

NOTES: ABSOLUTE VALUE EQUATIONS

DAY 6

(with extraneous solutions)

Textbook Chapter 1.7

OBJECTIVE: Today you will learn about extraneous solutions AND how to solve absolute value inequalities!

1. $|3x + 1| - 5 = -3$

$$\begin{array}{r} |3x + 1| - 5 = -3 \\ +5 \quad +5 \\ \hline \end{array}$$

$$|3x + 1| = 2$$

↙ ↘

$$\begin{array}{r} 3x + 1 = 2 \\ -1 \quad -1 \\ \hline \end{array}$$

$$\begin{array}{r} 3x + 1 = -2 \\ -1 \quad -1 \\ \hline \end{array}$$

$$\begin{array}{r} 3x = 1 \\ \frac{3}{3} \quad \frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 3x = -3 \\ \frac{3}{3} \quad \frac{-3}{3} \\ \hline \end{array}$$

$$x = \frac{1}{3}$$

$$x = -1$$

$$\boxed{x = \frac{1}{3}, -1}$$

2. $|2x + 12| = 4x$

$$\begin{array}{r} \swarrow \quad \searrow \\ 2x + 12 = 4x \quad 2x + 12 = -4x \\ -2x \quad -2x \quad -2x \quad -2x \\ \hline \end{array}$$

$$\frac{12}{2} = \frac{2x}{2}$$

$$\frac{12}{-6} = \frac{-6x}{-6}$$

$$\boxed{6 = x}$$

$$\cancel{-2 = x} \text{ ext.}$$

check: $|2(6) + 12| \stackrel{?}{=} 4(6)$
 $|24| = 24 \checkmark$

$|2(-2) + 12| \stackrel{?}{=} 4(-2)$
 $|8| \neq -8$

$$\boxed{x = 6}$$

3. $|x + 3| = 8$

↙ ↘

$$\begin{array}{r} x + 3 = 8 \\ -3 \quad -3 \\ \hline \end{array}$$

$$\begin{array}{r} x + 3 = -8 \\ -3 \quad -3 \\ \hline \end{array}$$

$$x = 5$$

$$x = -11$$

$$\boxed{x = 5, -11}$$

4. $\frac{-2|x + 7|}{-2} = \frac{-14}{-2}$

$$|x + 7| = 7$$

$$\begin{array}{r} x + 7 = 7 \\ -7 \quad -7 \\ \hline \end{array}$$

$$x = 0$$

$$\begin{array}{r} x + 7 = -7 \\ -7 \quad -7 \\ \hline \end{array}$$

$$x = -14$$

$$\boxed{x = 0, -14}$$

5. Interpret $|2x + 3| = -7$

A number times 2 plus 3 is
-7 units away from 0.

↖ hmmm...

NOTES: ABSOLUTE VALUE INEQUALITIES (AND)

Textbook Chapter 1.7

Example 1: $|x| \leq 3$

- a. What values will satisfy the inequality?
- b. Graph.



- c. Write a **COMPOUND** inequality that is equivalent to $|x| \leq 3$: $-3 \leq x \leq 3$

Example 2: My credit card balance ranges from -\$300 to \$300 at any given time.

- a. Write this situation as a **COMPOUND** inequality: $-300 \leq x \leq 300$
- b. How can you write this inequality as an absolute value? $|x| \leq 300$

Example 3: $|x + 1| \leq 3$

$$\begin{array}{r} -3 \leq x+1 \leq 3 \\ \underline{-1 \quad -1 \quad -1} \\ -4 \leq x \leq 2 \end{array}$$

Example 4: $|2x + 4| \leq 12$

$$\begin{array}{r} -12 \leq 2x+4 \leq 12 \\ \underline{-4 \quad -4 \quad -4} \\ -16 \leq 2x \leq 8 \\ \underline{\frac{-16}{2} \quad \frac{2x}{2} \quad \frac{8}{2}} \\ -8 \leq x \leq 4 \end{array}$$



NOTES: ABSOLUTE VALUE INEQUALITIES (OR)

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Example 4: $|x| \geq 5$

- What values will satisfy the inequality?
- Solve and graph.

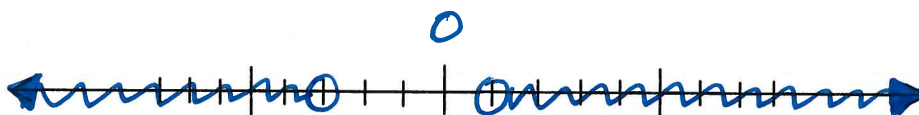


- Write a **DISJOINT** inequality that is equivalent to $|x| \geq 5$: _____

Example 5: $|x + 1| > 2$

$$\begin{array}{r} x+1 > 2 \\ -1 \quad -1 \\ \hline x > 1 \end{array}$$

$$\begin{array}{r} x+1 < -2 \\ -1 \quad -1 \\ \hline x < -3 \end{array}$$

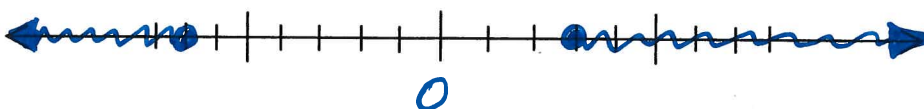


Example 6: $|2x + 4| + 2 \geq 12$

$$\begin{array}{r} -2 \quad -2 \\ \hline |2x+4| \geq 10 \end{array}$$

$$\begin{array}{r} 2x+4 \geq 10 \\ -4 \quad -4 \\ \hline 2x \geq 6 \\ \frac{2x}{2} \geq \frac{6}{2} \\ x \geq 3 \end{array}$$

$$\begin{array}{r} 2x+4 \leq -10 \\ -4 \quad -4 \\ \hline 2x \leq -14 \\ \frac{2x}{2} \leq \frac{-14}{2} \\ x \leq -7 \end{array}$$



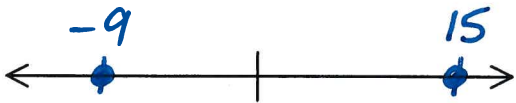
COMPARE (in as many ways as possible): $|x| < 3$ and $|x| \geq 3$

PRACTICE: ABSOLUTE VALUE EQUATIONS AND INEQUALITIES

Solve and graph.

1. $|x-3| = 12$

$$\begin{array}{r} x-3=12 \\ +3 \quad +3 \\ \hline x=15 \end{array} \qquad \begin{array}{r} x-3=-12 \\ +3 \quad +3 \\ \hline x=-9 \end{array}$$



2. $2|y+7| + 13 = 15$

$$\begin{array}{r} 2|y+7| + 13 = 15 \\ -13 \quad -13 \\ \hline 2|y+7| = 2 \\ \frac{2}{2} \quad \frac{2}{2} \\ |y+7| = 1 \\ y+7 = 1 \qquad y+7 = -1 \\ y = -6 \qquad y = -8 \end{array}$$



3. $|5x+2| \leq 3$

$$\begin{array}{r} -3 \leq 5x+2 \leq 3 \\ -2 \quad -2 \quad -2 \\ \hline -5 \leq 5x \leq 1 \\ \frac{-5}{5} \leq \frac{5x}{5} \leq \frac{1}{5} \\ -1 \leq x \leq \frac{1}{5} \end{array}$$



4. $|3y+4| > 8$

$$\begin{array}{r} 3y+4 > 8 \\ -4 \quad -4 \\ \hline 3y > 4 \\ \frac{3y}{3} > \frac{4}{3} \\ y > 4/3 \\ y > 1\frac{1}{3} \end{array} \qquad \begin{array}{r} 3y+4 < -8 \\ -4 \quad -4 \\ \hline 3y < -12 \\ \frac{3y}{3} < \frac{-12}{3} \\ y < -4 \end{array}$$



5. $2|m+5| + 10 \leq 16$

$$\begin{array}{r} 2|m+5| + 10 \leq 16 \\ -10 \quad -10 \\ \hline 2|m+5| \leq 6 \\ \frac{2}{2} \quad \frac{6}{2} \\ |m+5| \leq 3 \\ -3 \leq m+5 \leq 3 \\ -5 \quad -5 \quad -5 \\ -8 \leq m \leq -2 \end{array}$$



6. $\left|\frac{x}{5} + 3\right| > 2$

$$\begin{array}{r} \frac{x}{5} + 3 > 2 \\ -3 \quad -3 \\ \hline \frac{x}{5} > -5 \\ 5\left(\frac{x}{5}\right) > (-1)5 \\ x > -5 \end{array} \qquad \begin{array}{r} \frac{x}{5} + 3 < -2 \\ -3 \quad -3 \\ \hline \frac{x}{5} < -5 \\ 5\left(\frac{x}{5}\right) < -5 \cdot 5 \\ x < -25 \end{array}$$

