

NOTES: GRAPHING RATIONAL FUNCTIONS 1

DAY 6

Textbook Chapter 8.2

OBJECTIVE: Today you will learn how to graph rational functions!

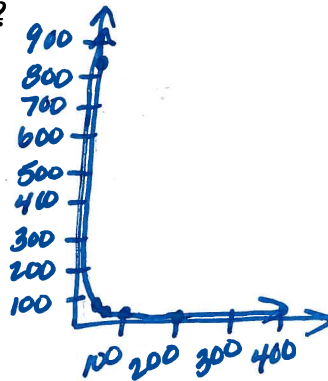
For a fundraising project, you decide to create t-shirts for the Run for Hope. The cost of the artwork and permission to use it costs \$850. In addition to these "one-time" charges, the unit cost of printing each t-shirt is \$3.25.

- a. Write a model that gives the average cost per t-shirt as a function of the number of t-shirts made.

$$\text{Average Cost} = \frac{850 + 3.25x}{x}$$

- b. Graph the model and use the graph to estimate the number of t-shirts you need to print before the average cost drops to \$5 a t-shirt. What happens to the average cost as the number of t-shirts printed increases?

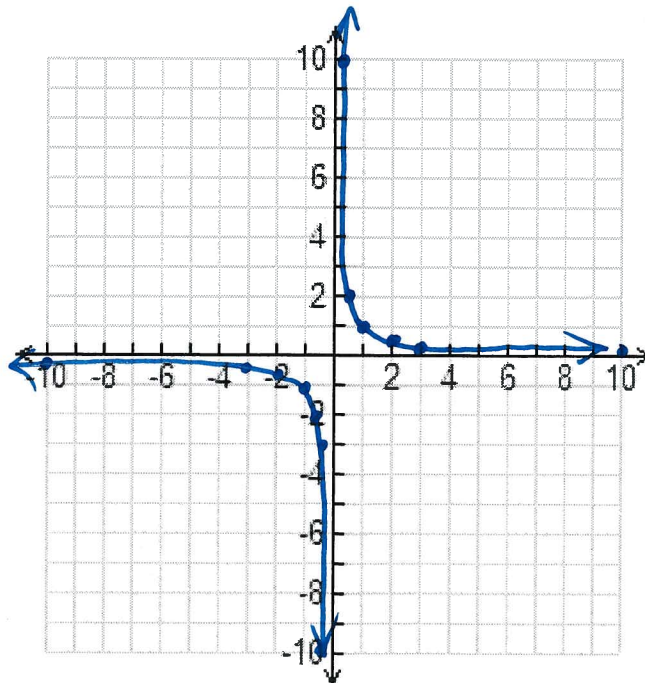
# Shirts	Avg Cost
0	Error
1	853.25
10	88.25
50	30.25
100	11.75
200	7.5



GRAPHING RATIONAL FUNCTIONS

$$f(x) = \frac{1}{x}$$

Key Features:



x	y
1	1
2	1/2
3	1/3
10	1/10
1/2	2
1/3	3
1/10	10
-1	-1
-2	-1/2
-3	-1/3
-10	-1/10
-1/2	-2
-1/3	-3
-1/10	-10
0	ERR

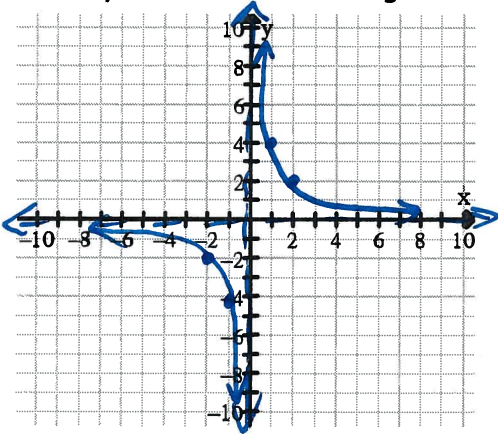
Graph a Rational Function of the Form $y = \frac{a}{x}$

1. **Draw asymptotes** (dotted lines) at $y = 0$ and $x = 0$
2. **Plot 2 points** on the left and right side of the vertical asymptote (use easy numbers)
3. **Draw the branches** of the hyperbola through the plotted points.

1. Graph $y = \frac{4}{x}$.

VA: $x=0$
HA: $y=0$

Identify the domain and range.



x	y
-2	-2
-1	-4
0	ERR
1	4
2	2

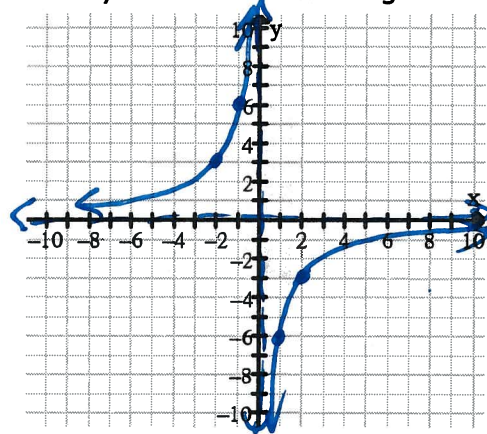
Domain: $\mathbb{R}, x \neq 0$

Range: $\mathbb{R}, y \neq 0$

2. Graph $y = \frac{-6}{x}$.

VA: $x=0$
HA: $y=0$

Identify the domain and range.



x	y
-2	3
-1	6
0	ERR
1	-6
2	-3

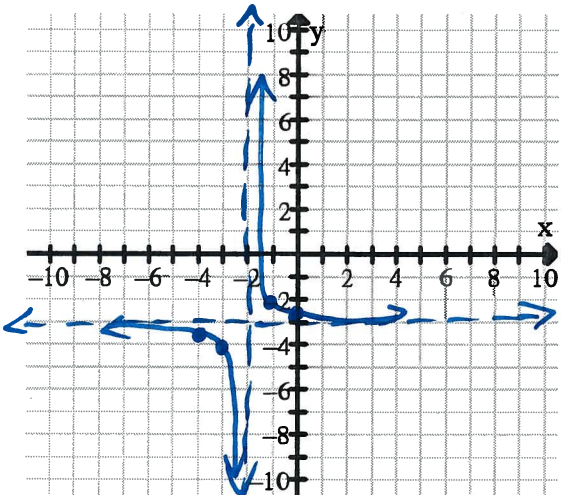
Domain: $\mathbb{R}, x \neq 0$

Range: $\mathbb{R}, y \neq 0$

Graph a Rational Function of the Form $y = \frac{a}{x-h} + k$

3. Graph $y = \frac{1}{x+2} - 3$

Asymptotes: $x = -2$ $y = -3$



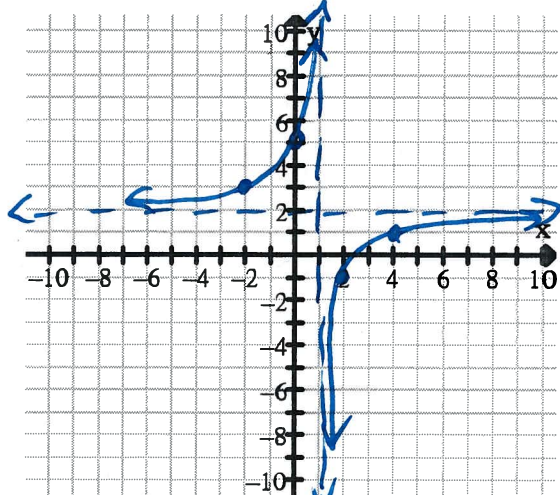
x	y
-4	-3.5
-3	-4
-2	ERR
-1	-2
0	-2.5

Domain: $\mathbb{R}, x \neq -2$

Range: $\mathbb{R}, y \neq -3$

4. Graph $y = \frac{-3}{x-1} + 2$

Asymptotes: $x = 1$ $y = 2$



x	y
-2	3
0	5
1	ERR
2	-1
4	1

Domain: $\mathbb{R}, x \neq 1$

Range: $\mathbb{R}, y \neq 2$

Find the vertical and horizontal asymptotes of the graph of the function.

1. $f(x) = \frac{4}{x-2} + 1$ VA: $x=2$ HA: $y=1$	2. $f(x) = \frac{2x+2}{3x-4}$ VA: $x=4/3$ HA: $y=2/3$	3. $f(x) = \frac{x+1}{2x-3}$ VA: $x=3/2$ HA: $y=1/2$
4. $f(x) = \frac{4x}{2x+3}$ VA: $x=-3/2$ HA: $x=2$	5. $f(x) = \frac{2x-1}{x-2}$ VA: $x=2$ HA: $y=2$	6. $f(x) = \frac{6x-1}{3x+6}$ VA: $x=-2$ HA: $y=2$

7. $f(x) = \frac{x+1}{x-3}$

VA: $x-3=0$ $x=3$	Domain: $\mathbb{R}, x \neq 3$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>1</td><td>-1</td></tr> <tr><td>2</td><td>-3</td></tr> <tr><td>3</td><td>ELL</td></tr> <tr><td>4</td><td>5</td></tr> <tr><td>5</td><td>3</td></tr> </tbody> </table>	x	y	1	-1	2	-3	3	ELL	4	5	5	3	Graph
x	y														
1	-1														
2	-3														
3	ELL														
4	5														
5	3														
HA: $y=1$ $y=1$	Range: $\mathbb{R}, y \neq 1$														

8. $f(x) = \frac{4x^2-4}{(x+1)(x-2)} = \frac{4(x^2-1)}{(x+1)(x-2)} = \frac{4(x+1)(x-1)}{(x+1)(x-2)} = \frac{4(x-1)}{(x-2)}$

Removable Discontinuities: $x+1=0, x=-1, y = \frac{4(-1-1)}{-1-2} = \frac{4(-2)}{-3} = \frac{-8}{-3} = \frac{8}{3}$

VA: $x-2=0$ $x=2$	Domain: $\mathbb{R}, x \neq 2$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-1</td><td>2.6</td></tr> <tr><td>0</td><td>2</td></tr> <tr><td>1</td><td>0</td></tr> <tr><td>2</td><td>ELL</td></tr> <tr><td>3</td><td>8</td></tr> <tr><td>4</td><td>6</td></tr> </tbody> </table>	x	y	-1	2.6	0	2	1	0	2	ELL	3	8	4	6	Graph
x	y																
-1	2.6																
0	2																
1	0																
2	ELL																
3	8																
4	6																
HA: $y=4$	Range: $\mathbb{R}, y \neq 4$																

PRACTICE: GRAPHING RATIONAL FUNCTIONS 1 DAY 23

Find the vertical and horizontal asymptotes of the graph of the function.

1. $f(x) = \frac{6}{x-1} + 5$ VA: $x=1$ HA: $y=5$	2. $f(x) = \frac{2x+4}{3x-1}$ VA: $x=1/3$ HA: $y=2/3$	3. $f(x) = \frac{x+2}{2x-1}$ VA: $x=1/2$ HA: $y=1/2$
4. $f(x) = \frac{3x}{x+3}$ VA: $x=-3$ HA: $y=3$	5. $f(x) = \frac{3x-1}{x+3}$ VA: $x=-3$ HA: $y=3$	6. $f(x) = \frac{10x-1}{3x+9}$ VA: $x=-3$ HA: $y=10/3$

7. $f(x) = \frac{2}{x+3} + 0$

VA: $x+3=0$ $x=-3$	Domain: $\mathbb{R}, x \neq -3$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-5</td><td>-1</td></tr> <tr><td>-4</td><td>-2</td></tr> <tr><td>-3</td><td>ECR</td></tr> <tr><td>-2</td><td>2</td></tr> <tr><td>-1</td><td>1</td></tr> </tbody> </table>	x	y	-5	-1	-4	-2	-3	ECR	-2	2	-1	1	Graph
x	y														
-5	-1														
-4	-2														
-3	ECR														
-2	2														
-1	1														
HA: $y=0$	Range: $\mathbb{R}, y \neq 0$														

8. $f(x) = \frac{x^2 - x - 6}{x^2 + x - 2} = \frac{(x-3)(x+2)}{(x+2)(x-1)} = \frac{(x-3)}{(x-1)}$

VA: $x-1=0$ $x=1$	Domain: $\mathbb{R}, x \neq 1$	Table <table border="1"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr><td>-2</td><td>-1/3</td></tr> <tr><td>-1</td><td>2</td></tr> <tr><td>0</td><td>3</td></tr> <tr><td>1</td><td>ECR</td></tr> <tr><td>2</td><td>-1</td></tr> <tr><td>3</td><td>0</td></tr> </tbody> </table>	x	y	-2	-1/3	-1	2	0	3	1	ECR	2	-1	3	0	Graph
x	y																
-2	-1/3																
-1	2																
0	3																
1	ECR																
2	-1																
3	0																
HA: $y=1$	Range: $\mathbb{R}, x \neq 1$																

Hole: $x+2=0$
 $x=-2$

$y = \frac{-2+3}{-2-1}$
 $y = \frac{1}{-3}$
 $y = -1/3$