NOTES: POLYNOMIAL DIVISION

DAY 7

Textbook Chapter 5.5

OBJECTIVE: Today you will learn about how to find zeros using polynomial division!

1. Remainder and Factor Theorem

KEY CONCEPT *For Your Notebook*

Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

Given the number: 20, $\frac{20}{5} = 4$ If one factor is 5, what is the other factor? 4

Given the function: $y = 3x^2 + 10x + 8$ If one factor is $(x + 2)$, how do you find the other factor?

$y = 3x^2 + 4x + 6x + 8$

$y = x(3x+4) + 2(3x+4)$

$y = (x+2)(3x+4)$

24: 1 24, 2 12, 3 8; +4+6

$\frac{3x^2+10x+8}{x+2} = 3x+4$

DIVIDE!

Long Division

There are 2 ways to divide polynomials: long division and synthetic division.

2. $x-2 \overline{) x^3 + 2x^2 - 6x - 9}$

$x^2 + 4x + 2$

$-(x^3 - 2x^2)$

$4x^2 - 6x$

$-(4x^2 - 8x)$

$2x - 9$

$-(2x - 4)$

-5

$x^2 + 4x + 2 + \frac{-5}{x-2}$

3. Divide $f(x) = x^4 + 4x^3 + 16x - 35$ by $x + 5$

$\frac{x^4}{x} = x^3$ zero = −5

−5	1	4	0	16	−35
	↓	−5	5	−30	70
	1	−1	5	−14	35

$1x^3 - 1x^2 + 6x - 14 + \frac{35}{x+5}$

SYNTHETIC DIVISION: Synthetic division works differently!

1. Divide $x^3+2x^2-6x-9 \div (x-2)$ (2,0)

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -6 & -9 \\ & \downarrow & 2 & 8 & 4 \\ \hline & 1 & 4 & 2 & -5 \end{array}$$

a. First, the divisor must be in the form: (x - k) What is k in the divisor (x - 2)?___

b. Write down the coefficients of each term. If a term is missing, you must use a zero as a place holder.

c. Then drop, multiply and add, multiply and add, etc.

d. The answer is found by filling in the terms backwards. The last term is the remainder.

$x^2 + 4x + 2 + \dfrac{-5}{x-2}$

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2. **Factor** ***f(x)*** **= x^3 - $2x^2$ - 40x - 64 completely given that (x - 8) is a factor.** (8,0)

$$\begin{array}{r|rrrr} 8 & 1 & -2 & -40 & -64 \\ & \downarrow & 8 & 48 & 64 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

x^2+6x+8

a. **Synthetic Division** - *because x - 8 is a factor, f(8) = 0, and x = 8 is a solution.*

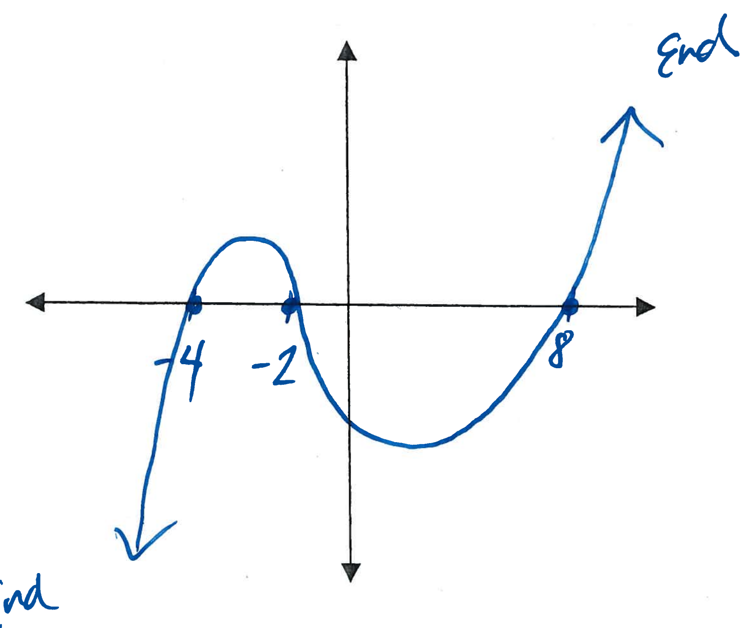
b. Write as a product of 2 factors. $f(x) = (x-8)(x^2+6x+8)$

c. Then completely factor.

f(x) = ($x-8$) ($x+4$) ($x+2$)

$x = 8, -4, -2$

d. Graph: ***f(x)*** = x^3 - $2x^2$ - 40x - 64



Degree = 3
LC = +1

PRACTICE: POLYNOMIAL DIVISION

DAY 7

1. Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $(x + 2)$ is a factor.

a. **Synthetic Division** - because x+2 is a factor, f(-2) = 0, and x = -2 is a solution.

$$\begin{array}{r|rrrr} -2 & 3 & -4 & -28 & -16 \\ & \downarrow & -6 & 20 & 16 \\ \hline & 3 & -10 & -8 & 0 \end{array}$$

b. Write as a product of 2 factors. $f(x) = (x+2)(3x^2-10x-8)$

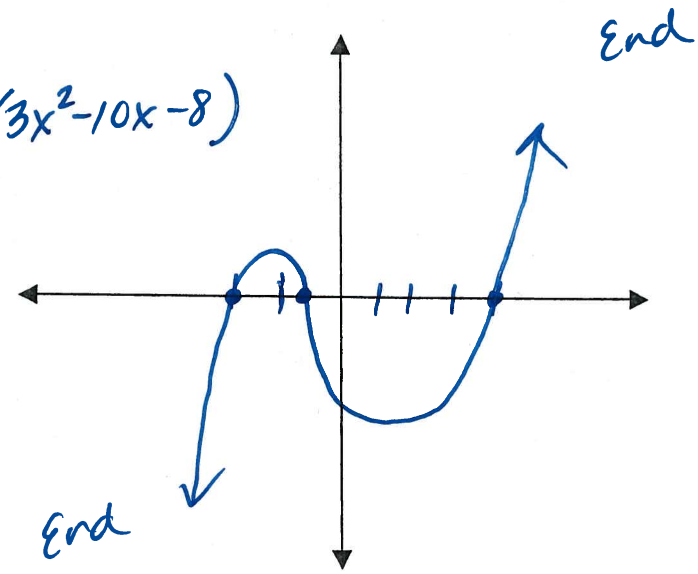
c. Then completely factor.

$f(x) = (\ x+2\ )(3x+2)(\ x-4\ )$

$x = -2,\ -\frac{2}{3},\ 4$

d. Graph!

Degree = 3
LC = +3



2. One zero of $f(x) = x^3 + x^2 - 16x - 16$ is 4. Find all the zeros.

a. **Synthetic Division** - *because f(4) = 0, x = 4 is a zero/solution, and (x - 4) is a factor.*

$$\begin{array}{r|rrrr} 4 & 1 & 1 & -16 & -16 \\ & \downarrow & 4 & 20 & 16 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$

b. Write as a product of 2 factors. $f(x) = (x-4)(x^2+5x+4)$

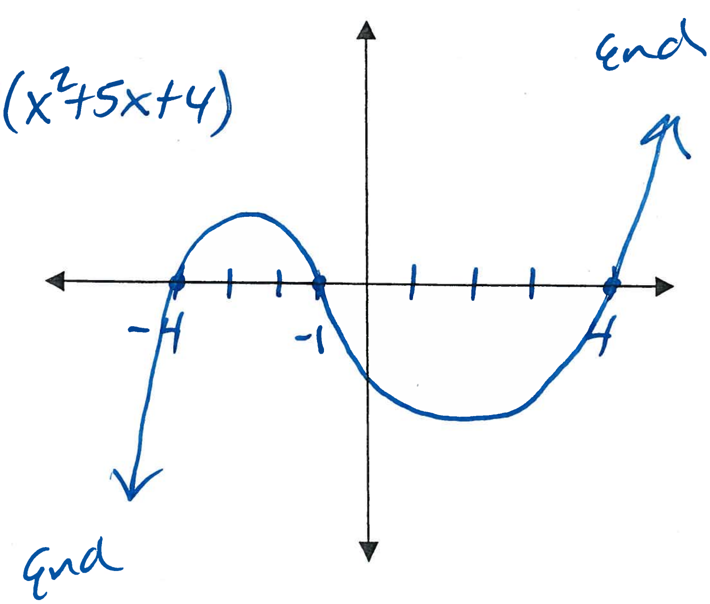
c. Then completely factor.

$f(x) = (\ x-4\ )(\ x+4\ )(\ x+1\ )$

$x = 4,\ -4,\ -1$

d. Graph!

Degree = 3
LC = +1

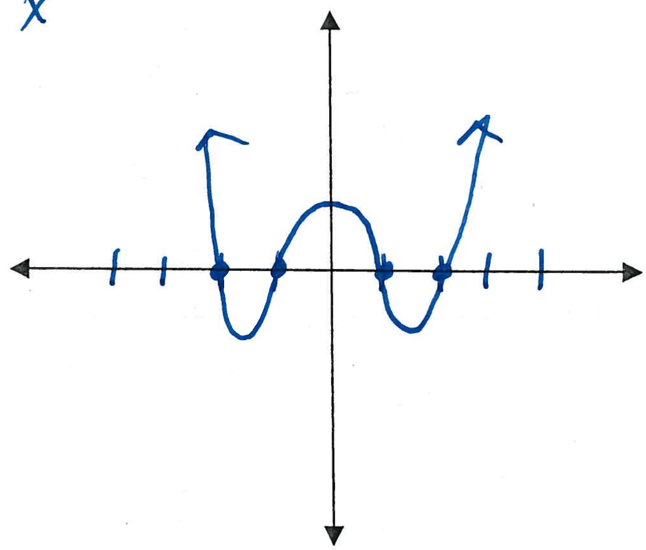


3. **GRAPH**: ~~$f(x) = x^4 + 4x^3 + 16x - 35$ by $x + 5$ using synthetic division.~~

One zero of $f(x) = x^4 + 3x^2 - 4$ is 1. Find all the zeros.

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 3 & 0 & -4 \\ & \downarrow & 1 & 1 & 4 & 4 \\ \hline & 1 & 1 & 4 & 4 & 0 \end{array}$$

$$\frac{x^4}{x} = x^3$$



$$f(x) = (x-1)(x^3+x^2+4x+4)$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & 4 & 4 \\ & \downarrow & -1 & 0 & -4 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$f(x) = (x-1)(x+1)(x^2-4)$$

$$f(x) = (x-1)(x+1)(x-2)(x+2)$$

Degree = 4

LC = 1

4. Divide $f(x) = x^4 + 4x^3 + 16x - 35$ by $x + 5$ using synthetic division.

$(-5, 0)$

$$\frac{x^4}{x} = x^3$$

$$\begin{array}{r|rrrrr} -5 & 1 & 4 & 0 & 16 & -35 \\ & \downarrow & -5 & 5 & -25 & 45 \\ \hline & 1 & -1 & 5 & -9 & 10 \end{array}$$

$$\frac{x^4+4x^3+16x-35}{x+5} = x^3-1x^2+5x-9+\frac{10}{x+5}$$