Solving Trigonometric Equations Part Two

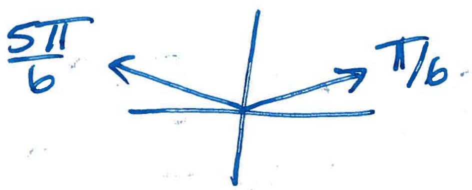
DAY 7

Solve the equation on the interval $0 \le x < 2\pi$.

1. $\sin(2x) = \frac{1}{2}$

Let $\theta = 2x$

$\sin\theta = \frac{1}{2}$



$\theta = \frac{\pi}{6} + 2\pi k$

$2x = \frac{\pi}{6} + 2\pi k$

$x = \frac{\pi}{12} + \pi k$

$x = \frac{\pi}{12} + \frac{12\pi}{12} k$

$x = \frac{\pi}{12} + \frac{12\pi}{12}(1)$

$\theta = \frac{5\pi}{6} + 2\pi k$

$2x = \frac{5\pi}{6} + 2\pi k$

$x = \frac{5\pi}{12} + \pi k$

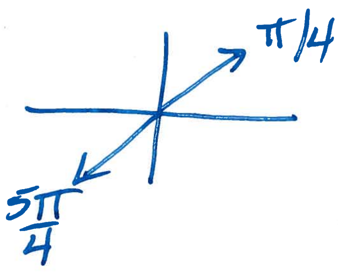
$x = \frac{5\pi}{12} + \frac{12\pi}{12} k$

$$\boxed{x = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}}$$

2. $\tan\left(\frac{1}{2}\theta\right) = 1$

Let $x = \frac{1}{2}\theta$

$\tan x = 1$



$x = \frac{\pi}{4} + \pi k$

$\frac{1}{2}\theta = \frac{\pi}{4} + \pi k$

$\theta = \frac{2\pi}{4} + 2\pi k$

$\theta = \frac{\pi}{2} + \frac{4\pi}{2} k$

$\theta = \frac{\pi}{2} + \frac{4\pi}{2}(1)$

$$\boxed{\theta = \frac{\pi}{2}}$$

Solving Trig Equations (Complex Angle)

General Solutions:

1. Substitute θ for the angle.
2. Isolate the trig.
3. Find the unit circle values.
   Write as a general solution (combine into one statement if you can)
   (ie: $\theta =$ ______ $+2\pi k$ and $\theta =$ ______ $+2\pi k$)
4. Back substitute.
5. Solve for the variable.

Find the solutions on the interval $[0,2\pi)$:

1. Follow the steps above (find the general solutions first)
2. Substitute 1 for k.
3. Combine the terms and find the magic number (numerator)
4. Use the magic number to GENERATE all the solutions on the interval $[0,2\pi)$.
5. Check for extraneous solutions.

EXAMPLE ONE→ Solve the following trigonometric functions on the interval $0 \le x < 2\pi$.

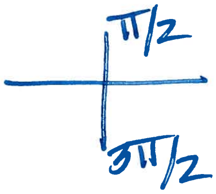
$$1 - \sin^2\left(2x + \frac{\pi}{2}\right) = 0$$

Let $\theta = 2x + \frac{\pi}{2}$

$1 - \sin^2\theta = 0$

$\sqrt{\cos^2\theta} = \sqrt{0}$

$\cos\theta = 0$



$\theta = \frac{\pi}{2} + \pi k$

$2x + \frac{\pi}{2} = \frac{\pi}{2} + \pi k$

$-\frac{\pi}{2} \quad -\frac{\pi}{2}$

$\frac{2x}{2} = \frac{0 + \pi k}{2}$

$x = 0 + \frac{\pi}{2}k$

$x = 0 + \frac{\pi}{2}(1)$

$\boxed{x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}}$

EXAMPLE TWO→ Write the general solution to the trigonometric equation.

$$2\sin^2\left(\frac{\theta}{3}\right) - \sin\left(\frac{\theta}{3}\right) - 1 = 0$$

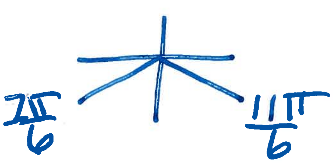
Let $x = \frac{\theta}{3}$

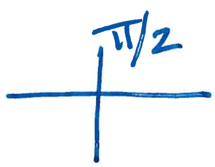
$2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$2\sin x + 1 = 0 \qquad \sin x - 1 = 0$

$\sin x = -\frac{1}{2} \qquad \sin x = 1$





$x = \frac{7\pi}{6} + 2\pi k$	$x = \frac{11\pi}{6} + 2\pi k$	$x = \frac{\pi}{2} + 2\pi k$
$\frac{\theta}{3} = \frac{7\pi}{6} + 2\pi k$	$\frac{\theta}{3} = \frac{11\pi}{6} + 2\pi k$	$\frac{\theta}{3} = \frac{\pi}{2} + 2\pi k$
$\theta = \frac{21\pi}{6} + 6\pi k$	$\theta = \frac{33\pi}{6} + 6\pi k$	$\theta = \frac{3\pi}{2} + 6\pi k$
$\theta = \frac{7\pi}{3} + 6\pi k$	$\theta = \frac{11\pi}{3} + 6\pi k$	

$$\boxed{\theta = \frac{7\pi}{3} + 6\pi k,\ \frac{11\pi}{3} + 6\pi k,\ \frac{3\pi}{2} + 6\pi k}$$

EXAMPLE THREE→ Find the general solution to the trigonometric equation.

$\cos(4x) + \sin(4x)\tan(4x) = 2$, $\theta = 4x$

$\cos\theta + \sin\theta\tan\theta = 2$

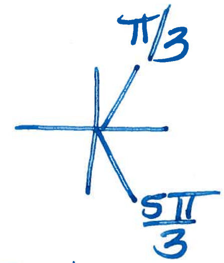
$\cos\theta + \dfrac{\sin\theta\sin\theta}{\cos\theta} = 2$

$\dfrac{\cos^2\theta}{\cos\theta} + \dfrac{\sin^2\theta}{\cos\theta} = 2$

$\dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta} = 2$

$\dfrac{1}{\cos\theta} = 2$

$\cos\theta = \frac{1}{2}$



$\theta = \frac{\pi}{3} + 2\pi k$	$\theta = \frac{5\pi}{3} + 2\pi k$
$4x = \frac{\pi}{3} + 2\pi k$	$4x = \frac{5\pi}{3} + 2\pi k$
$x = \frac{\pi}{12} + \frac{\pi}{2}k$	$x = \frac{5\pi}{12} + \frac{\pi}{2}k$
$x = \frac{\pi}{12} + \frac{6\pi}{12}(1)$	$x = \frac{5\pi}{12} + \frac{6\pi}{2}(1)$

$$\boxed{x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{23\pi}{12}}$$

Quick Check→ Solve the following equations.

$9\sin^2\alpha = \sin^2\alpha + 8\sin\alpha - 2$

$9\sin^2\alpha - \sin^2\alpha - 8\sin\alpha + 2 = 0$

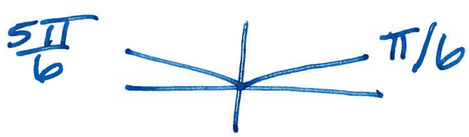
$\dfrac{8\sin^2\alpha - 8\sin\alpha + 2}{2} = \dfrac{0}{2}$

$4\sin^2\alpha - 4\sin\alpha + 1 = 0$

$(2\sin\alpha - 1)^2 = 0$

$2\sin\alpha - 1 = 0$

$\sin\alpha = \frac{1}{2}$



General solution:
$\alpha = \frac{\pi}{6} + 2\pi k$
$\alpha = \frac{5\pi}{6} + 2\pi k$

Solution on the interval $[0, 2\pi]$:
$\alpha = \frac{\pi}{6}, \frac{5\pi}{6}$