Comparing & Ordering Fractions

To compare and order fractions:

- Find the least common denominator (LCD) of the fractions which is the least common multiple of the denominators.
- Rewrite each fraction as an equivalent fraction whose denominator is the LCD.
- Compare the numerators.

**EXAMPLE 1** Replace ● with <, >, or = to make $\frac{1}{3} \ ● \frac{5}{12}$ true.

- The LCM of 3 and 12 is 12. So, the LCD is 12.
- Rewrite each fraction with a denominator of 12.

\[
\begin{align*}
\frac{1}{3} &= \frac{\bullet}{12}, & \frac{5}{12} &= \frac{5}{12} \\
\frac{4}{12} &= \frac{4}{12}
\end{align*}
\]

- Now, compare. Since $4 < 5$, $\frac{4}{12} < \frac{5}{12}$. So $\frac{1}{3} < \frac{5}{12}$.

**You Try**

Replace each ● with <, >, or = to make a true sentence.

1. $\frac{5}{12} \ ● \frac{3}{8}$

2. $\frac{6}{8} \ ● \frac{3}{4}$

3. $\frac{2}{7} \ ● \frac{1}{6}$

4. $\frac{5}{7} \ ○ \frac{3}{9}$

5. $\frac{1}{3} \ ○ \frac{8}{9}$

6. $\frac{3}{5} \ ○ \frac{1}{2}$
Ordering Fractions in a Set

**Example 2** Order $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{3}{8}$ from least to greatest.

The LCD of the fractions is 24. So, rewrite each fraction with a denominator of 24.

\[
\begin{align*}
\frac{1}{6} & = \frac{4}{24}, \quad \frac{2}{3} = \frac{16}{24}, \\
\frac{1}{3} & = \frac{8}{24}, \quad \frac{1}{4} = \frac{6}{24}, \\
\frac{1}{4} & = \frac{3}{24}, \quad \frac{3}{8} = \frac{9}{24},
\end{align*}
\]

The order of the fractions from least to greatest is $\frac{1}{6}, \frac{1}{4}, \frac{3}{8}, \frac{2}{3}$.

**You Try**

Order the fractions from least to greatest.

7. $\frac{3}{4}, \frac{3}{8}, \frac{1}{2}, \frac{1}{4}$

8. $\frac{2}{3}, \frac{1}{6}, \frac{5}{18}, \frac{7}{9}$

9. $\frac{1}{2}, \frac{5}{6}, \frac{5}{8}, \frac{5}{12}$

Order the fractions from greatest to least.

10. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

11. $\frac{2}{3}, \frac{1}{6}, \frac{3}{8}, \frac{1}{12}$

12. $\frac{5}{7}, \frac{1}{4}, \frac{1}{2}, \frac{3}{14}$
Ordering Fractions on a Number Line

Order the following fractions on the number line below:

\[ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4} \]

A = _____  B = _____  C = _____  D = _____
Ordering Mixed Numbers on a Number Line

A = _____  B = _____  C = _____  D = _____

A = _____  B = _____  C = _____  D = _____

A = _____  B = _____  C = _____  D = _____

A = _____  B = _____  C = _____  D = _____

A = _____  B = _____  C = _____  D = _____

A = _____  B = _____  C = _____  D = _____
Replace each • with <, >, or = to make a true sentence.

1. \( \frac{2}{3} \), \( \frac{3}{4} \)
2. \( \frac{3}{8} \), \( \frac{6}{16} \)
3. \( \frac{5}{8} \), \( \frac{7}{12} \)
4. \( \frac{1}{2} \), \( \frac{6}{7} \)
5. \( \frac{3}{9} \), \( \frac{1}{3} \)
6. \( \frac{1}{6} \), \( \frac{9}{10} \)
7. \( \frac{5}{6} \), \( \frac{7}{8} \)
8. \( \frac{5}{8} \), \( \frac{5}{12} \)
9. \( \frac{4}{5} \), \( \frac{2}{3} \)

Order the fractions from least to greatest.

10. \( \frac{3}{4} \), \( \frac{2}{5} \), \( \frac{5}{8} \), \( \frac{1}{2} \)
11. \( \frac{1}{3} \), \( \frac{2}{7} \), \( \frac{3}{14} \), \( \frac{1}{6} \)
12. \( \frac{2}{3} \), \( \frac{4}{9} \), \( \frac{5}{6} \), \( \frac{7}{12} \)

Order the fractions from greatest to least.

13. \( \frac{4}{5} \), \( \frac{2}{3} \), \( \frac{13}{15} \), \( \frac{7}{9} \)
14. \( \frac{11}{12} \), \( \frac{5}{6} \), \( \frac{3}{4} \), \( \frac{9}{16} \)
15. \( \frac{7}{15} \), \( \frac{3}{5} \), \( \frac{5}{12} \), \( \frac{1}{2} \)

Order the following fractions on the number line below: \( \frac{1}{3} \), \( \frac{2}{3} \), \( \frac{11}{3} \), \( \frac{12}{3} \)
1. **SHOES**: Tanya is looking in her closet. If $\frac{1}{3}$ of her shoes are black and $\frac{2}{3}$ are brown, does she have more black shoes or more brown shoes? How do you know?

2. **BUDGET**: Daniel spends $\frac{3}{7}$ of his money on rent and $\frac{4}{9}$ of his money on food. Does he spend more money of food or on rent? How do you know?

3. **FOOD**: In a recent survey, $\frac{2}{5}$ of the people surveyed said their favorite food is pizza, $\frac{1}{4}$ said it is hot dogs, and $\frac{3}{10}$ said it was popcorn. Which food was favored by the greatest number of people? Explain your answer.

4. **OFFICE SUPPLIES**: A blue paper clip is $\frac{1}{6}$ inch wide. A silver paper clip is $\frac{3}{8}$ inch wide, and a red paper clip is $\frac{1}{3}$ inch wide. What color paper clip has the smallest width? How do you know?

5. **GUM**: A red gumball is $\frac{5}{8}$ inch across. A green gumball is $\frac{5}{9}$ inch across, and a blue gumball is $\frac{7}{9}$ inch across. List the gumballs in order from largest to smallest.
Adding Fractions & Mixed Numbers

Like Fractions are fractions that have the _________ denominator.

To add or subtract like fractions, add the numerators and write the result over the denominator.

Simplify if necessary.

Unlike Fractions do not have the same _________________________.

To add unlike fractions, rename the fractions with a common denominator.

Then add the numerator and write the result over the denominator.

Simplify.

Example 1

Add: $\frac{1}{8} + \frac{3}{8}$. Write in simplest form.

The fractions have the same denominator, so we are ready to add the numerators.

$\frac{1}{8} + \frac{3}{8} = \frac{4}{8} \rightarrow \frac{1}{2}$

Example 2

Add $\frac{2}{3} + \frac{1}{12}$. Write in simplest form.

The least common denominator of 3 and 12 is 12.

$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ Rename $\frac{2}{3}$ using the LCD.

$\frac{2}{3} \rightarrow \frac{8}{12}$

$+ \frac{1}{12} \rightarrow + \frac{1}{12}$

Add the numerators and simplify.

$\frac{9}{12}$ or $\frac{3}{4}$
Practice (2.1a)
Solve and Model

1. \( \frac{5}{8} + \frac{1}{8} \)
2. \( \frac{1}{2} + \frac{3}{4} \)
3. \( \frac{5}{9} + \frac{5}{6} \)
4. \( \frac{3}{10} + \frac{7}{12} \)
5. \( \frac{7}{15} + \frac{5}{6} \)
6. \( \frac{5}{6} + \frac{5}{12} \)
Adding Mixed Numbers

To add mixed numbers:

1. Rename the fractions to include a common denominator.
2. Add the fractions.
3. Then add the whole numbers.
4. Simply the fraction is necessary.

**EXAMPLE 1** Find $2 \frac{1}{3} + 4 \frac{1}{4}$.

Estimate $2 + 4 = 6$

The LCM of 3 and 4 is 12.

$\frac{2}{3} \times \frac{4}{4} \quad \rightarrow \quad \frac{2}{12}$

$\frac{1}{4} \times \frac{3}{3} \quad \rightarrow \quad + \frac{4}{12}$

$\frac{7}{12} \quad \rightarrow \quad + \frac{3}{12}$

$\frac{6}{12} \quad \rightarrow \quad + \frac{7}{12}$

$2 \frac{1}{3} + 4 \frac{1}{4} = 6 \frac{7}{12}$. Compared to the estimate, the answer is reasonable.

**You Try** Add the mixed numbers. Write in simplest form.

1. $\frac{4}{3} \frac{3}{4} + 1 \frac{3}{4}$
2. $\frac{5}{2}$
3. $\frac{4}{3} + 2 \frac{1}{4}$
Practice (2.1a)

1.
\[
\begin{align*}
2\frac{1}{4} & \quad + \quad 2\frac{3}{7} & \quad + \quad 6\frac{2}{3} \\
3\frac{3}{4} & \quad + \quad 4\frac{2}{7} & \quad + \quad 3\frac{4}{9}
\end{align*}
\]

2.
\[
\begin{align*}
2\frac{1}{8} & \quad + \quad 5\frac{7}{8} \\
1\frac{2}{3} & \quad + \quad 4\frac{8}{9} \\
6\frac{5}{6} & \quad + \quad 3\frac{3}{8}
\end{align*}
\]

3.
\[
\begin{align*}
9\frac{4}{5} & \quad + \quad 2\frac{2}{3} \\
4\frac{3}{5} & \quad + \quad 9\frac{1}{3} \\
8\frac{3}{4} & \quad + \quad 6\frac{2}{5}
\end{align*}
\]
1. $\frac{1}{6} + \frac{2}{3}$

2. $\frac{1}{3} + \frac{7}{12}$

3. $2 \frac{1}{4} + 1 \frac{5}{6}$

4. $3 \frac{2}{3} + 1 \frac{2}{3}$

5. $\frac{2}{3} + \frac{2}{9}$

6. $\frac{2}{5} + \frac{2}{15}$

7. $1 \frac{1}{2} + 1 \frac{1}{3}$

8. $3 \frac{2}{3} + 2 \frac{2}{3}$

9. $\frac{1}{5} + \frac{7}{15}$

10. $\frac{15}{16} + \frac{7}{8}$

11. $1 \frac{3}{4} + 1 \frac{1}{4}$

12. $2 \frac{1}{2} + 5 \frac{1}{2}$
MAPS For Exercises 1-2 use the drawing at the right that shows distances between major sites on the Avenue of the Americas in New York City.

Avenue of the Americas, New York City

1. Carla walked from the Empire State Building to the Museum of Modern Art. How far did she walk?

2. Dion walked from Central Park South to the Empire State Building. How far did he walk?

3. CONSTRUCTION Mr. Hayashi is repairing his sidewalk. He mixed \( \frac{5}{9} \) pound of cement with sand and water to make concrete. The next day he mixed \( \frac{7}{9} \) pound of cement with sand and water. How many pounds of cement altogether did Mr. Hayashi use?
Subtracting Fractions & Mixed Numbers

**Example 1**

Find the difference of $\frac{3}{4}$ and $\frac{1}{4}$.

Estimate $1 - 0 = 1$

$\frac{3}{4} - \frac{1}{4} = \frac{3 - 1}{4}$ Subtract the numerators.

$= \frac{2}{4}$ or $\frac{1}{2}$ Simplify.

Compared to the estimate, the answer is reasonable.

\[
\begin{align*}
\frac{11}{12} - \frac{9}{12} &= \frac{2}{12} \\
\frac{4}{9} - \frac{1}{9} &= \frac{3}{9} \\
\frac{9}{11} - \frac{7}{11} &= \frac{2}{11}
\end{align*}
\]

**Example 2**

Find $\frac{2}{3} - \frac{1}{4}$.

The LCD of $\frac{2}{3}$ and $\frac{1}{4}$ is 12.

Write the problem. Rename $\frac{2}{3}$ as $\frac{8}{12}$ and $\frac{1}{4}$ as $\frac{3}{12}$.

Subtract the fractions.

\[
\begin{align*}
\frac{2}{3} - \frac{1}{4} &= \frac{2}{3} \times \frac{4}{4} = \frac{8}{12} \\
\frac{1}{4} \times \frac{3}{3} &= \frac{3}{12} \\
\frac{8}{12} - \frac{3}{12} &= \frac{5}{12}
\end{align*}
\]

\[
\begin{align*}
\frac{2}{3} - \frac{1}{2} &= \frac{9}{10} - \frac{3}{5} \\
\frac{5}{6} - \frac{1}{12} &= \frac{5}{12}
\end{align*}
\]
Example 1

Find $6 \frac{1}{2} - 2 \frac{1}{3}$.

Estimate $6 \frac{1}{2} - 2 = 4 \frac{1}{2}$

The LCM of 2 and 3 is 6.

$\frac{6\frac{1}{2}}{2} \times \frac{3}{3} \quad \rightarrow \quad \frac{6\frac{3}{6}}{6} \quad \rightarrow \quad \frac{6\frac{3}{6}}{6} \quad \rightarrow \quad \frac{6\frac{3}{6}}{6}$

$-2 \frac{1}{3} \times \frac{2}{2} \quad \rightarrow \quad -\frac{2\frac{2}{6}}{6} \quad \rightarrow \quad -\frac{2\frac{2}{6}}{6} \quad \rightarrow \quad -\frac{2\frac{2}{6}}{6}$

$\frac{1}{6}$

$4 \frac{1}{6}$

$6 \frac{1}{2} - 2 \frac{1}{3} = 4 \frac{1}{6}$. Compared to the estimate, the answer is reasonable.

---

$3 \frac{2}{3} - 2 \frac{1}{3}$

$6 \frac{7}{8} - 3 \frac{1}{2}$

$5 \frac{1}{3} - 2 \frac{1}{4}$

$3 \frac{2}{3} - 1 \frac{1}{2}$

$10 \frac{3}{5} - 2 \frac{1}{2}$

$5 \frac{2}{3} - \frac{1}{6}$
Subtracting Fractions with Renaming

Sometimes it is necessary to rename the whole number or the fraction part of a mixed number as an improper fraction before you can subtract.

**EXAMPLE 1** Find $5 - 2\frac{1}{4}$.

Write the problem. Rename 5 as $4\frac{4}{4}$.

\[
\begin{array}{c}
5 \\
-2\frac{1}{4}
\end{array}
\rightarrow
\begin{array}{c}
4\frac{4}{4} \\
-2\frac{1}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{4}{4} \\
\frac{1}{4}
\end{array}

\text{Subtract.}

\[
\begin{array}{c}
\frac{4}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{1}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{3}{4}
\end{array}

\text{Then cross out } \frac{1}{2}.

So, $5 - 2\frac{1}{4} = 2\frac{3}{4}$.

**EXAMPLE 2** Find $6\frac{1}{2} - 2\frac{3}{4}$.

Write the problem. The LCM of 2 and 4 is 4. Since $\frac{3}{4}$ is greater than $\frac{2}{4}$, rename $6\frac{2}{4}$ as $5\frac{6}{4}$.

\[
\begin{array}{c}
6\frac{1}{2} \\
-2\frac{3}{4}
\end{array}
\rightarrow
\begin{array}{c}
6\frac{2}{4} \\
-2\frac{3}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{6}{4} \\
\frac{3}{4}
\end{array}

\text{Subtract.}

\[
\begin{array}{c}
\frac{6}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{3}{4}
\end{array}
\rightarrow
\begin{array}{c}
\frac{3}{4}
\end{array}

So, $6\frac{1}{2} - 2\frac{3}{4}$ is $3\frac{3}{4}$.

1. $6 - 1\frac{1}{3}$
2. $5 - 3\frac{2}{9}$
3. $6\frac{1}{3} - 2\frac{2}{3}$
4. $4\frac{1}{6} - 1\frac{1}{3}$
Practice (2.1b)

a. \[\frac{2}{3} - \frac{1}{6} = \frac{1}{2}\] \[\frac{4}{7} - \frac{1}{2} = \frac{1}{14}\] \[\frac{1}{6} - \frac{1}{12} = \frac{1}{12}\]

b. \[\frac{711}{12} - \frac{37}{12} = \frac{674}{12} = \frac{337}{6}\] \[\frac{58}{9} - \frac{31}{6} = \frac{174}{18} - \frac{93}{18} = \frac{81}{18} = \frac{27}{6} = \frac{9}{2}\] \[\frac{103}{5} - \frac{21}{2} = \frac{206}{10} - \frac{105}{10} = \frac{101}{10}\]

c. \[\frac{13}{2} - \frac{74}{5} = \frac{65}{10} - \frac{148}{10} = -\frac{83}{10}\] \[\frac{8}{3} - \frac{25}{9} = \frac{24}{9} - \frac{25}{9} = -\frac{1}{9}\] \[\frac{97}{10} - \frac{64}{5} = \frac{97}{10} - \frac{128}{10} = -\frac{31}{10}\]
Subtract. Write in simplest form.

a. \[ \frac{5}{8} - \frac{1}{4} \quad \frac{1}{2} - \frac{2}{5} \quad \frac{11}{12} - \frac{3}{4} \]

b. \[ \frac{4\frac{5}{6}}{6} - 3\frac{1}{6} \quad \frac{8\frac{7}{12}}{12} - 5\frac{5}{12} \quad \frac{8\frac{5}{12}}{12} - 1\frac{1}{12} \]

c. \[ 8\frac{1}{2} - 5\frac{3}{10} \quad 9\frac{3}{4} - 7\frac{1}{2} \quad 5\frac{2}{3} - \frac{1}{6} \]
1. \( \frac{4}{7} - \frac{6}{7} = -\frac{1}{7} \)

2. \( 4 - 3\frac{1}{3} = -\frac{2}{3} \)

3. \( 7\frac{1}{6} - 3\frac{5}{6} = -\frac{2}{3} \)

4. \( 6\frac{1}{5} - 2\frac{7}{10} = -\frac{5}{2} \)

5. \( 10\frac{5}{9} - 2\frac{2}{3} = -\frac{1}{3} \)

6. \( 11\frac{1}{4} - 5\frac{3}{8} = -\frac{5}{8} \)

7. \( 3\frac{1}{4} - 1\frac{5}{8} = -\frac{2}{8} \)

8. \( 15\frac{1}{3} - 6\frac{1}{2} = -\frac{1}{2} \)

9. \( 12\frac{2}{5} - 4\frac{3}{4} = -\frac{1}{4} \)