Example 1: Relate side length and angle measure

Mark the largest angle, longest side, smallest angle, and shortest side of the triangle shown at the right. What do you notice?

THEOREM 5.10
If one side of a triangle is longer than another side, then the angle opposite the longer side is ________ than the angle opposite the shorter side.

$AB > BC$, so $m \angle \underline{\underline{}} > m \angle \underline{\underline{}}$.

THEOREM 5.11
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is ______________ than the side opposite the smaller angle.

$m \angle A > m \angle C$, so ______ > ______.

Example 2: Find angle measures

Boating A long-tailed boat leaves a dock and travels 2500 feet to a cave, 5000 feet to a beach, then 6000 feet back to the dock as shown below. One of the angles in the path is about 55° and one is about 24°. What is the angle measure of the path made at the cave?

Checkpoint: Complete the following exercises.

1. List the sides of $\triangle PQR$ in order from shortest to longest.

2. Another boat makes a trip whose path has sides of 1.5 miles, 2 miles, and 2.5 miles long and angles of 90°, about 53°, and about 37°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the right.

THEOREM 5.12: TRIANGLE INEQUALITY THEOREM
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$\underline{\underline{}} + \underline{\underline{}} > AC$

$AC + \underline{\underline{}} > \underline{\underline{}}$

$\underline{\underline{}} + AC > \underline{\underline{}}$
Example 3:  \textit{Find possible side lengths}

A triangle has one side of length 14 and another of length 10. Describe the possible lengths of the third side.

\textbf{Checkpoint: Complete the following exercise.}

3. A triangle has one side of 23 meters and another of 17 meters. Describe the possible lengths of the third side.

\textbf{THEOREM 5.13: HINGE THEOREM}
If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is \underline{\text{______}} than the third side of the second.

\textbf{THEOREM 5.14: CONVERSE OF THE HINGE THEOREM}
If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is \underline{\text{______}} than the included angle of the second.

\textbf{Example 4: Use the Converse of the Hinge Theorem}

Given that $\overline{AD} \cong \overline{BC}$, how does $\angle 1$ compare to $\angle 2$?

\textbf{Example 5: Solve a multi-step problem}

\textbf{Travel} Car A leaves a mall, heads due north for 5 mi and then turns due west for 3 mi. Car B leaves the same mall, heads due south for 5 mi and then turns 80° toward east for 3 mi. Which car is farther from the mall?