Chap.6 Motion in the Presence of Resistive Forces

1. Introduction
If an object is dropped and free-falls in a vacuum, then the speed of the object will continuously increase linearly with time. That is, since \( v = gt \) (choosing downward as positive), the speed is a linear function with \( t \) as the variable.

However, if an object is dropped in a medium of liquid or gas, the object will encounter a resistive force \( R \). For example, in a medium of air, the object will encounter air resistance (or air drag) and when it moves through liquid, viscous forces will act on the object. In this case the speed will not continue to increase but reach a maximum velocity. Hence, the speed is no longer linear with time.

2. Math Review
Find \( x \) for the following equation
i) \( 4^x = 16 \)  
ii) \( 3^{2x} = 9 \)  
iii) \( 3^{2x-1} = 27 \)
iv) \( e^{2x} = 4 \)  
v) \( e^{2x-1} = 3 \)  
vi) \( 1 - e^{-2/x} = 0.5 \)

3. Two Types of Resistive Forces
i) For objects falling very slowly through a liquid, or for small objects, the resistive force \( R \) is proportional to \( v \).
   ex) billiard ball falling through water, dust particles falling in the air

   ii) For large objects moving through air, the resistive force \( R \) is proportional to \( v^2 \).
   ex) sky divers free-falling in the air
4. Model 1: Resistive Force Proportional to Speed

The resistive force acting on an object moving through a liquid or gas that is proportional to the object’s speed can be expressed as

$$R = bv \quad [\text{N}]$$

where ‘$v$’ is the speed of the object and ‘$b$’ is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object.

A small sphere of mass $m$ is released from rest in a liquid. There are only two forces acting on the object, $F_s$ and $R$, so

$$\Sigma F_y = mg - bv = ma$$

The terminal speed $v_T$ can be obtained by setting $a=0$,

$$v_T = \frac{mg}{b}$$

However, we are also interested in finding the speed at any time before reaching terminal speed. That requires solving the differential equation above, that is, setting $v_i=0$ at $t=0$ and rewriting the equation gives

$$mg - bv = m \frac{dv}{dt}$$

$$\frac{dv}{dt} = g - b \frac{v}{m}$$

Solving the above equation gives

$$v(t) = \frac{mg}{b} \left(1 - e^{-\frac{bt}{m}}\right)$$

If we replace with $v_T = \frac{mg}{b}$ and $\tau = m/b$, then

$$v(t) = v_T \left(1 - e^{-t/\tau}\right)$$

where $\tau$ is called the time constant.

p.163 Ex) 6.8 Sphere Falling in Oil

A small sphere of mass 2g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00cm/s. Determine the time constant $\tau$ and the time at the sphere reached 90% of its terminal speed.
5. **Model 2: Resistive Force Proportional to Speed Squared**

For objects moving at high speeds through air, such as airplanes, sky divers, cars and baseballs, the resistive force is approximately proportional to the square of the speed.

\[ R = bv^2 \text{ [N]} \]

**Example**

A rubber ball of mass \( m \) is dropped from a cliff. As the ball falls, it is subject to air drag. The drag force on the ball has magnitude \( bv^2 \), where \( b \) is a constant drag coefficient and \( v \) is the instantaneous speed of the ball. The drag coefficient \( b \) is directly proportional to the cross-sectional area of the ball and the density of the air and does not depend on the mass of the ball. As the ball falls, its speed approaches a constant value called the terminal speed.

(a) On the figure below, draw and label all the forces on the ball at some instant before it reaches terminal speed.

(b) State whether the magnitude of the acceleration of the ball of mass \( m \) increases, decreases, or remains the same as the ball approaches terminal speed. **Explain.**

(c) Write, but do NOT solve, a differential equation for the instantaneous speed \( v \) of the ball in terms of time \( t \), the given quantities, and fundamental constants.

(d) Determine the terminal speed \( v_T \) in terms of the given quantities and fundamental constants.