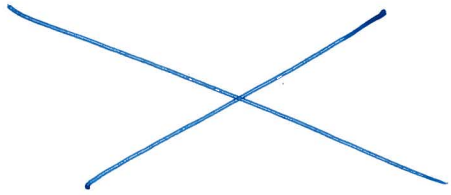
Solving Logarithmic and Exponential Equations
PART TWO

DAY 6

	Exponential Equations	Logarithmic Equations
Re-writing/Converting	Convert to log form OR: log both sides use power rule Ex: $2^x = 10$	Convert to exponential OR: Exponentiate both sides. Simplify & solve. Ex: $\log_2 x = 3$
Same Bases (equating exp/logs)	Set exponents equal. Solve. Ex: $5^{2x-1} = 5^{4x}$	Set log values equal Solve. Ex: $\log(x-3) = \log 7$
Similar Bases (changing the bases)	Rewrite Bases. Set exponents equal Ex: $(\frac{1}{2})^{x+1} = 4^{x-3}$	Use change of base Simplify Combine logs. Equate logs. Ex: $\log_2 x = \log_8 x+3$
Different bases (difficult!)	Log both sides. Use power rule. Distribute. Put x-terms on one side. Factor! Ex: $2^{x+1} = 5^{3x-2}$ Solve.	
Using exp/log properties	Ex: $8^x = (2^3)^x = 2^{3x}$ Ex: $(\frac{1}{2})^{x+1} = 2^{-(x+1)}$	$\log x^n = n \log x$ $\log x + \log y = \log xy$ $\log x - \log y = \log(\frac{x}{y})$
Applications (Interest)	$A = P(1 + \frac{r}{n})^{nt}$ $A = Pe^{rt}$	← Same !!

1. Changing the logarithmic base:

a) $\log_2(x+1) - \log_4(x) = 1$

$\log_4(x+1)^2 - \log_4(x) = 1$

$\log_4 \frac{(x+1)^2}{x} = 1$

$4^1 = \frac{(x+1)^2}{x}$

$4x = (x+1)^2$

$4x = x^2+2x+1$

$0 = x^2-2x+1$

$0 = (x-1)^2$

$x = 1$

b) $\log_2(3x+2) - \log_4 x = 3$

$\log_4(3x+2)^2 - \log_4 x = 3$

$\log_4 \frac{(3x+2)^2}{x} = 3$

$4^3 = \frac{(3x+2)^2}{x}$

$64x = (3x+2)^2$

$64x = 9x^2+12x+4$

$0 = 9x^2-52x+4$

2. Exponential Equations:

a) $2^{2x} - 2^x - 12 = 0$

$(2^x)^2 - (2^x) - 12 = 0$

$a^2 - a - 12 = 0$

$(a-4)(a+3) = 0$

$(2^x-4)(2^x+3) = 0$

$2^x - 4 = 0$ $\quad 2^x + 3 = 0$

$2^x = 4$ $\quad 2^x = -3$ No Soln.

$x = 2$

b) $\left(\frac{6}{5}\right)^x = \left(\frac{1}{2}\right)^{-x}$

$\left(\frac{6}{5}\right)^x = \left(\frac{2}{1}\right)^x$

3. Properties: Condense and expand.

a) $2\log_3 5 + 4\log_3 a + \frac{1}{2}\log_3 b$

$\log_3 5^2 + \log_3 a^4 + \log_3 b^{1/2}$

$\log_3 25a^4\sqrt{b}$

b) $\log_4 \left(\frac{(a-1)^2 b}{a^3 (b+3)^5} \right)$

$\log_4 (a-1)^2 + \log_4 b - \log_4 a^3 - \log_4 (b+3)^5$

$2\log_4(a-1) + \log_4 b - 3\log_4 a - 5\log_4(b+3)$

4. Rewrite Bases:

(a) $\left(\frac{4}{7}\right)^{x+2}=\left(\frac{16}{49}\right)$

$\left(\frac{4}{7}\right)^{x+2}=\left(\frac{4}{7}\right)^{2}$

$x+2=2$

$\boxed{x=0}$

(b) $\left(\frac{8}{27}\right)^{5x}=\left(\frac{4}{9}\right)$

$\left(\frac{2^3}{3^3}\right)^{5x}=\frac{4}{9}$

$\frac{2^{15x}}{3^{15x}}=\frac{2^2}{3^2}$

$\left(\frac{2}{3}\right)^{15x}=\left(\frac{2}{3}\right)^{2}$

$15x=2$

$\boxed{x=\frac{2}{15}}$

5. Equations with Radicals:

a) $2^x=32\sqrt{2}$

$2^x=32\cdot 2^{1/2}$

$2^x=2^5\cdot 2^{1/2}$

$2^x=2^{11/2}$

$\boxed{x=\frac{11}{2}}$

b) $3^x=27\sqrt{3}$

$3^x=3^3\cdot 3^{1/2}$

$3^x=3^{7/2}$

$\boxed{x=\frac{7}{2}}$