Objective: Today you will learn how to find the inverse of a function and verify if the two functions are inverses of one another.

Graph the function $f(x) = (x + 2)^2$ on the set of axes below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

How could you graphically test to determine if $f(x)$ is a function?

In order to determine if $f(x)$ has an inverse function we can use the

Extension: Let’s restrict the domain on the function $f(x)$ where $x \geq -2$.

a) Complete the table of values for both $f(x)$ and $f^{-1}(x)$. Then graph both relations.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f^{-1}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Give a reason why you think restricting the domain is beneficial to the idea of inverse functions.
Inverse Functions

- An inverse relation interchanges the _____ and _____ values of the original relation.
- The notation for the inverse of a function $f(x)$ is written as $f^{-1}(x)$.

Example #1 – Graph the inverse for the function shown below. Then state if the inverse relation is a function.

Is $f^{-1}(x)$ a function? **YES** or **NO**

Example #2 – Find the inverse for each of the following. Be sure to use proper notation.

a) $f(x) = -3x + 1$  

b) $g(x) = -7x^5 + 7$  

c) $h(x) = 7x^{1/5}$

In general the steps to follow when finding an inverse function are:

Switch the variables, then solve for $y$ and finally rewrite the inverse using the correct notation.

Example #3 – Given $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x + 2$. Find $f(g(x))$ and $g(f(x))$.

What do you notice about $f(g(x))$ and $g(f(x))$? ________________________________
**Definition** – Functions \( f \) and \( g \) are inverses function of each other provided that the following are true:

\[
f(g(x)) = x \quad \text{and} \quad g(f(x)) = x
\]

**Example #4** – Verify that the functions \( f(x) \) and \( g(x) \) are inverses.

\[f(x) = \frac{1}{4}x + 4; \quad g(x) = 4x - 16\]

**Example #5** – Verify that the functions \( f(x) \) and \( g(x) \) are inverses where \( x \geq -3 \).

\[f(x) = \sqrt{x+3} - 5; \quad g(x) = (x+5)^2 - 3\]
1. Graph the inverse for the function shown below. Then state if the inverse relation is a function.

Is \( f^{-1}(x) \) a function? **YES or NO**

2. Find the inverse for each of the following functions.
   
   a) \( f(x) = 9x - 6 \)  
   
   b) \( f(x) = (4x - 8)^2 \) where \( x \leq 2 \)

   c) \( y = -\frac{3}{5}x + \frac{7}{5} \)

   d) \( g(x) = \frac{16}{25}x^2 \) where \( x \leq 0 \)

5. Use compositions to determine if \( f(x) \) and \( g(x) \) are inverses.

   a) \( f(x) = \frac{1}{2}x^2 + 3; \ g(x) = \sqrt{2x - 6} \) where \( x \geq 3 \)  
   
   b) \( f(x) = \frac{x^3 + 3}{4}; \ g(x) = \sqrt[3]{4x - 3} \)
6. In order for \( f(x) \) and \( g(x) \) to be inverses, what must be true about their composition?

7. If the relations \( f(x) \) and \( g(x) \) are inverses, what is the relationship between the graphs?

8. Decide whether the functions are inverses of each other.
   
a) \( f(x) = 3x - 4 \), \( g(x) = \frac{x+3}{4} \)   
b) \( f(x) = 5x + 25 \), \( g(x) = \frac{1}{5}x - 5 \)

9. Which of the following represents the inverse of the function \( f(x) = 8x + 7 \)?

   (A) \( f^{-1}(x) = \frac{x-7}{8} \)  
   (B) \( f^{-1}(x) = \frac{x}{8} + 7 \)  
   (C) \( f^{-1}(x) = \frac{x}{8} - 7 \)  
   (D) \( f^{-1}(x) = \frac{x+7}{8} \)

10. Which of the following represents the inverse of the function \( f(x) = x^2 - 1 \) where \( x \geq 0 \)?

    (A) \( f^{-1}(x) = \sqrt{x - 1} \)  
    (B) \( f^{-1}(x) = \sqrt{x + 1} \)  
    (C) \( f^{-1}(x) = \sqrt{x} + 1 \)  
    (D) \( f^{-1}(x) = \sqrt{x} - 1 \)

11. Find the inverse of the graph shown below.

   ![Graph Image]
Composition Practice

1. Given the functions \( f(x) = x^2 - 8 \) and \( g(x) = -2 - x^2 \), determine the equation for the combined function \( y = f(x) + g(x) \).
   (A) \(-6 - 2x^2\)
   (B) \(-10\)
   (C) \(16 - x^2\)
   (D) \(4 - x\)

2. Given the functions \( f(x) = x - 3 \) and \( g(x) = \frac{1}{x+3} \), determine the simplified form of \( g(f(x)) \).
   (A) \(-\frac{8}{x}, x \neq 0\)
   (B) \(\frac{3x-8}{x-3}, x \neq 3\)
   (C) \(\frac{3x-8}{x+3}, x \neq -3\)
   (D) \(\frac{1}{x}, x \neq 0\)

3. Given the functions \( f(x) = x^2 + 3 \) and \( g(x) = x - 2 \), determine \( f(g(-2)) \).
   (A) \(-4\)          (B) \(7\)          (C) \(19\)          (D) \(-13\)

4. Given the functions \( f(x) = x^2 - 8 \) and \( g(x) = -5 - x \), determine an equation for the combined function \( h(x) = f(g(x)) \).
   (A) \(y = x^2 + 10x + 17\)
   (B) \(y = -x^2 - 13\)
   (C) \(y = -x^2 + 3\)
   (D) \(y = x^2 - 10x + 3\)

5. Given the function \( f(x) = x^2 - 5 \), determine the value of \( f(f(-1)) \).
   (A) \(-8\)          (B) \(16\)          (C) \(11\)          (D) \(-4\)

6. Given the functions \( f(x) = 7x + 9 \) and \( g(x) = 4x - 1 \) find \( f(g(x)) \).
   (A) \(28x + 8\)
   (B) \(28x + 16\)
   (C) \(28x + 2\)
   (D) \(28x + 35\)

7. Given the functions \( f(x) = x^2 - 1 \) and \( g(x) = x^2 + 3 \) find \( f(g(x)) \).
   (A) \(x^4 - 2x^2 + 4\)
   (B) \(x^4 + 4\)
   (C) \(x^4 + 6x^2 + 8\)
   (D) \(x^4 + 8\)