#1-8: Solve each triangle described. If there are two triangles, solve both. Round to the thousandths place.

1. \( \alpha = 50^\circ, \ \beta = 17^\circ, \ a = 10 \)

   \[ \gamma = \ldots; \quad \gamma = \ldots \]

   \[ b = \ldots; \quad b = \ldots \]

   \[ c = \ldots; \quad c = \ldots \]

2. \( \alpha = 71^\circ, \ \beta = 21^\circ, \ c = 5 \)

   \[ \gamma = \ldots; \quad \gamma = \ldots \]

   \[ a = \ldots; \quad a = \ldots \]

   \[ b = \ldots; \quad b = \ldots \]
3. \( a = 10, \ b = 12, \ c = 15 \)

\[
\begin{align*}
\alpha &= \_\_\_\_\_\_\_\; ; \; \alpha &= \_\_\_\_\_\_\_ \\
\beta &= \_\_\_\_\_\_\_\; ; \; \beta &= \_\_\_\_\_\_\_ \\
\gamma &= \_\_\_\_\_\_\_\; ; \; \gamma &= \_\_\_\_\_\_\_ \\
\end{align*}
\]

4. \( \alpha \ = 40^\circ, \ b = 15, \ c = 25 \)

\[
\begin{align*}
\beta &= \_\_\_\_\_\_\_\; ; \; \beta &= \_\_\_\_\_\_\_ \\
\gamma &= \_\_\_\_\_\_\_\; ; \; \gamma &= \_\_\_\_\_\_\_ \\
\alpha &= \_\_\_\_\_\_\_\; ; \; \alpha &= \_\_\_\_\_\_\_ \\
\end{align*}
\]
5. $\alpha = 40^\circ$, $a = 3$, $b = 2$

$\beta =$ ______; $\beta =$ ______

$\gamma =$ ______; $\gamma =$ ______

$c =$ ______; $c =$ ______

6. $a = 2$, $c = 1$, $\gamma = 100^\circ$

$\beta =$ ______; $\beta =$ ______

$\gamma =$ ______; $\gamma =$ ______

$a =$ ______; $a =$ ______
7. $\alpha = 40^\circ$, $a = 6$, $b = 8$

$\beta = \boxed{\_\_\_\_\_\_}$; $\beta = \boxed{\_\_\_\_\_\_}$

$\gamma = \boxed{\_\_\_\_\_\_}$; $\gamma = \boxed{\_\_\_\_\_\_}$

$c = \boxed{\_\_\_\_\_\_}$; $c = \boxed{\_\_\_\_\_\_}$

8. $\beta = 74^\circ$, $a = 61$, and $c = 100$

$\alpha = \boxed{\_\_\_\_\_\_}$; $\alpha = \boxed{\_\_\_\_\_\_}$

$\gamma = \boxed{\_\_\_\_\_\_}$; $\gamma = \boxed{\_\_\_\_\_\_}$

$b = \boxed{\_\_\_\_\_\_}$; $b = \boxed{\_\_\_\_\_\_}$
1. **Measuring the Length of a Lake**  
   From a stationary hot-air balloon 500 feet above the ground, two sightings of a lake are made (see the figure). How long is the lake?

2. **Smokestack**  
   At a point 75 feet from the base of a building, the angle of elevation to the bottom of a smoke stack is 40°, the angle of elevation to the top is 44°. Find the height of the smoke stack alone.
3. **Leaning Wall**  
A leaning wall is inclined $6^\circ$ from the vertical. At a distance of 40 feet from the wall, the angle of elevation to the top is $22^\circ$. Find the height of the wall to the nearest foot.

![Diagram of a leaning wall]

4. **Constructing a Highway**  
A highway whose primary directions are north-south is being constructed along the west coast of Florida. Near Naples, a bay obstructs the straight path of the road. Since the cost of a bridge is prohibitive, engineers decide to go around the bay. The illustration shows the path that they decide on and the measurements taken. What is the length of highway needed to go around the bay?

![Diagram of a highway绕行示意图]
5. **Surveying**
Two homes are located on opposite sides of a small hill. To measure the distance between them, a surveyor walks a distance of 50 feet from house A to point C, uses a transit to measure the angle ACB, which is found to be 80°, and then walks to house B, a distance of 60 feet. How far apart are the houses?

![Surveying Diagram](image)

6. **Guy Wires**
The height of a radio tower is 500 feet and the ground on one side of the tower slopes upward at an angle of 10°.

   a) How long should a guy wire be if it is to connect to the top of the tower and be secured at a point on the sloped side 100 feet from the base of the tower?

   b) How long should the second guy wire be if it is to connect to the middle of the tower and be secured at a point 100 ft from the base on the flat side?
7. **Correcting a Navigation Error**

Two cities A and B are 300 miles apart. In flying from city A to city B, a pilot accidentally took a course that was 5° in error.

a) If the error was discovered after flying 10 minutes at a constant speed of 420 miles per hour, through what angle should the pilot turn to correct the course?

b) What new constant speed should be maintained so that no time is lost due to the error? (assume that the speed would have been a constant 420 miles per hour if no error had occurred).

8. The dimensions of a triangular lot are 50ft by 70ft by 100ft. If the price of such land is $3 per square foot, how much does the lot cost?
**CHAPTER 7 SUMMARY SHEET**

Solve the triangle: Find all 3 sides and all 3 angles

Angle of Elevation = Angle of Depression

<table>
<thead>
<tr>
<th><strong>Right Triangle Trig</strong></th>
<th><strong>Right Triangle Trig</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given an angle and a side:</strong></td>
<td><strong>Given two sides:</strong></td>
</tr>
<tr>
<td>1. Label the sides opposite, adjacent, or hypotenuse (relative to the angle).</td>
<td>1. Label the sides opposite, adjacent, or hypotenuse (relative to the angle).</td>
</tr>
<tr>
<td>2. Decide which trig function to use.</td>
<td>2. Decide which trig function to use.</td>
</tr>
<tr>
<td>3. Set up a trig equation.</td>
<td>3. Set up a trig equation.</td>
</tr>
<tr>
<td>4. Cross multiply and solve.</td>
<td>4. Use inverse trig to solve for the angle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Law of Sines (ASA or AAS)</strong></th>
<th><strong>Law of Cosines (SSS or SAS)</strong></th>
</tr>
</thead>
</table>
| \[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\] | In any \(\triangle ABC\), with sides \(a\), \(b\), and \(c\), |
| 1. Find matching side and angle. | \(a^2 = b^2 + c^2 - 2bc \cos \alpha\) |
| 2. Set up the ratio with the matching side and angle. | \(b^2 = a^2 + c^2 - 2ac \cos \beta\) |
| 3. Make another ratio between a known and an unknown. | \(c^2 = a^2 + b^2 - 2ab \cos \gamma\) |
| 4. Set up an equation and solve! | 1. If you have 2 sides and the angle in between, use a formula to find the third side. |
|                             | 2. If you have 3 sides, use a formula to find an angle. Solve for the angle using inverse trig. |
**Ambiguous Case (SSA)**

You will always be given an angle, its opposite side, and a second side.

<table>
<thead>
<tr>
<th>If the given angle is obtuse:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. There is <strong>1 TRIANGLE</strong>.</td>
</tr>
<tr>
<td>2. Use the law of sines to solve the triangle.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>If the given angle is acute and:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Opposite side &gt; the second side means <strong>1 TRIANGLE</strong>. Use the law of sines to solve the triangle.</td>
</tr>
<tr>
<td>2. Opposite side &lt; the second side:</td>
</tr>
<tr>
<td>Calculate the triangle height = bsinα (second side sin angle)</td>
</tr>
<tr>
<td>a. The opposite side &lt; height means <strong>NO TRIANGLE</strong>. Write “No Triangle” and you are finished!</td>
</tr>
<tr>
<td>b. Opposite side = height means <strong>1 TRIANGLE</strong>. Solve the triangle: Use the law of sines to find the second angle. Find the third angle. Use the law of sines to find the third side.</td>
</tr>
<tr>
<td>c. The opposite side &gt; height means <strong>2 TRIANGLES</strong>. Draw two triangles (one acute and one obtuse). Find β₁ (In triangle 1, the angle opposite the second side). Find β₂ (In triangle 2), take 180° − β₁.</td>
</tr>
<tr>
<td>Solve the triangle following the steps from part b.</td>
</tr>
</tbody>
</table>

**Area of a Triangle (SAS)**

\[ A = \frac{1}{2}bc \sin \alpha \quad A = \frac{1}{2}ab \sin \gamma \quad A = \frac{1}{2}ac \sin \beta \]