NOTES: COUNTING PRINCIPLES

DAY 3

Textbook Chapter 10.1, 10.2

OBJECTIVE: Today you will learn about various methods of counting!

Fundamental Counting Prinicple

The fundamental counting principle can be used to determine the number of possible outcomes when there are two or more characteristics.

> **Fundamental Counting Principle**
>
> **If an event has *m* possible outcome and another independent event has *n* possible outcomes, then there are *m·n* possible outcomes for the two events to occur together.**

1. For Christmas you want to get your parents a framed family picture. At the framing store, there are 4 different styles each available in 5 different colors. You decide to use a blue mat board and there are 3 different shades of blue to choose from. How many different frames can you create?

   $4 \cdot 5 \cdot 3 =$ 60 frames

2. In one Austrian province, a vehicular plate number consists of 4 letters and 3 numbers. (example: BACK_301) Using the fundamental principle of counting (one more time!), how many plate numbers are available?

   26 26 26 26 – 10 10 10

   456,976,000 plate numbers

The factorial symbol (!)

$4! = 4 \cdot 3 \cdot 2 \cdot 1$

$0! = 1$

Password

Permutation: (Picky about order)

The ordering of n objects

Permutation of n Objects taken r at a time:

The number of permutations of r objects taken from a group of n distinct objects is denoted

$${}_nP_r = \frac{n!}{(n-r)!}$$

OR: _ _ _ _

EXAMPLE ONE→ There are eight teams competing in a bobsleigh skeleton race.

A) How many different ways can each team finish?

$${}_8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = 40{,}320 \text{ ways}$$

B) How many ways can teams be awarded the gold, silver, and bronze metals?

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot7\cdot6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 8\cdot7\cdot6 = 336 \text{ ways}$$

EXAMPLE TWO → You are burning a CD demo for your band. Your band has 12 songs stored on your computer, but room enough on the demo for only 4 songs. How many different orders can you burn the 4 songs onto the demo?

$${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 12\cdot11\cdot10\cdot9 = 11{,}880 \text{ CD's}$$

Group

Combination: (don't Care about order)

The combination on n objects taken r at a time is denoted

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

EXAMPLE THREE → There are eight teams competing in the bobsleigh skeleton race. Only 6 teams make it to the next round.

A) How many different COMBINATIONS of teams can make it to the next round?

$${}_8C_6 = \frac{8!}{(8-6)!\,6!} = \frac{8!}{2!\,6!} = \frac{8\cdot7}{2\cdot1} = \frac{56}{2} = 28 \text{ teams}$$

B) How many ways can the top three be chosen?

$${}_8C_3 = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = \frac{8\cdot7\cdot6}{3\cdot2\cdot1} = \frac{336}{6} = 56 \text{ ways}$$

EXAMPLE FOUR → You are burning a CD demo for your band. Your band has 12 songs stored on your computer, but room enough on the demo for only 4 songs. How many different combinations of songs can you put on the demo?

$${}_{12}C_4 = \frac{12!}{(12-4)!\,4!} = \frac{12!}{8!\,4!} = \frac{12\cdot11\cdot10\cdot9}{4\cdot3\cdot2\cdot1} = \frac{11{,}880}{24} = 495$$

PRACTICE: COUNTING PRINCIPLES DAY 3

DIRECTIONS:

- ✓ Indicate whether this problem is an example of permutations or combinations.
- ✓ Justify this answer.
- ✓ Solve the problem.

P 2. In a race in which six automobiles are entered and there are no ties, in how many ways can the first three finishers come in?

$$_6P_3 = \frac{6!}{3!} = \underline{6}\cdot\underline{5}\cdot\underline{4} = 120 \text{ ways}$$

C 3. A book club offers a choice of 8 books from a list of 40. In how many ways can a member make a selection?

$$_{40}C_8 = \frac{40!}{(40-8)!\,8!} = \frac{40!}{32!\,8!} = \frac{\cancel{40}\cdot\cancel{39}\cdot\cancel{38}\cdot 37\cdot\cancel{36}\cdot\cancel{35}\cdot\cancel{34}\cdot\cancel{33}}{\cancel{8}\cdot 7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 76{,}904{,}685$$

C 4. A medical researcher needs 6 people to test the effectiveness of an experimental drug. If 13 people have volunteered for the test, in how many ways can 6 people be selected?

$$_{13}C_6 = \frac{13!}{(13-6)!\,6!} = \frac{13!}{7!\,6!} = \frac{13\cdot 12\cdot 11\cdot 10\cdot 9\cdot 8}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 1716 \text{ ways}$$

C 5. From a club of 20 people, in how many ways can a group of three members be selected to attend a conference?

$$_{20}C_3 = \frac{20!}{17!\,3!} = \frac{20\cdot 19\cdot 18}{3\cdot 2\cdot 1} = \frac{6840}{6} = 1140 \text{ groups}$$

P 6. How many different four-letter passwords can be formed from the letters A, B, C, D, E, F, and G if no repetition of letters is allowed?

$$_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \underline{7}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4} = 840 \text{ passwords}$$

7. For Christmas you want to get your parents a framed family picture. At the framing store, there are 4 different styles each available in 5 different colors. You decide to use a blue mat board and there are 3 different shades of blue to choose from. How many different frames can you create?

$$4\cdot 5\cdot 3 = 60 \text{ frames}$$

DIRECTIONS: Solve each of the following applications. Show the formula used.

8. An election ballot asks voters to select three city commissioners from a group of six candidates. In how many ways can this be done?

9. A four-person committee is to be elected from an organizations' membership of 11 people. How many different committees are possible?

10. Of 12 possible books, you plan to take 4 with you on vacation. How many different collections of 4 books can you take?

11. Of the 100 people in the U.S. Senate, 18 serve on the Foreign Relations Committee. How many ways are there to select Senate members for this committee (assuming party affiliation is not a factor in selection)?

12. A mathematics exam consists of 10 multiple-choice questions and 5 open-ended problems in which all work must be shown. If an examinee must answer 8 of the multiple-choice questions and 3 of the open-ended problems, in how many ways can the questions and problems be chosen?

13. How many different six-letter arrangements can be made from the letters in the word HOCKEY.

14. Your local ice cream store has 5 flavors, 2 types of cones, and 10 toppings. How many different cones are possible?

15. Write a word problem that can be solved by evaluating ${}_{21}C_3$. Now solve it!