

1.

$$\frac{x^2 - x - 20}{2x + 3} \cdot \frac{6x^2 + 7x - 3}{5 - x}$$

$$\frac{(\cancel{x-5})(x+4)}{(\cancel{2x+3})} \cdot \frac{(3x-1)(\cancel{2x+3})}{(-1)(\cancel{x-5})}$$

$$\boxed{-(x+4)(3x-1)}$$

2.

$$\frac{x^{\cancel{3}}}{4x+12} \cdot \frac{(x+3)}{\cancel{x^2}}$$

$$\frac{x^3}{4(x+3)} \cdot \frac{(x+3)}{1}$$

$$\boxed{\frac{x^3}{4}}$$

3.

$$\frac{2x-14}{3x+4} \div \frac{x^2-49}{27x^3+64}$$

$$\frac{2x-14}{3x+4} \cdot \frac{27x^3+64}{x^2-49}$$

$$\frac{2(\cancel{x-7})}{\cancel{3x+4}} \cdot \frac{(\cancel{3x+4})(9x^2-12x+16)}{(\cancel{x-7})(x+7)}$$

$$\boxed{\frac{2(9x^2-12x+16)}{(x+7)}}$$

4.

$$\frac{8x^3}{x^2-9} \div \frac{4x^3+12}{1}$$

$$\frac{8x^3}{(x+3)(x-3)} \cdot \frac{1}{4x^3+12x^2}$$

$$\frac{\cancel{2}x^3}{(x+3)(x-3)} \cdot \frac{1}{\cancel{4}x^2(x+3)}$$

$$\boxed{\frac{2x}{(x+3)^2(x-3)}}$$

1.

$$\frac{(x+4)2x}{(x+4)x-3} + \frac{(x-5)(x-3)}{(x+4)(x-3)} \quad \text{LCD}$$

$$\frac{2x(x+4)}{(x-3)(x+4)} + \frac{(x-5)(x-3)}{(x+4)(x-3)}$$

$$\frac{(2x^2+8x) + (x^2-8x+15)}{(x-3)(x+4)}$$

$$\frac{3x^2+15}{(x-3)(x+4)}$$

$$\boxed{\frac{3(x^2+5)}{(x-3)(x+4)}}$$

2.

$$\frac{x^2}{x-4} - \frac{3x+4}{x-4}$$

$$\frac{x^2-3x-4}{x-4}$$

$$\frac{(x-4)(x+1)}{(x-4)}$$

$$\boxed{x+1}$$

3.

$$\frac{x}{x-2} - \frac{3x-15}{x^2-7x+10}$$

$$\frac{(x-5)x}{(x-5)(x-2)} - \frac{3(x-5)}{(x-5)(x-2)}$$

$$\frac{x(x-5) - 3(x-5)}{(x-5)(x-2)}$$

$$\frac{x(x-5) - 3(x-5)}{(x-5)(x-2)}$$

$$\frac{x^2-5x-3x+15}{(x-5)(x-2)}$$

$$\frac{x^2-8x+15}{(x-5)(x-2)}$$

$$\frac{(x-5)(x-3)}{(x-5)(x-2)} = \boxed{\frac{x-3}{x-2}}$$

4.

$$5. \quad \frac{2x^2}{x^2+1} + \frac{2}{3x^2+3} =$$

$$3 \cdot \frac{2x^2}{(x^2+1)} + \frac{3}{3(x^2+1)} \quad \begin{matrix} (x) \\ (x) \end{matrix}$$

$$\frac{6x^2}{3(x^2+1)} + \frac{3}{3(x^2+1)}$$

$$\frac{6x+3}{3(x^2+1)}$$

$$\frac{\cancel{3}(2x+1)}{\cancel{3}(x^2+1)} = \boxed{\frac{2x+1}{x^2+1}}$$

1	$y = \frac{4}{x+2} + 1$	$x = -2$	$y = 1$
5	$y = \frac{3x-2}{x+1}$	$x = -1$	$y = 3$
6	$y = \frac{4x+1}{(x+3)(x-5)}$	$x = -3, 5$	$y = 0$

Graph the following function:

4.  $g(x) = \frac{3x+6}{x-1} = \frac{3(x+2)}{(x-1)}$

Removable Discontinuities: None

VA: $x-1=0$ $x=1$	Domain: $\mathbb{R}, x \neq 1$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>-1.5</td> </tr> <tr> <td>0</td> <td>-6</td> </tr> <tr> <td>1</td> <td><del>2.5</del></td> </tr> <tr> <td>2</td> <td>12</td> </tr> <tr> <td>3</td> <td>7.5</td> </tr> <tr> <td>4</td> <td>6</td> </tr> </tbody> </table>	x	y	-1	-1.5	0	-6	1	<del>2.5</del>	2	12	3	7.5	4	6	Graph 
x	y																
-1	-1.5																
0	-6																
1	<del>2.5</del>																
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3	7.5																
4	6																
HA: $y = \frac{3}{1}$	Range: $\mathbb{R}, y \neq 3$																

7	$y = \frac{(x+4)}{(x-3)(x+3)}$	$x = -3, 3$	$y = 0$	$( \quad , y )$
8	$y = \frac{5(x-1)(x+2)}{1(x-5)(x+2)}$	$x = 5$	$y = 5$	$(-2, y)$
9	$y = \frac{1(x+3)(x+6)}{1(x+2)(x+3)}$	$x = -2$	$y = 1$	$( \quad , y )$

Graph the following function:

$$h(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3}$$

$$4. \quad h(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3} = \frac{(x-4)(x-1)}{(x+3)(x-1)} = \frac{x-4}{x+3}$$

$$\begin{aligned} x-1 &= 0 \\ x &= 1 \\ \hline y &= \frac{1-4}{1+3} \\ y &= -3/4 \end{aligned}$$

Removable Discontinuities:  $(1, -3/4)$

VA: $x+3=0$ $x = -3$	Domain: $\mathbb{R},$ $x \neq -3, 1$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-5</td> <td>-9/2</td> </tr> <tr> <td>-4</td> <td>-8/1</td> </tr> <tr> <td>-3</td> <td>ERR</td> </tr> <tr> <td>-2</td> <td>-6/1</td> </tr> <tr> <td>-1</td> <td>-5/2</td> </tr> </tbody> </table>	x	y	-5	-9/2	-4	-8/1	-3	ERR	-2	-6/1	-1	-5/2	Graph 
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HA: Same Degree $y = 1$	Range: $\mathbb{R},$ $y \neq 1, -3/4$														

a. Y varies inversely with the square of x.

$$y = \frac{k}{x^2}$$

b. Z varies directly with y and inversely with x.

$$z = \frac{ky}{x}$$

c. Z varies directly with x and inversely with y.

$$z = \frac{kx}{y}$$

d. Y varies jointly with x and y and inversely with z.

$$y = \frac{kxy}{z}$$

e. R is inversely proportional to s and directly proportional to the square of t.

$$r = \frac{kt^2}{s}$$

f. M is directly proportional to n and inversely proportional to s cubed.

$$M = \frac{kn}{s^3}$$

**EXAMPLE FOUR → WRITE A JOINT VARIATION EQUATION**

z varies directly with x and y. Write the equation that relates the variables if  $z = -75$ ,  $x = 3$ , and  $y = -5$ . Then find z when  $x = 2$  and  $y = 6$

$$\begin{aligned} z &= kxy \\ -75 &= k(3)(-5) \\ -75 &= -15k \\ \frac{-75}{-15} &= \frac{-15k}{-15} \\ 5 &= k \end{aligned}$$

$$\begin{aligned} z &= 5xy \\ z &= 5(2)(6) \\ \boxed{z} &= \boxed{60} \end{aligned}$$

Circle the following equations that represent direct variation:

$y = 2x$	$y = \frac{3}{4}x + 3$	$y = \frac{3}{4}x$	$\frac{5y}{5} = \frac{-4x}{5}$	$3y - 2 = x$	$4 = xy$
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$$y = -\frac{4}{5}x$$

4. An equation shows  $m$  is directly proportional to  $n$  and inversely proportional to  $s$  cubed. When  $m = 5$ , then  $n = 160$  and  $s = 2$ . What is the constant of proportionality? Write your answer as a fraction.

$$m = \frac{Kn}{s^3}$$

$$5 = \frac{K(160)}{2^3}$$

$$5 = \frac{160K}{8}$$

$$\frac{40}{160} = \frac{160K}{160}$$

$$\boxed{\frac{1}{4} = K}$$

2. A farmer pumps water from an irrigation well to water his field. The time it takes to water the field varies inversely with the rate at which the pump operates. It takes 20 hours to water the field when the pumping rate is 600 gallons per minute. If he adjusts the pump so that it pumps at a rate of 400 gallons per minute, how long will it take to water the field?

- a. 12.5 hours
- b. 15 hours
- c. 30 hours
- d. 40 hours

$$T = \frac{K}{r}$$

$$20 = \frac{K}{600}$$

$$12,000 = K$$

$$T = \frac{12,000}{r}$$

$$T = \frac{12,000}{400}$$

$$\boxed{T = 30 \text{ hours}}$$

- j. The radiation,  $r$ , from the decay of plutonium is directly proportional to the mass,  $m$ , of the sample tested and inversely proportional to the square of the distance,  $d$ , from the detector to the sample.  $r = \frac{Km}{d^2}$