Objective: Today you will be able to
- divide polynomials through long division.
- use long division to factor completely.
- use the Remainder Theorem to evaluate a polynomial.

**POLYNOMIAL LONG DIVISION**

Numerical Long Division

Example #1 – Divide each of the following using long division.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a) 84 ÷ 3</td>
<td>b) 7</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
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<td>24</td>
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Polynomial Long Division

The method of dividing a polynomial by something other than a monomial is a process that can simplify a polynomial. Simplifying the polynomial helps in the process of factoring and therefore can help in finding the roots/zeros of the function.

Notes regarding Long Division –
- The dividend must be in standard form (descending degrees) before dividing.
- Include the degrees not present by putting 0x^n.

Example #2 – Divide each of the following by the given factor.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>a) 8x² + 10x - 7 by x - 3</td>
<td>b) 6x² - 30 + 9x³ by 3x - 4</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>8x² + 10x - 7</td>
<td>3x² + 6x + 8 + 2</td>
</tr>
<tr>
<td>8x² - 24x</td>
<td>3x² + 6x + 8 + 2</td>
</tr>
<tr>
<td>34x - 7</td>
<td>-q x³ + 12x²</td>
</tr>
<tr>
<td>-34x + 102</td>
<td>9x³ - 12x²</td>
</tr>
<tr>
<td>34(x - 3)</td>
<td>18x² - 24x</td>
</tr>
<tr>
<td>34x - 102</td>
<td>6(x - 4x)</td>
</tr>
<tr>
<td></td>
<td>18x² - 24x</td>
</tr>
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</tbody>
</table>
Example #3 – Factor each of the following completely using long division.

a) \( x^3 - 6x^2 + 5x + 12 \) by \( x - 4 \)

\[
\begin{array}{c|ccccc}
& x^2 & -2x & -3 \\
\hline
x & x^3 & -6x^2 & +5x & +12 \\
-4 & -4x^2 & +16x & -48 \\
\hline
x^2 & -2x & +5x \\
-4 & -4x & +12 \\
\hline
-3 & +12 \\
\hline
\end{array}
\]

b) \( x^3 - 12x^2 + 12x + 80 \) by \( x - 10 \)

\[
\begin{array}{c|cccc}
& x^2 & +12x & -8 \\
\hline
x & x^3 & -12x^2 & +12x & +80 \\
-10 & -10x^2 & -120x & -800 \\
\hline
x^2 & 12x & -8 \\
-10 & -10x & -80 \\
\hline
-8 & +80 \\
\hline
\end{array}
\]

What do you notice about the remainders in Examples 3a & 3b? **No remainder**

When the remainder is found to be **0** through polynomial division, this is a proof that the specific factor is indeed a root to the polynomial...meaning \( P(x) = 0 \) where \( c \) is the root to the divisor.

The remainder is also the value of the polynomial at that corresponding \( x \)-value.

**The Remainder Theorem**

If a polynomial \( P(x) \) is divided by a linear binomial \( (x - c) \), then the remainder equals \( P(c) \).

**NOTE** – This theorem evaluates a function at a specific \( x \)-value.
Example #4 – Given the function \( P(x) = x^3 - 7x^2 + 4 \) find \( P(-3) \) using direct substitution. Then use long division to verify the value of \( P(-3) \).

\[
P(-3) = (-3)^3 - 7(-3)^2 + 4
\]

\[
= -27 - 63 + 4
\]

\[
= -90 + 4
\]

\[
= -86
\]

\[
\begin{array}{c}
\frac{x^2 - 10x + 30}{x + 3} \quad \frac{x^3 - 7x^2 + 10x + 4}{x^3 - 3x^2}
\end{array}
\]

\[
\begin{array}{c}
-10x^2 + 10x
\end{array}
\]

\[
\begin{array}{c}
-10x + 30
\end{array}
\]

\[
\begin{array}{c}
30x + 4
\end{array}
\]

\[
\begin{array}{c}
30x - 90
\end{array}
\]

\[
\begin{array}{c}
-86
\end{array}
\]

Example #5 – Find \( P(1) \) given \( P(x) = x^2 + 9x - 16 \). Then conclude what this implies about \( P(x) \) at \( x = 1 \).

\[
P(1) = (1)^2 + 9(1) - 16
\]

\[
= 1 + 9 - 16
\]

\[
= -6
\]

\[
\begin{array}{c}
\frac{x + 10}{x - 1} \quad \frac{x^2 + 9x - 16}{x^2 + x}
\end{array}
\]

\[
\begin{array}{c}
10x - 16
\end{array}
\]

\[
\begin{array}{c}
10x + 10
\end{array}
\]

\[
\begin{array}{c}
-6
\end{array}
\]

\( x - 1 \) is NOT a factor of \( P(x) \) because there is a remainder.
POLYNOMIALS HW #2

1. Find each quotient by using long division.
   
a) \((5x^4 - 3x^3 + 2x - 7) + (x - 5)\)
   
\[
\begin{array}{c|ccccc}
\multicolumn{6}{c}{5x^4 - 3x^3 + 2x - 7} \\
\hline
x - 5 & 5x^3 - 10x^2 - 5x - 7 \\
\hline
\end{array}
\]

\[
5x^3 - 25x^2 \\
5x^4 - 25x^2 \\
-22x^2 + 110x \\
-110x^2 + 550x \\
11x^2 - 100x + 11 \\
11x^2 - 100x + 11 \\
\hline
\]

b) \(\frac{48x^3 - 72x^2 + 16}{6x^2 - 2}\)

\[
\begin{array}{c|cccc}
\multicolumn{5}{c}{48x^3 - 72x^2 + 16} \\
\hline
6x^2 - 2 & 8x^3 - 12x^2 + 8x - 16 \\
\hline
\end{array}
\]

\[
8x^3 - 40x - 27 \\
48x^3 + 10x^2 - 16x \\
72x^3 + 16x + 16 \\
16x - 8 \\
\hline
\]

2. Factor each of the following polynomials completely given a factor.

   a) \((x - 5)\) is one factor of \(2x^3 + x^2 - 43x - 60\)

\[
\begin{array}{c|cccc}
\multicolumn{5}{c}{2x^3 + x^2 - 43x - 60} \\
\hline
x - 5 & 2x^2 + 11x + 12 \\
\hline
\end{array}
\]

\[
2x^2 + 10x \\
2x^3 - 10x^2 \\
11x - 55x \\
11x^2 - 55x \\
12(x - 5) \\
12x - 60 \\
\hline
\]

   b) \((x + 5)\) is one factor of \(x^3 + 6x^2 + 3x - 10\)

\[
\begin{array}{c|cccc}
\multicolumn{5}{c}{x^3 + 6x^2 + 3x - 10} \\
\hline
x + 5 & x^2 + 3x + 2 \\
\hline
\end{array}
\]

\[
x^2 + 5x^2 \\
x^3 + 5x^2 \\
x^2 + 5x^2 \\
x^2 + 5x \\
-2(x + 5) \\
(x^2 + x - 1) \\
\hline
\]

\[
(x + 5)(x + 2)(x - 1) \\
\]

Answer: \(x^2 + 4\)
Question #2 Continued...

c) \(x^3 + 15x^2 + 56x \) by \(x + 7\)

\[
\begin{array}{c}
\text{x}^2 + 8x \\
\text{x}^2 + 7x^2 \\
8x(x + 7) \\
8x^2 + 56x \\
x(x + 8) \\
\hline
\end{array}
\]

\[\text{x}(x + 7)(x + 8)\]

3. Using the Remainder Theorem and Long Division evaluate each polynomial. Check your answers using Direct Substitution on Desmos.

a) Find \(P(-5)\) given \(P(x) = -x^3 - 4x^2 + x - 2\).

\[
P(-5) = -(-5)^3 - 4(-5)^2 - 5 - 2
\]

\[
= -125 - 100 - 5 - 2
\]

\[
= 18
\]

b) Given \(g(x) = -3x^3 - 5x^2 + 4\), find \(g(2)\).

\[
g(2) = -3(2)^3 - 5(2)^2 + 4
\]

\[
= -24 - 20 + 4
\]

\[
= -40
\]