Exploration

Example #1 – Graph each of the following functions using Desmos. Then discuss any differences and/or similarities.

\[ y = x^2 (x - 3)^2 + 2 \quad \text{and} \quad y = (x + 2)^2 (x - 3) \]

<table>
<thead>
<tr>
<th>Differences</th>
<th>Similarities</th>
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Example #2 – Given the function \( f(x) = (x - 6)(x - 1)^2 (x + 2) \).

a) Determine the degree and leading coefficient.

b) Determine the end behavior of \( f(x) \).
   \[ x \to -\infty, \quad f(x) \to \_ \_ \_ \_ \_ \]
   \[ x \to \infty, \quad f(x) \to \_ \_ \_ \_ \_ \]

c) What are the zeros of \( f(x) \)?

Example #3 – Given the function \( f(x) = x^2 (x + 3)^2 (x - 2) \).

a) Determine the degree and leading coefficient.

b) What are the zeros of \( f(x) \)?

c) At which zero(s) will the graph of \( f(x) \) cross through the \( x \)-axis?

d) At which zero(s) will the graph of \( f(x) \) hit the \( x \)-axis but not cross through?
Objective: Today you will be able to
✦ determine the multiplicity of a polynomial’s zero.
✦ write a polynomial function given its graph and/or zeros.
✦ Determine possible rational zeros of a polynomial.

Multiplicities of Polynomial Functions

Definition of Multiplicity
The multiplicity of a graph refers to the ________________ that its associated factor appears in
the polynomial function.

Example #1 – Given $f(x) = (x-3)^4(x-5)(x-8)^2$, determine the zeroes and the multiplicity of each zero.

Notes
✦ The multiplicity of a zero determines whether the graph crosses the $x$-axis at that zero or if it
instead turns back the way it came (bounced off the $x$-axis).

✦ If a zero has an odd multiplicity, then the graph will cross through the $x$-axis at that zero.

✦ If a zero has an even multiplicity, then the graph will not cross through (bounced off) the $x$-axis at
that zero.

Example #2 – Each of the following graphs have zeros at $x = -6$ and $x = 7$. Write the equation for each
using the multiplicity of each root. Be sure to note the sign of the $a$-value for each.

GRAPH A

Equation: ____________________

GRAPH B

Equation: ____________________

GRAPH C

Equation: ____________________

GRAPH D

Equation: ____________________
Example #3 – Without using a calculator, write and then sketch the graph of a least degree polynomial using the information provided. Then state the degree and end behavior for each polynomial.

a) \( f(x) \) has a negative leading coefficient and zeros of \( x = -2 \) with multiplicity 2 and \( x = 1 \).

b) \( g(x) \) has a positive leading coefficient and zeros of \( x = -2 \) with multiplicity 3, \( x = 0 \), and \( x = 3 \) with multiplicity 2.

---

Equation: ______________________________

Degree: ________________

\[ x \to -\infty, \quad f(x) \to \text{________} \]

\[ x \to \infty, \quad f(x) \to \text{________} \]

---

Equation: ______________________________

Degree: ________________

\[ x \to -\infty, \quad f(x) \to \text{________} \]

\[ x \to \infty, \quad f(x) \to \text{________} \]

---

**Definition of the Fundamental Theorem of Algebra**

If \( P(x) \) is a polynomial of degree \( n \geq 1 \) with complex coefficients, then \( P(x) = 0 \) has at least one complex root.

**NOTE:** A polynomial can have, at most, the same number of real roots as its highest degree. Last unit we have worked quadratics that had either ________, ________, or ________ real roots. If a quadratic function had no real roots we said that this function had __________________________ roots.

**Remember:** Imaginary roots of any polynomial must be a complex set... in other words imaginary roots must be in pairs. For example, if a polynomial has a root of \( x = 3 + 2i \), then the second root that is paired with this is ________________.

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Example #4 – Determine all combinations of real and non-real unique roots for each type of polynomial.

### Quadratic

<table>
<thead>
<tr>
<th>Real Roots</th>
<th>Non-Real, Complex Roots</th>
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### Cubic

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### Quartic

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**Rational Root Theorem**

Given a polynomial \( f(x) \), the only possible rational solutions of the equation are \( x = \frac{p}{q} \). Where \( p \) is a factor of the ______________ and \( q \) is a factor of the ____________________.

**Example #5** – List all possible rational zeros of the given functions using the rational root theorem.

a) \( f(x) = 2x^2 - 7x - 10 \)  

b) \(-3x^3 + 3x^2 - 11x - 6 = 0\)
Polynomials HW #1

1. Determine the zeros for each of the functions graphed. Then determine each zero’s multiplicity and finally write a possible equation of least degree for each function.

   a) 
   
   ![Graph of function a) with Zeros and Multiplicity table]

<table>
<thead>
<tr>
<th>Zeros</th>
<th>Multiplicity</th>
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   Equation: ________________

   b) 
   
   ![Graph of function b) with Zeros and Multiplicity table]

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   Equation: ________________

   c) 
   
   ![Graph of function c) with Zeros and Multiplicity table]

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<th>Multiplicity</th>
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   Equation: ________________

   d) 
   
   ![Graph of function d) with Zeros and Multiplicity table]

<table>
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   Equation: ________________

2. How can you tell if the multiplicity is even or odd by looking at the graph of a polynomial?

3. How does the number of bends/turns relate to the degree of an equation?
4. Write an equation with a positive leading coefficient that could represent a polynomial that has 
\(x\)-intercepts that cross the \(x\)-axis at \(-2, 4, 5\), and an \(x\)-intercept at \(1\) that does not cross through the \(x\)-axis.
   a) Equation: _____________________________
   b) What is the degree of the polynomial? _____________
   c) What is the end behavior of the polynomial? 
      \(x \to -\infty, f(x) \to \) _______
      \(x \to \infty, f(x) \to \) _______

5. Write an equation that could represent a polynomial that opens downward and has \(x\)-intercepts of 4 and 5, in which both intercepts do not cross through the \(x\)-axis.
   a) Equation: _____________________________
   b) What is the degree of the polynomial? _____________
   c) What is the end behavior of the polynomial? 
      \(x \to -\infty, f(x) \to \) _______
      \(x \to \infty, f(x) \to \) _______

6. Match each of the following descriptions with its appropriate graph.
   _____ A function that has a positive leading coefficient and an even degree.
   _____ A function that has a negative leading coefficient and an even degree.
   _____ A function that has a positive leading coefficient and an odd degree.
   _____ A function that has a negative leading coefficient and an odd degree.

7. Use the Rational Root Theorem to find all possible rational zeros of the functions:
   a) \(h(x) = 2x^3 + x^2 - 5x - 21\).
   b) \(f(x) = -x^3 - 2x^2 + x - 24\).
   c) \(y = x^3 + 2x^2 + 3x + 6\).
   d) \(f(x) = x^4 - 7x^2 + 12\).