**Introduction to Arithmetic and Geometric Sequences & Series**

**Arithmetic Sequences and Series**

An arithmetic sequence is one whose consecutive terms have a common **difference**.

For example: 1, 5, 9, 13, 17. The common difference, \(d\), is **4**.

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**Formula for an Arithmetic Sequence**

Every Arithmetic Sequence has an \(n^\text{th}\) term given by the following formula: \(a_n = a_1 + (n-1)d\), where \(a_1\) is the first term, \(d\) is the common difference, and \(n\) represents the number of the term you are looking for.

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**Examples**

1. Find a formula for the \(n^\text{th}\) term of the arithmetic sequence whose first term is 2 and whose common difference is 3.

   \[ a_1 = 2 \quad d = 3 \]

   \[ a_n = a_1 + (n-1)d \]

   \[ a_n = 2 + (n-1)(3) \quad a_n = 2 + 3n - 3 \]

   \[ a_n = 3n - 1 \]

2. Find the formula for the \(n^\text{th}\) term of an arithmetic sequence if the common difference is 5 and the 2\(^\text{nd}\) term is 12. Find the 50\(^\text{th}\) term.

   \[ d = 5 \quad a_1, a_2 \]

   \[ a_2 = 12 \quad a_1, 12 \]

   \[ n = 50 \]  

   \[ a_n = a_1 + (n-1)d \]

   \[ a_{50} = 7 + (50-1)(5) \]

   \[ a_{50} = 7 + 49(5) \]

   \[ a_{50} = 7 + 245 \]

   \[ a_{50} = 252 \]

3. The number -28 is the \(\_\_\) \(\text{th}\) term of 7, 2, -3, ...

   \[ a_1 = 7 \quad d = -5 \]

   \[ a_n = a_1 + (n-1)d \]

   \[ a_n = 7 + (n-1)(-5) \]

   \[ a_n = 7 - 5n + 5 \]

   \[ a_n = -5n + 12 \]

   \[ -28 = -5n + 12 \]

   \[ -12 \]

   \[ -40 = -5n \]

   \[ 8 = n \]

   \[ 8^{\text{th}} \text{ term} \]
Geometric Sequences and Series
A geometric sequence is one whose consecutive terms have a common ratio.
For example, $1, 5, \frac{25}{2}, \frac{125}{2}, \frac{625}{2}$ has a common ratio, $r$, of $\frac{5}{2}$.
\[
r = \frac{5}{2} \quad \frac{5}{2} \cdot \frac{5}{2} = \frac{25}{2} \quad \frac{25}{2} \cdot \frac{5}{2} = \frac{125}{2} \quad \frac{125}{2} \cdot \frac{5}{2} = \frac{625}{2}
\]

Formula for an Geometric Sequence
Every geometric sequence has an $n^{th}$ term given by the following formula: $a_n = a_1 r^{n-1}$, where $a_1$ is the first term, $n$ is the number of terms, and $r$ is the common ratio of the sequence.

Finding a term of a geometric sequence.
Examples –

4. Find the 20$^{th}$ term of the geometric sequence given $a_1 = 4$ and $r = 2$.
\[
a_1 = a_1 
\]
\[
a_{20} = a_1 r^{n-1} = 4 (2)^{20-1} = 4 (2)^{19}
\]
\[
= 2,097,152
\]

5. Find the 11$^{th}$ term of the geometric sequence whose first 3 terms are 5, 15, and 45.
\[
a_1 = a_1 
\]
\[
a_n = a_1 r^{n-1} = 5 (3)^{11-1} = 5 (3)^{10}
\]
\[
= 2,95,245
\]

8. One term of a geometric sequence is $a_3 = 5$. The common ratio is 2. Write a rule for the $n^{th}$ term.
\[
a_1 \quad \frac{a_1}{a_2} \quad \frac{a_2}{a_3} = \frac{5}{2} \quad \frac{5}{2} \quad \frac{1}{2}
\]
\[
a_1 = 5 
\]
\[
a_n = \frac{5}{2} (2)^{n-1}
\]