HOMEWORK: CONSTRUCTING FUNCTIONS

NAME: _____ DAY 6 DUE: _____

1. The volume, V, of a right circular cylinder of height, h, and radius, r, is: $V = \pi r^2 h$. If the height is twice the radius, express the volume, V, as a function of r.

$V = \pi r^2 h$ $\quad h = 2r$

$V(r) = \pi r^2 (2r)$

$\boxed{V(r) = 2\pi r^3}$

2. The price, p, and the quantity, x, sold of a certain product obey the demand equation $x = -5p + 100$, $0 \le p \le 20$.

a) Express the revenue, R, as a function of x.

Revenue = Price x Quantity Sold

$R = (p)(x)$

$R(x) = (-\frac{1}{5}x + 20)(x$

$\boxed{R(x) = -\frac{1}{5}x^2 + 20x}$

$x = -5p + 100$

$\frac{x - 100}{-5} = \frac{-5p}{-5}$

$-\frac{1}{5}x + 20 = p$

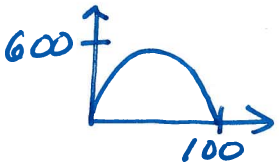
b) What is the revenue if 15 units are sold?

$R(15) = -\frac{1}{5}(15)^2 + 20(15)$

$= -45 + 300$

$= \boxed{\$225}$

c) Create a graph.



d) What quantity maximizes revenue? $x = 50$

What is the maximum revenue? $R(50) = -\frac{1}{5}(50)^2 + 20(50)$

$= -500 + 1000$

$= \boxed{\$500}$

e) What price should the company charge to maximize revenue?

$p = -\frac{1}{5}x + 20$

$p = -\frac{1}{5}(50) + 20$

$p = -10 + 20$

$\boxed{p = \$10}$

3. Let P = (x, y) be a point on the graph of $y = x^2 - 8$.

x_1, y_1
$(0,0)$

a) Express the distance, d, from P to the origin as a function of x.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$$

$$d = \sqrt{x^2 + y^2}$$

$$d = \sqrt{x^2 + (x^2 - 8)^2}$$

$$(x^2 - 8)^2$$
$$(x^2 - 8)(x^2 - 8)$$
$$x^4 - 8x^2 - 8x^2 + 64$$
$$x^4 - 16x^2 + 64$$

$$d = \sqrt{x^2 + x^4 - 16x^2 + 64}$$

$$\boxed{d = \sqrt{x^4 - 15x^2 + 64}}$$

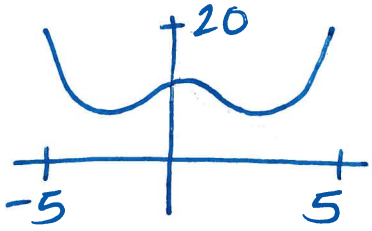
b) What is d if x = 0?

$$d(0) = \sqrt{0^4 - 15(0) + 64}$$
$$= \sqrt{64}$$
$$= \boxed{8}$$

c) What is d if x = 1?

$$d(1) = \sqrt{1^4 - 15(1) + 64}$$
$$= \sqrt{50}$$
$$= \boxed{5\sqrt{2}}$$

d) Graph.



e) For what values is d smallest?

$x = -2.74$ and $x = 2.74$

4. David has available 400 yards of fencing and wishes to enclose a rectangular area.

a) Express the area, A, of the rectangle as a function of the width, x, of the rectangle.

$P = 2x + 2y$

$400 = 2x + 2y$

$-2x \quad -2x$

$\frac{400 - 2x}{2} = \frac{2y}{2}$

$200 - x = y$

$A = \ell \cdot w$

$A = x \cdot y$

$A(x) = x(200 - x)$

$A(x) = 200x - x^2$

$\boxed{A(x) = -x^2 + 200x}$

b) What is the domain of A?

$x > 0$ and $y > 0$

$200 - x > 0$

$200 > x \rightarrow$ $\boxed{D: 0 < x < 200}$

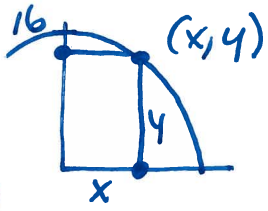
c) Graph A = A(x).

(Quadratic)

For what value of x is the area largest?

$\boxed{x = 100 \text{ yards}}$

5. A rectangle has one corner on the graph of $y = 16 - x^2$, another at the origin, a third on the positive y-axis and a fourth on the positive x-axis.



a) Express the area, A, of the rectangle as a function of x.

$A = \ell \cdot w$

$A = x \cdot y$

$A(x) = x(16 - x^2)$

$A(x) = 16x - x^3$ → $\boxed{A(x) = -x^3 + 16x}$

b) What is the domain of A?

$x > 0$ and $y > 0$

$16 - x^2 > 0$

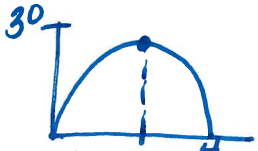
$16 > x^2$

$-4 \quad 4$

$-4 < x < 4$

$\boxed{0 < x < 4}$

c) Graph A = A(x)



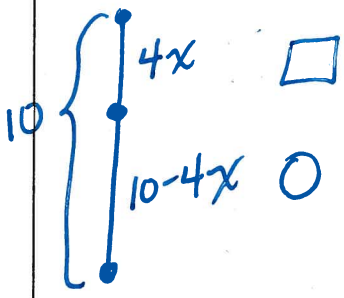
(Cubic)

d) For what values is A the largest?

$\boxed{x = 2.31}$

6. A wire 10 meters long is to be cut into two pieces. One piece will be shaped as a square and the other piece will be shaped as a circle.

a) Express the total area, A, enclosed by the pieces of wire as a function of the length, x, of a side of the square.



$A = s^2 + \pi r^2$

$A(x) = x^2 + \pi\left(\frac{5-2x}{\pi}\right)^2$

$A(x) = x^2 + \pi \cdot \frac{(5-2x)^2}{\pi^2}$

$$\boxed{A(x) = x^2 + \frac{(5-2x)^2}{\pi}}$$

$C = 2\pi r$

$\frac{10-4x}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{10-4x}{2\pi} = r$

$\frac{5-2x}{\pi} = r$

b) What is the domain of A? $x > 0$

$y > 0$

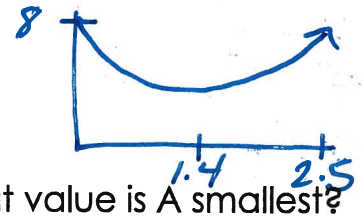
$10 - 4x > 0$

$\frac{10}{4} > \frac{4x}{4}$

$2.5 > x$

$$\boxed{0 < x < 2.5}$$

c) Graph.

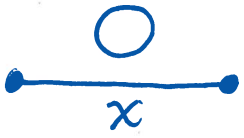


For what value is A smallest?

$x = 1.4$

7. A wire of length x is bent into the shape of a circle.

a) Express the circumference of the circle as a function of x.



$$\boxed{C(x) = x}$$

b) Express the area of the circle as a function of x.

$A = \pi r^2$

$A(x) = \pi\left(\frac{x}{2\pi}\right)^2$

$A(x) = \pi \cdot \frac{x^2}{4\pi^2}$

$$\boxed{A(x) = \frac{x^2}{4\pi}}$$

$C = 2\pi r$

$\frac{x}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{x}{2\pi} = r$