2019 Algebra 2 Summer Assignment

Please look over the following review topics and complete the exercises. These topics will help you begin your study of Algebra 2. They are review topics and will not be covered in length during the start of the school year. You should also look into buying a graphing calculator for this course. It is suggested that you purchase a TI-84. These can be bought for around $90 during back to school sales.

The following packet contains topics and definitions that you will be required to know in order to succeed in Algebra 2 this year. You are advised to be familiar with each of the concepts and to complete the included problems by Thursday, September 6, 2018. All of these topics were discussed in either Algebra I or Geometry and will be used frequently throughout the year. All problems are expected to be completed.

Functions

**Function:** A function is a relation in which each element of the domain is paired with exactly one element of the range.

**Vertical line test:** If you draw a vertical line through any graph and it only crosses the graph once, then the relation is a function.

Equations

A linear function is a function where the highest power of x is 1. You have seen these functions in many forms. Some of the common forms are \( y = mx + b \) (slope-intercept form) and \( Ax + By = C \) (standard form). Notice in both forms the exponent on x is 1.

Every linear function has an x and y intercept.

**x - Intercept:** Where a function crosses the x - axis.  
**y - Intercept:** Where a function crosses the y - axis.

Slope is a key concept to consider when thinking of linear functions. Slope is the “m” in the \( y = mx + b \) and is defined to be \(-A/B\) for standard form of a line. Here are some definitions of slope.

\[
slope = m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Positive slopes increase from left to right. Negative slopes decrease from left to right.
Section 1: Linear Equations

1. Graph each equation.

a) \( y = 3x - 2 \)

b) \( y + 3 = \frac{-3}{4}x \)  
\( y = \frac{-3}{4}x - 3 \)

c) \( 5x - 4y = 20 \)  
\( 4y = -5x + 20 \)  
\( y = \frac{5}{4}x - 5 \)

d) \( y - 2x = -5 \)  
\( y = 2x - 5 \)

2. Find the slope of the line through the following points. \( slope = m = \frac{y_2 - y_1}{x_2 - x_1} \)

a) (-2, 5) and (8, -3)  
\( m = \frac{-3 - 5}{8 - (-2)} \)  
\( m = \frac{-8}{10} \)  
\( m = -\frac{4}{5} \)

b) (2, 5) and (2, -7)  
\( m = \frac{-7 - 5}{2 - 2} \)  
\( m = \frac{-12}{0} \)  
\( m = undefined \)

c) (4, -3) and (-2, -3)  
\( m = \frac{-3 - (-3)}{-2 - 4} \)  
\( m = \frac{0}{-6} \)  
\( m = 0 \)
3. Find the slope of the line given the following equations
   a) \( y = -2x - 7 \)
   \[ m = -2 \]
   b) \( y - 3x = -2 \)
   \[ y = 3x - 2 \]
   \[ m = 3 \]
   c) \( x = -4 \) \( \text{Vox} \)
   \[ m = \text{undefined} \]
   d) \( y = 9 \) \( \text{Hoy} \)
   \[ m = 0 \]
   e) \( 4x + 2y = -6 \)
   \[ 2y = -4x - 6 \]
   \[ y = -2x - 3 \]
   \[ m = -2 \]
   f) \( 5x - y = -8 \)
   \[ y = 5x + 8 \]
   \[ m = 5 \]

4. Write the equation of the line given the following. Write the equation in slope-intercept form.
   a) slope = \( \frac{2}{5} \) through the point (-1, -6)
   \[
   y - 6 = \frac{2}{5}(x + 1) \\
   y + 6 = \frac{2}{5}x + \frac{2}{5} \\
   y = \frac{2}{5}x + \frac{2}{5} - \frac{30}{5} \\
   y = \frac{2}{5}x - \frac{28}{5}
   \]
   b) Through the points (-2, 5) and (5, 8)
   \[
   m = \frac{8 - 5}{5 + 2} = \frac{3}{7} \\
   y - 8 = \frac{3}{7} (x - 5) \\
   y = \frac{3}{7}x - \frac{15}{7} + \frac{56}{7} \\
   y = \frac{3}{7}x + \frac{41}{7}
   \]

Section 2: Simplifying Algebraic Expressions
The difference between an expression and an equation is that an expression doesn’t have an equal sign. Expressions can only be simplified, not solved. Simplifying an expression often involves combining like terms. Terms are like if and only if they have the same variable and power or if they are constants. Simplifying expressions also refers to substituting values to get a resultant value of the expression.

5. Simplify the following expressions by combining like terms.

<table>
<thead>
<tr>
<th>a) ( 3 + 2y^2 - 7 - 5x - 4y^3 + 6x )</th>
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</thead>
<tbody>
<tr>
<td>(-4y^3 + 2y^2 + x - y)</td>
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<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>b) ( x^2 + x^2 + x + x )</td>
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<tr>
<td>------------------------------------------</td>
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<tr>
<td>( 2x^2 + 2x )</td>
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<td>------------------------------------------</td>
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<tr>
<td>c) ( 4(3x - 2x^3 + 5) - 6x )</td>
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<td>------------------------------------------</td>
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<tr>
<td>((12x - 8x^3 + 20) - 6x )</td>
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<tr>
<td>(-8x^3 + 6x + 20 )</td>
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<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>d) ( x(2x - 3x^4 + 2y - 5xy) )</td>
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<tr>
<td>------------------------------------------</td>
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<tr>
<td>( 2x^2 - 3x^5 + 2xy - 5x^2y )</td>
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<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>e) ( x^3(5x^2 - 3y + 2 - 5x^5y) )</td>
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<td>------------------------------------------</td>
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<tr>
<td>( 5x^5 - 3x^3y + 2x^3 - 5x^8y )</td>
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<td>------------------------------------------</td>
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<tr>
<td>f) ( 8a - (7b - 4a) - 3(4a + 2b) )</td>
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<tr>
<td>------------------------------------------</td>
</tr>
<tr>
<td>( 8a - 7b + 4a - 12a - 6b )</td>
</tr>
<tr>
<td>(-13b )</td>
</tr>
</tbody>
</table>
6. Evaluate the following expressions by substituting the given values for the variables.

\[ a) \quad 3a + 2b - 6a + 5b - 8b; \quad a = -2 \text{ and } b = 6 \]
\[-6 + 12 + 12 + 30 - 48 \quad 54 - 54 \quad 0\]

\[ b) \quad 3(4x - 2y) + y(2xy + 7); \quad x = -2 \text{ and } y = 3 \]
\[ 12x - 6y + 2xy^2 + 7y \]
\[ 12(-2) - 6(3) + 2(-2)(3)^2 + 7(3) \]
\[ -24 - 18 - 36 + 21 \]
\[-42 + 15 \]
\[-57\]

**Section 3: Solving Equations**

7. Solve each equation and check your answer.

\[ a) \quad 3x + 4 = -2 \]
\[-4 - 4 \]
\[ \frac{3x}{3} = -6 \]
\[ x = -2 \]

\[ b) \quad 3(4 - 3t) = -2 \]
\[ 12 - 9t = -2 \]
\[-9t = -14 \]
\[ t = \frac{14}{9} \]

\[ c) \quad 5 - 2(3t + 4) = -1 \]
\[ 5 - 6t - 8 = -1 \]
\[-6t = -3 = -1 \]
\[ t = \frac{2}{3} \]
\[ t = \frac{2}{3} \]
\[ \frac{1}{3} \]

\[ d) \quad 2(3x - 4) = -3(x - 8) \]
\[ 6x - 8 = -3x + 24 \]
\[ 9x = 32 \]
\[ x = \frac{32}{9} \]

\[ e) \quad \frac{2}{3}s - 5 = 4 \]
\[ 3 \cdot \frac{2}{3}s = 9 \cdot 3 \]
\[ 2s = 27 \]
\[ s = \frac{27}{2} \]

\[ f) \quad \frac{1}{3}(2x - 1) = \frac{3}{4}(x + 2) \]
\[ 12 \left(\frac{2}{3}x - \frac{1}{3}\right) = \frac{9}{4}x + \frac{3}{2} \]
\[ 8x - 4 = 9x + 18 \]
\[-22 = x \]

8. Solve each formula by isolating the indicated variable.

\[ a) \quad A = bh, \text{ solve for } h \]
\[ (\text{Hint: Get "}h\text{" by itself}) \]
\[ \frac{A}{b} = h \]
\[ h = \frac{A}{b} \]

\[ b) \quad V = \pi r^2 h; \text{ solve for } h \]
\[ (\text{Hint: Get "}h\text{" by itself}) \]
\[ \frac{V}{\pi r^2} = h \]
\[ h = \frac{V}{\pi r^2} \]

\[ c) \quad s = \frac{1}{2}gt^2; \text{ solve for } g \]
\[ (\text{Hint: Get "}g\text{" by itself}) \]
\[ 2s = \left(\frac{1}{2} \cdot g \cdot t^2\right) \]
\[ \frac{2s}{t^2} = g \]
\[ g = \frac{2s}{t^2} \]
Section 4: Multiplying Binomials

\[(a + b)(c + d) = \frac{ac + ad + bc + bd}{\text{first outside inside last}}\]

\[(x + 3)(x + 5) = x \cdot x + x \cdot 5 + 3 \cdot x + 3 \cdot 5\]
\[= x^2 + 5x + 3x + 15\]
\[= x^2 + 8x + 15\]

9. Expand each of the following expressions.

a) \((x + 3)(2x - 5)\)
\[2x^2 - 5x + 6x - 15\]
\[= 2x^2 + x - 15\]

b) \((x - 11)(x + 5)\)
\[x^2 + 5x - 11x - 55\]
\[= x^2 - 6x - 55\]

c) \((3y^2 - 3y + 7)(4y^2 + 6y + 5)\)
\[12y^4 + 18y^3 + 15y^2 - 12y^3 - 18y^2 - 15y\]
\[+ 28y^2 + 42y + 35\]
\[= 12y^4 + 6y^3 + 25y^2 + 27y + 35\]

Section 5: Fractions

To multiply fractions:

- Multiply the numerators of the fractions.
- Multiply the denominators of the fractions.
- Place the product of the numerators over the product of the denominators.
- Simplify the fraction.

**Example:**
\[
\frac{3}{4} \times \frac{5}{6} = \frac{15}{24} = \frac{5}{4}
\]

To divide fractions:

- Invert (i.e. turn over) the denominator fraction and multiply the fractions as above.

**Example:**
\[
\frac{2}{9} \div \frac{3}{12} = \frac{2}{9} \times \frac{12}{3} = \frac{24}{27} = \frac{8}{9}
\]

To add or subtract:

- Find a common denominator, if needed.
- Then add or subtract the numerators.
- The denominator stays the same.

**Example:**
\[
\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12} = \frac{19}{12}
\]

10. Simplify each of the following expressions.

a) \(\frac{12 \times 3}{4} = 9\)

b) \(\frac{1}{2} \times \frac{20}{4} = \frac{2}{4} = \frac{1}{2}\)

c) \(\frac{2 \times 21}{3} = \frac{3}{5} = \frac{3}{5}\)

d) \(\frac{20}{3} + 4\)
\[= \frac{20}{3} + \frac{12}{3} = \frac{32}{3}\]

\[= \frac{5}{3}\]

e) \(\frac{1}{10} + \frac{3}{5}\)
\[= \frac{1}{10} + \frac{6}{10} = \frac{7}{10}\]

e) \(\frac{1}{10} + \frac{3}{5}\)
\[= \frac{2}{20} + \frac{8}{20} = \frac{10}{20} = \frac{1}{2}\]

f) \(\frac{2}{5} \div \frac{8}{10}\)
\[= \frac{2}{5} \times \frac{10}{8} = \frac{2}{4} = \frac{1}{2}\]

i) \(\frac{2}{3} - \frac{3}{7}\)
\[= \frac{14}{21} - \frac{9}{21} = \frac{5}{21}\]

h) \(\frac{1}{2} + \frac{2}{3}\)
\[= \frac{3}{6} + \frac{4}{6} = \frac{7}{6}\]

i) \(\frac{14}{21} - \frac{9}{21} = \frac{5}{21}\)
Section 6: LAWS OF EXPONENTS

The following laws of exponents are for multiplying and dividing monomials.

**PRODUCT RULE:** \(a^m \cdot a^n = a^{m+n}\) (when multiplying like bases, add the powers)

**Examples:**
1) \(x^4 \cdot x^5 = x^{4+5} = x^9\)
2) \(5^5 \cdot 5^8 = 5^{5+8} = 5^{13}\)

**POWER RULE:** \((a^m b^n)^n = a^{mn} b^{mn}\) (when taking a monomial to a power, multiply the powers including the coefficient)

**Examples:**
1) \((a^3b^2)^2 = a^{3\cdot2}b^{2\cdot2} = a^6b^4\)
2) \((3m^3n^5)^4 = 3^4m^{3\cdot4}n^{5\cdot4} = 81m^{12}n^{20}\)

**QUOTIENT RULE:** \(\frac{a^m}{a^n} = a^{m-n}\) (when dividing with like bases, subtract the powers)

(Note: It is always the numerator's power minus the denominator's power)

**Examples:**
1) \(\frac{x^6}{x^4} = x^{6-4} = x^2\)
2) \(\frac{m^7n^7}{m^4n^{10}} = m^{7-4}n^{7-10} = \frac{m^3}{n^3}\)

**ZERO POWER RULE:** \(a^0 = 1\) (any term to the zero power is one)

**Examples:**
1) \((m^3n^7)^0 = 1\)
2) \((4m^8n^2)(-2mn4)^0 = (4m^8n^2)(1) = 4m^8n^2\)

**NEGATIVE POWER RULE:** \(a^{-n} = \frac{1}{a^n}\) and \(\frac{1}{a^{-n}} = a^n\) (take the reciprocal of the variable to the negative power)

**Example:** \(3x^{-4} = \frac{3}{x^4}\)

**NOTE:** It is improper to leave negative powers in your final answer. All final answers should be written with positive powers. Therefore, you will need to follow the Negative Power Rule property.

11. Simplify each of the following expressions. Write your answers using positive exponents.

a) \(15^{-4}(15^9)\)

b) \(a^7(a^3)(a)\)

c) \((3m^4n^6)(2mn^2)(3m^2n)\)

d) \(-28a^6b^3c^5\)

\[
\frac{(3m^4n^6)(2mn^2)(3m^2n)}{7a^{11}b^{5}c^5}
\]

\[\frac{-4}{a^5b^2}\]

e) \((-x^3y^6)^{10}\)

f) \((5m^3n)(-2mn^3)\)

\[(-10m^4n^4)\]

g) \((7ab)(-2a^4b^3)(2a^3b^6)^{-1}\)

\[
\frac{(7ab)(-2a^{12}b^9)}{(2a^5b^6)} \cdot \frac{1}{2a^5b^6}
\]

\[\frac{-28a^8b^4}{2a^5b^6} = -28a^3b^4\]
Section 7: Simplifying Radicals

Perfect Squares:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^2$</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
</tr>
</tbody>
</table>

$\sqrt{25}$ is read "the square root of 25" $\sqrt{25} = 5$ because $5^2 = 25$

Examples:

$\sqrt{36}$

\[
\begin{align*}
\sqrt{36} &= \sqrt{9 \cdot 4} \\
6 &= 3 \sqrt{4}
\end{align*}
\]

$\sqrt{27}$

\[
\begin{align*}
\sqrt{27} &= \sqrt{9 \cdot 3} \\
3 \sqrt{3} &= \sqrt{25} \cdot \sqrt{2}
\end{align*}
\]

**Simplifying Square Roots**

12. Simplify each of the following expressions.

a) $\sqrt{4}$

\[
\begin{align*}
\sqrt{4} &= \sqrt{2 \cdot 2} \\
2 &= 2
\end{align*}
\]

b) $\sqrt{16}$

\[
\begin{align*}
\sqrt{16} &= \sqrt{4 \cdot 4} \\
4 &= 4
\end{align*}
\]

c) $\sqrt{100}$

\[
\begin{align*}
\sqrt{100} &= \sqrt{10 \cdot 10} \\
10 &= 10
\end{align*}
\]

d) $\sqrt{18}$

\[
\begin{align*}
\sqrt{18} &= \sqrt{9 \cdot 2} \\
3 \sqrt{2} &= \sqrt{9} \cdot \sqrt{2}
\end{align*}
\]

e) $\sqrt{125}$

\[
\begin{align*}
\sqrt{125} &= \sqrt{25 \cdot 5} \\
5 \sqrt{5} &= 5 \sqrt{5}
\end{align*}
\]

f) $\sqrt{180x^3y^6}$

\[
\begin{align*}
\sqrt{180x^3y^6} &= \sqrt{36 \cdot 5 \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y} \\
6xy \sqrt{5x} &= 6x \cdot y \cdot \sqrt{5x}
\end{align*}
\]

g) $\sqrt{72a^5b^3}$

\[
\begin{align*}
\sqrt{72a^5b^3} &= \sqrt{36 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b} \\
6a^2 \cdot b \sqrt{a \cdot b} &= 6a^2 \cdot b \sqrt{a \cdot b}
\end{align*}
\]

h) $\sqrt{98}$

\[
\begin{align*}
\sqrt{98} &= \sqrt{49 \cdot 2} \\
7 \sqrt{2} &= 7 \sqrt{2}
\end{align*}
\]

Perfect cubes:

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3$</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>216</td>
<td>342</td>
<td>512</td>
<td>729</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Simplifying Cube Roots**

13. Simplify each of the following expressions.

a) $\sqrt[3]{128}$

\[
\begin{align*}
\sqrt[3]{128} &= \sqrt[3]{64 \cdot 2} \\
4 \cdot 2 &= 4 \cdot 2
\end{align*}
\]

b) $\sqrt[3]{81}$

\[
\begin{align*}
\sqrt[3]{81} &= \sqrt[3]{27 \cdot 3} \\
3 \sqrt{3} &= 3 \sqrt{3}
\end{align*}
\]

c) $\sqrt[3]{375a^5b^4}$

\[
\begin{align*}
\sqrt[3]{375a^5b^4} &= \sqrt[3]{\frac{25 \cdot 3 \cdot a^5 \cdot b^4}{5a^2 \cdot b}} \\
5a^2 \cdot b \sqrt{3a^2 \cdot b} &= 5a^2 \cdot b \sqrt{3a^2 \cdot b}
\end{align*}
\]
Section 8: Factoring

Factoring Using a GCF:
To factor using a GCF, take the greatest common factor (GCF), for the numerical coefficient. When choosing the GCF for the variables, if all terms have a common variable, take the ones with the lowest exponent.
Example: $9x^4 + 3x^3 + 12x^2$
\[ \text{GCF: Coefficients } = 3 \]
\[ \text{Variables (x) } = x^2 \]
\[ \text{GCF } = 3x^2 \]
Next, you just divide each monomial by the GCF!
\[ \text{Answer } = 3x^2(3x^2 + x + 4) \]
Then, check by using the distributive property!

14. Factor each of the following expressions using a GCF and check your answer by using the distributive property.

a) $10x - 15x^3$
\[ 5x \left( 2x - 3x^2 \right) \]
\[ 5x \left( 2 - 3x^2 \right) \]

d) $8 - 4y$
\[ 4(2 - y) \]

e) $3ab^2 - 6a^3b$
\[ 3ab(6 - 2a) \]

f) $10xy - 15x^2y^2$
\[ 5xy \left( 2 - 3xy \right) \]

i) $6c^3d - 12c^2d^2 + 3cd$
\[ 3cd \left( 2c^2 - 4cd + 1 \right) \]

Factoring Trinomials
Example: $x^2 - 4x - 12$
\[ (x - 6)(x + 2) \]

15. Factor each of the following expressions.

a) $a^2 + 3a + 2$
\[ (a + 2)(a + 1) \]

d) $m^2 - 3m - 18$
\[ (m - 6)(m + 3) \]

b) $c^2 + 6c + 5$
\[ (c + 5)(c + 1) \]

e) $y^2 + 12y + 35$
\[ (y + 5)(y + 7) \]

f) $x^2 + 11x + 24$
\[ (x + 8)(x + 3) \]

g) $a^2 + 11a + 18$
\[ (a + 9)(a + 2) \]

h) $2x^2 + 15x + 7$
\[ (2x + 1)(x + 7) \]

i) $3x^2 - 5x - 12$
\[ (3x + 4)(x - 3) \]

d) $9x^2 + 11x + 2$
\[ (9x + 2)(x + 1) \]

k) $7x^2 - 22x + 3$
\[ (x - 3)(7x - 1) \]
\[ (x - 2)(x - 1) \]
\[ \frac{x^2 - 22x + 21}{7} \]