Multiple Choice

1. (calculator not allowed) If \( \frac{dy}{dx} = x^2 y \), then \( y \) could be

(A) \( 3 \ln \left( \frac{x}{3} \right) \)

(B) \( e^x + 7 \)

(C) \( 2e^x \)

(D) \( 3e^{x^2} \)

(E) \( \frac{x^3}{3} + 1 \)

\[ \int \frac{1}{y} \, dy = \int x^2 \, dx \]

\[ e^{\ln y} = \frac{x^3}{e^x} + C \]

\[ y = \pm ke^{x^3} \]

\[ |y| = e^{x^3} \cdot e^c \]

2. (calculator not allowed) Which of the following is the solution to the differential equation \( \frac{dy}{dx} = \frac{4x}{y} \), where \( y(2) = -2 \)?

(A) \( y = 2x \) for \( x > 0 \)

(B) \( y = 2x - 6 \) for \( x \neq 3 \)

(C) \( y = -\sqrt{4x^2 - 12} \) for \( x > \sqrt{3} \)

(D) \( y = \sqrt{4x^2 - 12} \) for \( x > \sqrt{3} \)

(E) \( y = -\sqrt{4x^2 - 6} \) for \( x > \sqrt{1.5} \)

\[ \int y \, dy = \int 4x \, dx \]

\[ \frac{1}{3} y^2 = 2x^2 + C \]

\[ y^2 = 4x^2 + C \]

\[ 4 = 16 + C \]

\[ -12 = C \]

\[ y^2 = 4x^2 - 12 \]

\[ y = \pm \sqrt{4x^2 - 12} \]
3. (calculator not allowed)

Which of the following is a slope field for the differential equation \( \frac{dy}{dx} = \frac{x}{y} \):

(A) [Diagram]

(B) [Diagram]

(C) [Diagram]

(D) [Diagram]

(E) [Diagram]

\[ y \, dy = x \, dx \]
\[ \frac{y^2}{2} = \frac{x^2}{2} + c \]
\[ y^2 - x^2 = C \]

Hyperbolas

\[ \frac{dy}{dx} = -\frac{x}{y} \]

Circles

\[ \frac{dy}{dx} = -\frac{2x}{y} \]

Ellipses
5. (calculator not allowed)
Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) \( \frac{3 \ln 3}{\ln 2} \)
(B) \( \frac{2 \ln 3}{\ln 2} \)
(C) \( \frac{\ln 3}{\ln 2} \)
(D) \( \ln \left( \frac{27}{2} \right) \)
(E) \( \ln \left( \frac{9}{2} \right) \)

6. (calculator not allowed)
If \( \frac{dy}{dt} = -2y \) and if \( y = -1 \) when \( t = 0 \), what is the value of \( t \) for which \( y = \frac{1}{2} \)?

(A) \( -\ln \frac{2}{3} \)
(B) \( -\frac{1}{4} \)
(C) \( -\ln \frac{2}{3} \)
(D) \( \frac{\sqrt{2}}{2} \)
(E) \( \ln 2 \)

7. (calculator not allowed)
If \( \frac{dy}{dx} = 2y^2 \) and if \( y = -1 \) when \( x = 1 \), then when \( x = 2, y = \)

(A) \( -\frac{2}{3} \)
(B) \( -\frac{1}{3} \)
(C) 0
(D) \( \frac{1}{3} \)
(E) \( \frac{2}{3} \)
8. (calculator not allowed)
At each point \((x,y)\) on a certain curve, the slope of the curve is \(3x^2y\). If the curve contains the point \((0,8)\), then its equation is

(A) \(y = 8e^{x^2}\)
(B) \(y = x^3 + 8\)
(C) \(y = e^{x^3} + 7\)
(D) \(y = \ln(x+1) + 8\)
(E) \(y^2 = x^3 + 8\)

\[
\frac{dy}{dx} = 3x^2y
\]

\[
\frac{dy}{y} = 3x^2\,dx
\]

\[
\ln|y| = e^{x^3} + C
\]

\[
y = 8e^{x^3}
\]

9. (calculator not allowed) If the graph of \(y = f(x)\) contains the point \((0, 2)\), \(\frac{dy}{dx} = \frac{-x}{y e^{x^2}}\) and \(f(x) > 0\) for all \(x\), then \(f(x) = \)

(A) \(3 + e^{-x^2}\)
(B) \(\sqrt{3 + e^{-x^2}}\)
(C) \(1 + e^{-x^2}\)
(D) \(\sqrt{3 + e^{-x^2}}\)
(E) \(\sqrt{3 + e^2}\)

\[
\int y\,dy = \frac{1}{2} \int \frac{2x}{e^{x^2}}\,dx
\]

\[
u = x^2
\]

\[
u' = 2x
\]

\[
\frac{\sqrt{y^2}}{2} = -\frac{1}{2} \int \frac{du}{e^u}
\]

\[
\frac{\sqrt{y^2}}{2} = -\frac{1}{2} \cdot e^{-u} + C
\]

\[
y^2 = e^{-x^2} + C
\]

\[
y^2 = \frac{e^{-x^2} + C}{2}
\]

\[
y = 1 + C
\]

10. (calculator not allowed)
If \(\frac{dy}{dx} = \tan x\), then \(y = \)

(A) \(\frac{1}{2} \tan^2 x + C\)
(B) \(\sec^2 x + C\)
(C) \(\ln|\sec x| + C\)
(D) \(\ln|\sec x + C|\)
(E) \(\sec x \tan x + C\)

\[
y = -\int \frac{du}{u}
\]

\[
y = -\ln|1 + C|
\]

\[
y = -\ln|\cos x| + C = \ln(|\cos x|^{-1} + C)
\]

\[
y = \ln|\cos x|^{-1} + C
\]

\[
y = \ln|\cos x|^{-1} + C
\]
11. (calculator not allowed)

Consider the differential equation \( \frac{dy}{dx} = -\frac{2x}{y} \).

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(b) Let \( y = f(x) \) be the particular solution to the differential equation with the initial condition \( f(1) = -1 \). Write an equation for the line tangent to the graph of \( f \) at \((1,-1)\) and use it to approximate \( f(1.1) \).

(c) Find the particular solution \( y = f(x) \) to the given differential equation with the initial condition \( f(1) = -1 \).
12. (calculator not allowed)

Consider the differential equation \( \frac{dy}{dx} = (y-1)^2 \cos(\pi x) \).

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(b) There is a horizontal line with equation \( y = c \) that satisfies this differential equation. Find the value of \( c \).

(c) Find the particular solution \( y = f(x) \) to the differential equation with the initial condition \( f(1) = 0 \).

\[
\int (y-1)^2 \, dy = \int \cos(\pi x) \, dx
\]

\[
\frac{-1}{y-1} = \frac{\sin(\pi x)}{\pi} + C
\]

\[
-1 = \frac{\sin(\pi x)}{\pi} + C
\]

\[
1 = C
\]

\[
\frac{-1}{y-1} = \frac{\sin(\pi x) + 1}{\pi} \quad \Rightarrow \quad y-1 = \frac{-1}{\sin(\pi x) + 1}
\]

\[
y = \frac{-1}{\sin(\pi x) + 1} + 1
\]
13. (calculator not allowed)
At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function \( W \) models the total amount of solid waste stored at the landfill. Planners estimate that \( W \) will satisfy the differential equation \( \frac{dW}{dt} = \frac{1}{25} (W - 300) \) for the next 20 years. \( W \) is measured in tons, and \( t \) is measured in years from the start of 2010.

(c) Find the particular solution \( W = W(t) \) to the differential equation \( \frac{dW}{dt} = \frac{1}{25} (W - 300) \) with initial condition \( W(0) = 1400 \).

\[
\int \frac{dW}{W-300} = \int \frac{1}{25} dt
\]

\[
\ln |W - 300| = \frac{1}{25} t + C
\]

\[
|W - 300| = e^{\frac{1}{25} t + C}
\]

\[
|W - 300| = ke^{\frac{1}{25} t}
\]

\[
W - 300 = \pm ke^{\frac{1}{25} t} + 300
\]

\[
1400 = ke^0 + 300
\]

\[
1100 = k
\]

\[
W = 1100e^{\frac{1}{25} t} + 300
\]