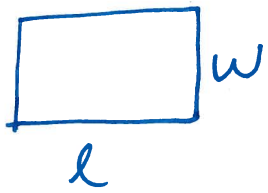
Chapter 2.6 – Constructing Functions

DAY 6

$A(w) = w$

1. The perimeter of a rectangle is 60 feet. Express its area A as a function of the width of a side.



$P = 2l + 2w$

$60 = 2l + 2w$

$-2w \qquad -2w$

$\frac{60 - 2w}{2} = \frac{2l}{2}$

$30 - w = l$

$A = l \cdot w$

$A(w) = (30 - w)w$

$A(w) = 30w - w^2$

$\boxed{A(w) = -w^2 + 30w}$

2. Let $P = (x, y)$ be a point on the graph of $y = x^2 - 2$.

Express the distance d from P to the origin as a function of x.

(0, 0)
x_1, y_1

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(x - 0)^2 + (y - 0)^2}$

$d = \sqrt{x^2 + y^2}$

$d = \sqrt{x^2 + (x^2 - 2)^2}$

$d = \sqrt{x^2 + (x^2 - 2)(x^2 - 2)}$

$d = \sqrt{x^2 + x^4 - 4x^2 + 4}$

$\boxed{d(x) = \sqrt{x^4 - 3x^2 + 4}}$

Recall:

$\sqrt{9+4} = \sqrt{13}$

$\sqrt{9+4} \neq \sqrt{9} + \sqrt{4}$

3. A rectangle has one corner on the graph of $y = 36 - x^2$, another at the origin, and the other two corners are on the positive x-axis and positive y-axis.

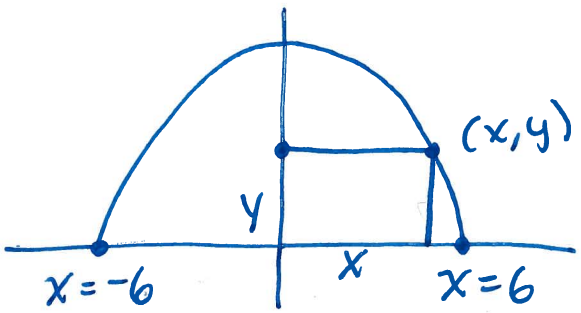
a) Express the area A of the rectangle as a function of x.

$A = \ell \cdot w$

$A = x \cdot y$

$A = x(36 - x^2)$

$\boxed{A(x) = 36x - x^3}$



b) What is the domain of A?

x and y are lengths, so $x > 0$ and $y > 0$.

$36 - x^2 > 0$

$-x^2 > -36$

$x^2 < 36$

$-6 < x < 6$

$\boxed{0 < x < 6}$

4. The volume V of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. If the height is 3 times the radius, express the volume V as a function of r.

$h = 3r$

$V = \frac{1}{3}\pi r^2 h$

$V(r) = \frac{1}{3}\pi r^2 (3r)$

$\boxed{V(r) = \pi r^3}$

5. In economics, revenue R is the amount of money received from the sale of a product. It is equal to the product of unit selling price p and the number of units x actually sold.

The price p, in dollars, and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{10}x + 20, \quad 0 \le x \le 20$$

Express the revenue R as a function of x.

$R = xp$

$R(x) = x\left(-\frac{1}{10}x + 20\right)$

$$\boxed{R(x) = -\frac{1}{10}x^2 + 20x}$$

What is the revenue if 15 units are sold?

$R(15) = -\frac{1}{10}(15)^2 + 20(15)$

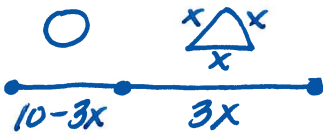
$R(15) = -\frac{1}{10}(225) + 300$

$R(15) = -22.5 + 300$

$$\boxed{R(15) = \$277.50}$$

6. A wire 10 meters long is to be cut into two pieces. One piece will be shaped as an equilateral triangle, and the other piece will be shaped as a circle.

Express the total area A enclosed by the pieces of wire as a function of the length x of a side of the equilateral triangle.



$A = \frac{s^2\sqrt{3}}{4} + \pi r^2$

$A(x) = \frac{x^2\sqrt{3}}{4} + \pi\left(\frac{10-3x}{2\pi}\right)^2$

$C = 2\pi r$

$\frac{10-3x}{2\pi} = \frac{2\pi r}{2\pi}$

$\frac{10-3x}{2\pi} = r$

What is the domain of A?

$$\boxed{A(x) = \frac{x^2\sqrt{3}}{4} + \frac{(10-3x)^2}{4\pi}}$$

Triangle: $3x$ is the length of wire.

$3x > 0$

$x > 0$

Circle: $10-3x$ is the length of wire.

$10 - 3x > 0$

$-3x > -10$

$x < 10/3$

$$\boxed{D: 0 < x < \frac{10}{3}}$$