a. Find \( LN \).

\[ LN = \frac{x + 9 + 4x}{2} = \frac{5x + 9}{2} \]

b. Find \( MR \).

\[ MR = \frac{4x - 12 + -2x + 21}{2} = \frac{2x + 9}{2} \]

Ex: Find the midpoint of the segments.

-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

\[ A \quad B \quad C \quad D \quad E \quad F \quad G \quad H \quad I \quad J \quad K \]

a. CE  

b. AK  

c. FI  

d. AI

Ex1: Given two endpoints can you find the midpoint?

The coordinates of a segment’s midpoint are

\( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \)

Here’s the math...

Find the coordinates of the midpoint of the segment with the given endpoints.

a. S (4, -1) and T (6, 0)

b. L (4, 2) and P (0, 2)

c. H (-5, 5) and K (7, 3)
Ex2: Given an endpoint and a midpoint, can you find the other endpoint?

Use the given endpoint \( R \) and midpoint \( M \) of \( \overline{RS} \) to find the coordinates of the other endpoints.

\[
a. \ R(6, \ 0), \ M(0, \ 2) \quad \quad \quad \quad \quad \quad \quad b. \ R(3, \ 4), \ M(3, \ -2)
\]

On Your Own:

\[
c. \ R(-3, \ -2), \ M(-1, \ -8) \quad \quad \quad \quad \quad \quad \quad d. \ R(11, \ -5), \ M(-4, \ -4)
\]

Ex3: Given two endpoints, can you find the distance?

Find the length of the segment. Leave answer in simplest radical form and round to the nearest tenth of a unit.

\[
a. \ J(-2, \ 4), \ K(1, \ 3)
\]

\[
b. \ S(4, \ 4), \ R(2, \ -1)
\]
Ex4: Given **two endpoints**, can you evaluate to see if they are **congruent**?

The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.

a. \( AB: A(2, 6), B(0, 3) \)
   \[ \overline{CD}: C(-1, 0), D(1, 3) \]

b. \( RS: R(5, 4), S(0, 4) \)
   \[ \overline{TU}: T(-4, -3), U(-1, 1) \]
1. **Angle** – figure formed by 2 __________ w/ same endpt.

2. Rays of the angle are called __________ of the angle; endpoint is called the ________________.

3. **Interior of angle** - _________ than 180°, **exterior of angle** - __________ than 180°

An angle is named 3 ways:

1) 3 capital letters, **vertex in middle**.
   \[ \angle \text{__________} \text{ or } \angle \text{__________} \]

2) # placed **inside the angle**.
   \[ \angle \text{__________} \]

3) Vertex...if there is only 1 \( \angle \) at that vertex.
   \[ \angle \text{__________} \]

**Adjacent** angles are 2 angles that

1. ______
2. ______
3. ______
4. ______

Are these angles adjacent?

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______
7. ______
8. ______
9. ______
10. ______

Definition: **Congruent angles** are angles that have the same ____________.

Reminders...

1. **Acute angle**- angle with measure between _____ and _____.
2. **Right angle**- angle with measure of _______.
3. **Obtuse angle**- angle with measure between_____ and_____.
4. **Straight angle**- angle with measure of ________.
Write three names for the angle shown. Then name the vertex and sides of the angle.

1. 

Classify the angle with the given measure as acute, obtuse, right, or straight.

4. \( m\angle A = 115^\circ \)  
5. \( m\angle A = 85^\circ \)  
6. \( m\angle A = 90^\circ \)  
7. \( m\angle A = 170^\circ \)

THREE BIG IDEAS (for angles):

1. Protractor Postulate - allows us to find the measure of an angle.

Ex. Let’s approximate by using fractions.

\[ \begin{align*} 
\text{m\angle ABC} &= \underline{} \\
\text{m\angle ABD} &= \underline{} \\
\text{m\angle ABE} &= \underline{} \\
\text{m\angle ABF} &= \underline{} 
\end{align*} \]

2. Angle Bisector Idea:

Ex. \( \overline{BT} \) bisects \( \angle ABC \). Find the value of \( x \).

Then find the measure of the angles to the nearest tenth.

On Your Own:

In the diagram, \( \overline{BD} \) bisects \( \angle ABC \). Find \( m\angle ABC \).
In the diagram, $\overline{BD}$ bisects $\angle ABC$. Find $x$ given the following:

A. $m\angle ABC = 98^\circ$  

B. $m\angle ABC = 9x + 87^\circ$

3. **Angle Addition Postulate:**

EX: $\angle ADC = 50^\circ$. What does $\angle ADB$ and $\angle BDC$ equal to the nearest tenth?

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**Find the indicated angle measure.**

18. $m\angle PRS = ?$  

19. $m\angle EFG = ?$  

20. $m\angle WXZ = ?$

---

**Use the given information to find the indicated angle measure.**

21. Given $m\angle ADC = 135^\circ$, find $m\angle BDC$.  

22. Given $m\angle NRQ = 78^\circ$, find $m\angle PRQ$.  

1.5 Angle Relationships

Vertical Angles:

Linear Pair:

Adjacent:

Determine if the angles are vertical angles, linear pair, or neither.

a) $\angle 1$ and $\angle 3$

b) $\angle 2$ and $\angle 3$

c) $\angle 4$ and $\angle 5$

d) $\angle 5$ and $\angle 8$

e) $\angle 4$ and $\angle 9$

f) $\angle 5$ and $\angle 6$

g) $\angle 1$ and $\angle 4$

Classify the angles as linear pair, vertical angles or neither.

a) $\angle 2$, $\angle 5$

b) $\angle 4$, $\angle 7$

c) $\angle 8$, $\angle 10$

d) $\angle 3$, $\angle 10$

Complementary Angles:

Supplementary Angles:

1) Find the complement of $27^\circ$.

2) Find the supplement of $115^\circ$.

3) Find the complement of $15^\circ$. 
4) The following angles are complementary. Find the measure of the angles to the nearest tenth.
   \[ m\angle A = 5x + 4, \quad m\angle B = 7x - 10 \]

5) The following angles are supplementary. Find the measure of the angles to the nearest tenth.
   \[ m\angle A = 11x + 2, \quad m\angle B = 8x + 7 \]

6) The measure of one angle is three times the measure of its complement. Find the measure of each angle to the nearest tenth.

7) Two angles form a linear pair. The measure of one angle is 8 times the measure of the other angle. Find the measure of each angle to the nearest tenth.

Find the value of each variable to the nearest tenth. Be sure to write an equation and circle the answer.

1. \[ (7x + 30)^\circ \] \[ (11x - 24)^\circ \]

2. \[ (25x + 8)^\circ \] \[ (9x + 2)^\circ \]

3. \[ (8x - 5)^\circ \] \[ (10x + 14)^\circ \] \[ (20y + 1)^\circ \]

4. \[ (10x + 3)^\circ \] \[ (16x - 18)^\circ \] \[ 12y^\circ \]