Solving Trigonometric Equations NOTES

DAY 6

Warm up→ Solve the following equations

1. $2x^2 + 1 = 3x$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$2x - 1 = 0 \qquad x - 1 = 0$

$x = \frac{1}{2} \qquad x = 1$

$\boxed{x = \frac{1}{2}, 1}$

2. $x^2 - 10 = 4x$

$x^2 - 4x - 10 = 0$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 + 40}}{2}$

$x = \frac{4 + \sqrt{56}}{2}$

$x = \frac{4 + \sqrt{4}\sqrt{14}}{2}$

$x = \frac{4 + 2\sqrt{14}}{2}$

$\boxed{x = 2 + \sqrt{14}}$

3. $x = 3x$

$-x \quad -x$

$\frac{0}{2} = \frac{2x}{2}$

$\boxed{0 = x}$

4. $x^2 = 3x$

$-3x \quad -3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0 \qquad x - 3 = 0$

$+3 \quad +3$

$x = 3$

$\boxed{x = 0, 3}$

Solving Trig Equations

Solving Trig Equations on the interval $[0,2\pi)$.

Step 1: Isolate the trig.

Step 2: Find the unit circle values.

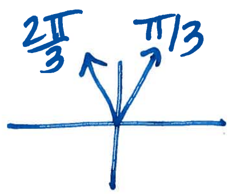
Step 3: Check for extraneous solutions.

1. $2\sin(\alpha) - \sqrt{3} = 0$

$$\quad\; +\sqrt{3} \quad +\sqrt{3}$$

$$\frac{2\sin\alpha}{2} = \frac{\sqrt{3}}{2}$$

$$\sin\alpha = \frac{\sqrt{3}}{2}$$



$$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$$

On the interval $[0, 2\pi)$:

$$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$$

General Solutions (all):

$$\alpha = \frac{\pi}{3} + 2\pi k, \frac{2\pi}{3} + 2\pi k$$

(k is an integer)

2. $9\tan^2(\theta) + 2 = 5$

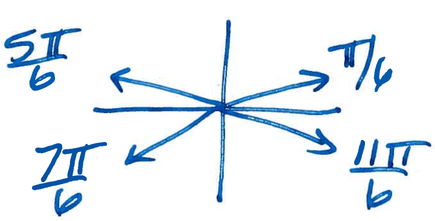
$$\quad -2 \quad -2$$

$$\frac{9\tan^2\theta}{9} = \frac{3}{9}$$

$$\sqrt{\tan^2\theta} = \sqrt{\frac{1}{3}}$$

$$\tan\theta = \pm\frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\tan\theta = \pm\frac{\sqrt{3}}{3}$$



On the interval $[0, 2\pi)$:

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

General Solutions (all):

$$\theta = \frac{\pi}{6} + \pi k, \frac{5\pi}{6} + \pi k$$

EXAMPLE THREE→ Find the general solution to the trigonometric equation.

$$\sin^3(\theta) = 4\sin^2(\theta)$$

$$-4\sin^2\theta \quad -4\sin^2\theta$$

$$\sin^3\theta - 4\sin^2\theta = 0$$

$$\sin^2\theta(\sin\theta - 4) = 0$$

$$\sin^2\theta = 0 \quad \Big| \quad \sin\theta - 4 = 0$$

$$\sin\theta = 0 \quad \Big| \quad \cancel{\sin\theta = 4}$$

π 0 No Solution

$$\boxed{\begin{array}{l}\text{on the interval } [0, 2\pi): \\ \theta = 0, \pi \\ \\ \text{General Solutions:} \\ \theta = 0 + \pi K\end{array}}$$

EXAMPLE FOUR→ Find the general solution to the trigonometric equation.

$$2\cos^2 x + 7\cos x = 4$$

$$-4 \quad -4$$

$$2\cos^2 x + 7\cos x - 4 = 0$$

$$(2\cos x - 1)(\cos x + 4) = 0$$

$$2\cos x - 1 = 0 \qquad \cos x + 4 = 0$$

$$\cos x = \frac{1}{2} \qquad \cancel{\cos x = -4}$$

$\pi/3$ $5\pi/3$

$$\boxed{\begin{array}{l}\text{on the interval } [0, 2\pi): \\ x = \frac{\pi}{3}, \frac{5\pi}{3} \\ \\ \text{General Solutions:} \\ x = \frac{\pi}{3} + 2\pi K, \\ \quad \frac{5\pi}{3} + 2\pi K\end{array}}$$

EXAMPLE FIVE→ Find the solution to the trigonometric equation on the interval $[0, 2\pi]$.

$$(1 + \cos\theta)^2 = (\sin\theta)^2$$

$$(1 + \cos\theta)(1 + \cos\theta) = \sin^2\theta$$

$$1 + 2\cos\theta + \cos^2\theta = \sin^2\theta$$

$$\cancel{1} + 2\cos\theta + \cos^2\theta = (\cancel{1} - \cancel{\cos^2\theta})$$

$$-\cancel{1} \qquad + \cos^2\theta \qquad -\cancel{1} + \cancel{\cos^2\theta}$$

$$2\cos\theta + 2\cos^2\theta = 0$$

$$2\cos^2\theta + 2\cos\theta = 0$$

$$2\cos\theta(\cos\theta + 1) = 0$$

$$2\cos\theta = 0 \qquad \cos\theta + 1 = 0$$

$$\cos\theta = 0 \qquad \cos\theta = -1$$

$\pi/2$ $3\pi/2$ π

$$\boxed{\begin{array}{l}\text{on the interval } [0, 2\pi): \\ \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \pi \\ \\ \text{General Solutions:} \\ \theta = \frac{\pi}{2} + \pi K, \\ \quad \pi + 2\pi K\end{array}}$$

EXAMPLE SIX→ Find the solution to the trigonometric equation on the interval $[0,\ 2\pi]$.

$\sin\beta = \tan\beta$

$\cos\beta\,(\sin\beta) = \left(\dfrac{\sin\beta}{\cancel{\cos\beta}}\right)\cancel{(\cos\beta)}$

$\sin\beta\cos\beta = \sin\beta$

$-\sin\beta \quad -\sin\beta$

$\sin\beta\cos\beta - \sin\beta = 0$

$\sin\beta(\cos\beta - 1) = 0$

$\sin\beta = 0$ — π, 0

$\cos\beta - 1 = 0$

$\cos\beta = 1$ — 0

$$\boxed{\begin{array}{l}\text{On the interval } [0, 2\pi): \beta = 0, \pi \\ \text{General Solution}: \beta = 0 + \pi K\end{array}}$$

EXAMPLE SEVEN→ Find the general solution to the trigonometric equation.

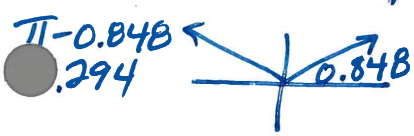
$12\sin(x) - 3 = 6$

$+3 \quad +3$

$\dfrac{12\sin x}{12} = \dfrac{9}{12}$

$\sin x = \dfrac{3}{4}$

$x = \sin^{-1}\left(\frac{3}{4}\right)$



$$\boxed{\begin{array}{l}\text{On the interval } [0, 2\pi): \\ \quad x \approx 0.848, 2.294 \\ \text{General Solution}: \\ \quad x \approx 0.848 + 2\pi K, \\ \quad\quad 2.294 + 2\pi K\end{array}}$$

EXAMPLE EIGHT→ Find the general solution to the trigonometric equation.

$\cos^2(\theta) - 5\cos(\theta) + 2 = 0$

Let $x = \cos\theta$

$x^2 - 5x + 2 = 0$

$x = \dfrac{+5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{25 - 8}}{2}$

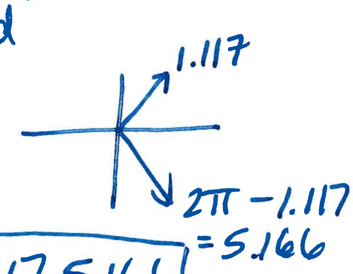
$x = \dfrac{5 \pm \sqrt{17}}{2}$

$\cos\theta = \dfrac{5 \pm \sqrt{17}}{2}$

$\theta = \cancel{\cos^{-1}\left(\dfrac{5+\sqrt{17}}{2}\right)}$, $\cos^{-1}\left(\dfrac{5-\sqrt{17}}{2}\right)$

undefined

$\theta \approx 1.117$



$$\boxed{\begin{array}{l}\text{On } [0, 2\pi): \theta = 1.117, 5.166 \\ \text{General Solution}: \theta = 1.117 + 2\pi K, \\ \quad\quad 5.166 + 2\pi K\end{array}}$$

Homework: Worksheet