Ch 4.1 and 4.2 Composition and Inverse Functions DAY 7

Find the domain of composition functions.

1. Suppose that $f(x)=\frac{1}{x+2}$ and $g(x)=\frac{4}{x-1}$

Find f(g(x)) and then state the domain of the composite function.

Step 1:
Find the composite function.

Step 2:
Find the domain of the composite function.

Step 3:
Check with the inside function. Apply any restrictions from the inside function to the domain of the composition function.

$f(g(x))$

$= \dfrac{1}{\frac{4}{x-1}+2}$

$= \dfrac{1}{\frac{4}{x-1}+\frac{2(x-1)}{x-1}}$

$= \dfrac{1}{\frac{4+2x-2}{x-1}}$

$= 1 \div \dfrac{2x+2}{x-1}$

$= 1 \cdot \dfrac{x-1}{2x+2}$

$= \boxed{\dfrac{x-1}{2x+2}}$

Domain:

$f(g(x))$: $2x+2 \neq 0$
$2x \neq -2$
$x \neq -1$

$g(x)$: $x \neq 1$

$\boxed{D: \mathbb{R},\ x \neq -1, 1}$
OR
$(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

2. Find $(f \circ f)(x)$ and then state the domain of the composite function.

$f(f(x))$

$= \dfrac{1}{\frac{1}{x+2}+2}$

$= \dfrac{1}{\frac{1}{x+2}+\frac{2(x+2)}{x+2}}$

$= \dfrac{1}{\frac{1+2x+4}{x+2}}$

$= 1 \div \dfrac{2x+5}{x+2}$

$= 1 \cdot \dfrac{x+2}{2x+5}$

$= \boxed{\dfrac{x+2}{2x+5}}$

Domain:

$f(f(x))$: $2x+5 \neq 0$
$2x \neq -5$
$x \neq -5/2$

$f(x)$: $x \neq -2$

$\boxed{D: \mathbb{R},\ x \neq -2, -\frac{5}{2}}$

Evaluate each expression using the values in the table.

x	-3	-2	-1	0	1	2	3
f(x)	-7	-5	-3	-1	3	5	5
g(x)	8	3	0	-1	0	3	8

3. $(f \circ g)(1)$

$f(g(1))$

$f(0)$

$\boxed{-1}$

4. f(g(−1))

$f(0)$

$\boxed{-1}$

5. f(f(−1))

$f(-3)$

$\boxed{-7}$

6. $(g \circ f)(0)$

$g(f(0))$

$g(-1)$

$\boxed{0}$

Finding the Components of a Composite Function

7. Find functions *f* and *g* such that $(f \circ g)$ = H, given $H(x)=(2x+3)^4$

f(x) = x^4

g(x) = $2x+3$

$f(g(x)) = H(x)$

$f(g(x)) = (2x+3)^4$

8. Find functions *f* and *g* such that $(f \circ g)$ = H, given $H(x)=\sqrt{5x-1}+2$.

f(x) = $\sqrt{x}+2$

g(x) = $5x-1$

$f(g(x)) = H(x)$

$f(g(x)) = \sqrt{5x-1}+2$

Inverse Functions

a) To verify that f^{-1} is the inverse of f, show that $f(g(x)) = g(f(x)) = x$

b) The graphs of f and f^{-1} are symmetric with respect to the line $y = x$.

c) Domain of f = Range of f^{-1}; Range of f = Domain of f^{-1}

d) To find the range of a one-to-one function f, find the domain of the inverse function f^{-1}

> How to find the inverse of a function
> 1. Switch variables x and y
> 2. Solve for y
> 3. Check the result by showing $f(g(x)) = g(f(x)) = x$

8. Given $f(x) = 4x + 2$, find its inverse. Sketch f and f^{-1} on the same coordinate axes.

$y = 4x + 2$

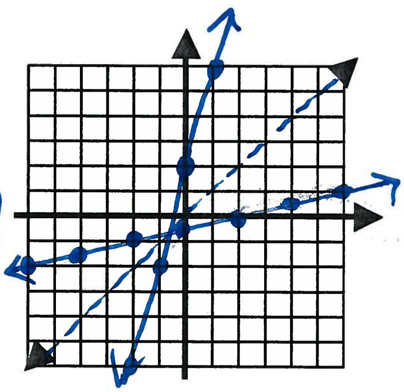
$x = 4y + 2$

$x - 2 = 4y$

$\frac{x-2}{4} = y$

$\frac{1}{4}x - \frac{1}{2} = y$

$\boxed{f^{-1}(x) = \frac{1}{4}x - \frac{1}{2}}$



One-to-One Functions

If a function is **one-to-one,** then the graph of the function will pass the **Horizontal Line test.**

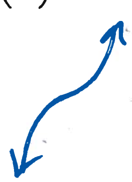
If a function f is one-to-one, then it has an inverse function f^{-1}

9. Determine whether the function is one-to-one.

a) $f(x) = x^2 + 4$

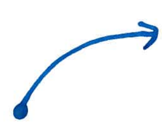
$\boxed{\text{No}}$

b) $f(x) = x^3 - 2$



$\boxed{\text{Yes}}$

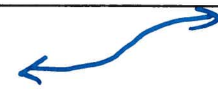
c) $f(x) = \sqrt{3x - 5}$



$\boxed{\text{Yes}}$

PRACTICE

Determine if each function is one-to-one.

1. $f(x)=x^4+3$  No	2. $f(x)=\sqrt[3]{x}-2$  yes
3. $f(x)=\lvert x-3\rvert+4$  No	4. $f(x)=5x$  yes

5. Verify that the functions f and g are inverses of each other.

f(x) = 3x – 8 and g(x) = $\dfrac{x+8}{3}$

f(g(x)) = $3\left(\dfrac{x+8}{3}\right)-8$

$= x+8-8$

$= x$ ✓

g(f(x)) = $\dfrac{3x-8+8}{3}$

$= \dfrac{3x}{3}$

$= x$ ✓

6. Given $f(x) = 3x + 5$, find $f^{-1}(x)$. $y = 3x+5$ $x = 3y+5$ $\frac{x-5}{3} = \frac{3y}{3}$ $\frac{x-5}{3} = y$ $\boxed{f^{-1}(x) = \frac{x-5}{3}}$	7. Given $f(x) = x^3 - 4$, find $f^{-1}(x)$ $y = x^3-4$ $x = y^3-4$ $x-4 = y^3$ $\sqrt[3]{x-4} = y$ $\boxed{f^{-1}(x) = \sqrt[3]{x+4}}$

8. Given the function $f(x)=(x-4)^2$, $x \geq 4$, find its inverse.

$(4,0)$

$y = (x-4)^2$

$x = (y-4)^2$

$\sqrt{x} = y-4$

$\sqrt{x}+4 = y$ → $\boxed{f^{-1}(x) = \sqrt{x}+4}$

Sketch f and f^{-1} on the same coordinate axes.