Solving Trigonometric Equations HOMEWORK day two

Name: _____

Date: _____ Block: _____

Find the solutions to the trigonometric equations on the interval $[0, 2\pi)$.

1. $\cos\left(2x - \frac{\pi}{3}\right) = 1$

$\theta = 2x - \frac{\pi}{3}$

$\cos\theta = 1$

$\theta = 0 + 2\pi k$

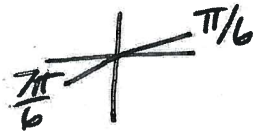
$\theta = 2\pi k$

$2x - \frac{\pi}{3} = 2\pi k$

$+\frac{\pi}{3} \quad +\frac{\pi}{3}$

$2x = \frac{\pi}{3} + 2\pi k$

$x = \frac{\pi}{6} + \pi k$



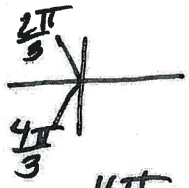
$x = \frac{\pi}{6}, \frac{7\pi}{6}$

2. $\sec\left(\frac{3\theta}{2}\right) = -2$

$x = \frac{3\theta}{2}$

$\sec x = -2$

$\cos x = -\frac{1}{2}$



$x = \frac{2\pi}{3} + 2\pi k$ $\quad x = \frac{4\pi}{3} + 2\pi k$

$\frac{3\theta}{2} = \frac{2\pi}{3} + 2\pi k$ $\quad \frac{3\theta}{2} = \frac{4\pi}{3} + 2\pi k$

$\theta = \frac{4\pi}{9} + \frac{4\pi}{3}k$ $\quad \theta = \frac{8\pi}{9} + \frac{4\pi}{3}$

$\theta = \frac{4\pi}{9}, \frac{16\pi}{9}, \frac{8\pi}{9}$

3. $2\sin^2(\beta + \pi) - 1 = 0$

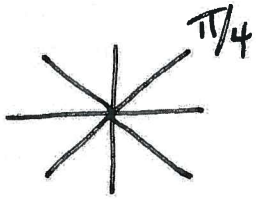
$x = \beta + \pi$

$2\sin^2(x) - 1 = 0$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

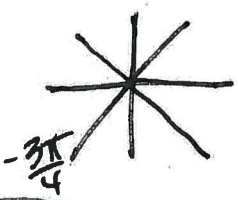
$\sin x = \pm\frac{\sqrt{2}}{2}$



$x = \frac{\pi}{4} + \frac{\pi}{4}k$

$\beta + \pi = \frac{\pi}{4} + \frac{\pi}{4}k$

$\beta = -\frac{3\pi}{4} + \frac{\pi}{4}k$



$\beta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4. $4\cos(\pi\beta) + 3\sqrt{3} = \sqrt{3}$

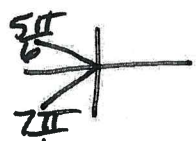
$x = \pi\beta$

$4\cos x + 3\sqrt{3} = \sqrt{3}$

$4\cos x = -2\sqrt{3}$

$\cos x = -\frac{2\sqrt{3}}{4}$

$\cos x = -\frac{\sqrt{3}}{2}$



$x = \frac{5\pi}{6} + 2\pi k$ $\quad x = \frac{7\pi}{6} + 2\pi k$

$\pi\beta = \frac{5\pi}{6} + 2\pi k$ $\quad \pi\beta = \frac{7\pi}{6} + 2\pi$

$\beta = \frac{5}{6} + 2k$ $\quad \beta = \frac{7}{6} + 2k$

$\beta = \frac{5}{6}, \frac{17}{6}, \frac{29}{6}, \frac{7}{6}, \frac{19}{6}, \frac{31}{6}$

Find the general solutions to the trigonometric equations.

7. $\tan\left(\frac{\alpha}{2}+\frac{\pi}{3}\right)=0$ $\quad x=\frac{\alpha}{2}+\frac{\pi}{3}$

$\tan x = 0$

$x = 0 + \pi K$

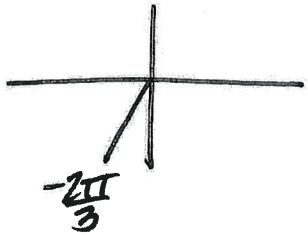
$x = \pi K$

$\frac{\alpha}{2} + \frac{\pi}{3} = \pi K$

$-\frac{\pi}{3} \quad -\frac{\pi}{3}$

$\frac{\alpha}{2} = -\frac{\pi}{3} + \pi K$

$\alpha = -\frac{2\pi}{3} + 2\pi K$



$\alpha = \frac{4\pi}{3}$

on $[0, 2\pi)$

8. $\sec(\pi x) = \tan(\pi x) + \cot(\pi x)$ $\quad \theta = \pi x$

$\sec\theta = \tan\theta + \cot\theta$

$\frac{1}{\cos\theta} = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\frac{1}{\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\frac{1}{\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

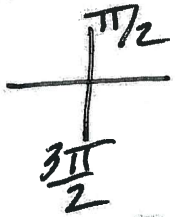
$\cos\theta = \sin\theta\cos\theta$

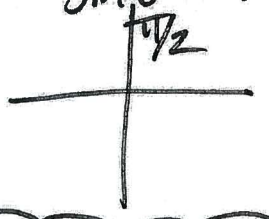
$0 = \sin\theta\cos\theta - \cos\theta$

$0 = \cos\theta(\sin\theta - 1)$

$\cos\theta = 0$ $\qquad$ $\sin\theta - 1 = 0$

$\sin\theta = 1$





$\theta = \frac{\pi}{2} + \pi K$ $\qquad$ $\theta = \frac{\pi}{2} + 2\pi K$

$\pi x = \frac{\pi}{2} + \pi K$

$x = \frac{1}{2} + K$

$\pi x = \frac{\pi}{2} + 2\pi K$

$x = \frac{1}{2} + 2K$

repeat

$x = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$

on $[0, 2\pi)$

5. $\csc^2\left(\frac{x}{3}\right) = \cot\left(\frac{x}{3}\right) - 1$ $\theta = \frac{x}{3}$

$1 + \cot^2\theta = \cot\theta - 1$

$\cot^2\theta - \cot\theta + 2 = 0$

$\theta = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(2)}}{2(1)}$

$\theta = \frac{1 \pm \sqrt{-7}}{2}$ ext.

No Solution

6. $1 - \sin(2\alpha) = \cos(2\alpha)$ $x = 2\alpha$

$(1 - \sin x)^2 = (\cos x)^2$

$1 - 2\sin x + \sin^2 x = \cos^2 x$

$1 - 2\sin x + \sin^2 x = 1 - \sin^2 x$

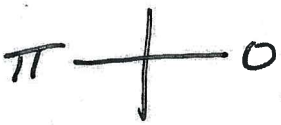
$-2\sin x + 2\sin^2 x = 0$

$-2\sin x(1 - \sin x) = 0$

$-2\sin x = 0$ $\quad$ $1 - \sin x = 0$

$\sin x = 0$ $\quad$ $-\sin x = -1$

$\sin x = 1$



$x = 0 + \pi k$ $\quad$ $x = \frac{\pi}{2} + 2\pi k$

$2\alpha = \pi k$ $\quad$ $2\alpha = \frac{\pi}{2} + 2\pi k$

$\alpha = \frac{\pi}{2}k$ $\quad$ $\alpha = \frac{\pi}{4} + \pi k$

$\alpha = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$

9. $(\sin(\alpha) + \sqrt{2}) = (\cos(\alpha))^2$

$\sin^2\alpha + 2\sqrt{2}\sin\alpha + 2 = \cos^2\alpha$

$\sin^2\alpha + 2\sqrt{2}\sin\alpha + 2 = 1 - \sin^2\theta$

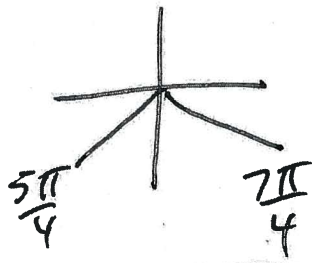
$2\sin^2\alpha + 2\sqrt{2}\sin\alpha + 1 = 0$

$\sin\alpha = \dfrac{-2\sqrt{2} \pm \sqrt{(2\sqrt{2})^2 - 4(2)(1)}}{2(2)}$

$\sin\alpha = \dfrac{-2\sqrt{2} \pm \sqrt{4(2) - 8}}{4}$

$\sin\alpha = \dfrac{-2\sqrt{2} \pm \sqrt{0}}{4}$

$\sin\alpha = -\dfrac{\sqrt{2}}{2}$



$\alpha = \dfrac{5\pi}{4} + 2\pi K$ ~~$\alpha = \dfrac{7\pi}{4} + 2\pi K$~~

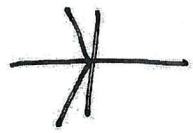
$\alpha = \dfrac{7\pi}{4} + 2\pi K$

10. $4\sec(\pi\theta) + 6 = -2$ $\quad x = \pi\theta$

$4\sec(x) = -8$

$\sec(x) = -2$

$\cos x = -\frac{1}{2}$



$x = \frac{2\pi}{3} + 2\pi K \quad x = \frac{4\pi}{3} + 2\pi K$

$\pi\theta = \frac{2\pi}{3} + 2\pi K \quad \pi\theta = \frac{4\pi}{3} + 2\pi K$

$\theta = \frac{2}{3} + 2K \quad \theta = \frac{4}{3} + 2K$