

# HOMWORK: SOLVING LOG EQUATIONS

NAME: \_\_\_\_\_ DAY 5 DUE: \_\_\_\_\_

Solve the exponential equations.

<p>1. <math>5^{x-4} = 25^{x-6}</math></p> $5^{x-4} = (5^2)^{x-6}$ $5^{x-4} = 5^{2x-12}$ $\begin{array}{r} x-4 = 2x-12 \\ -2x+4 \quad -2x+4 \\ \hline -x = -8 \end{array}$ $\boxed{x=8}$	<p>2. <math>8^{x-1} = 32^{3x-2}</math></p> $(2^3)^{x-1} = (2^5)^{3x-2}$ $2^{3x-3} = 2^{15x-10}$ $\begin{array}{r} 3x-3 = 15x-10 \\ -3x+10 \quad -3x+10 \\ \hline 7 = 12x \end{array}$ $\frac{7}{12} = \frac{12x}{12}$ $\boxed{x = \frac{7}{12}}$
<p>3. <math>36^{5x+2} = \left(\frac{1}{6}\right)^{11-x}</math></p> $(6^2)^{5x+2} = \left(\frac{1}{6}\right)^{11-x}$ $6^{10x+4} = (6^{-1})^{11-x}$ $6^{10x+4} = 6^{-11+x}$ $\begin{array}{r} 10x+4 = -11+x \\ -x-4 \quad -4-x \\ \hline 9x = -15 \end{array}$ $x = \frac{-15}{9}$ $\boxed{x = -\frac{5}{3}}$	<p>4. <math>25^{10x+8} = \left(\frac{1}{125}\right)^{4-2x}</math></p> $(5^2)^{10x+8} = (5^{-3})^{4-2x}$ $5^{20x+16} = 5^{-12+6x}$ $\begin{array}{r} 20x+16 = -12+6x \\ -6x-16 \quad -16-6x \\ \hline 14x = -28 \end{array}$ $\frac{14x}{14} = \frac{-28}{14}$ $\boxed{x = -2}$

Solve the logarithmic equations.

<p>5. <math>\log_5(5x+9) = \log_5 6x</math></p> $\begin{array}{r} 5x+9 = 6x \\ -5x \quad -5x \\ \hline 9 = x \end{array}$ $\boxed{x=9}$	<p>6. <math>\ln(4x-7) = \ln(x+11)</math></p> $\begin{array}{r} 4x-7 = x+11 \\ -x+7 \quad -x+7 \\ \hline 3x = 18 \end{array}$ $\frac{3x}{3} = \frac{18}{3}$ $\boxed{x=6}$
<p>7. <math>\ln(x+19) = \ln(7x-8)</math></p> $\begin{array}{r} x+19 = 7x-8 \\ -x+8 \quad -x+8 \\ \hline 27 = 6x \end{array}$ $\frac{27}{6} = \frac{6x}{6}$ $\boxed{x = \frac{27}{6}}$	<p>8. <math>\log_5(2x-7) = \log_5(3x-9)</math></p> $\begin{array}{r} 2x-7 = 3x-9 \\ -2x+9 \quad -2x+9 \\ \hline 2 = x \end{array}$ $\boxed{x=2}$

Solve the equations. Check for extraneous solutions.

9.  $\log_4 x = -1$

$$4^{-1} = x$$

$$\boxed{x = \frac{1}{4}}$$

10.  $\log_2(x-4) = 6$

$$2^6 = x - 4$$

$$64 = x - 4$$

$$+4 \quad +4$$

$$\boxed{68 = x}$$

11.  $\log_2 x + \log_2(x-2) = 3$

$$\log_2(x)(x-2) = 3$$

$$2^3 = (x)(x-2)$$

$$8 = x^2 - 2x$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, -2$$

$$\boxed{x = 4}$$

12.  $\log_4(-x) + \log_4(x+10) = 2$

$$\log_4(-x)(x+10) = 2$$

$$4^2 = (-x)(x+10)$$

$$16 = -x^2 - 10x$$

$$x^2 + 10x + 16 = 0$$

$$(x+8)(x+2) = 0$$

$$\boxed{x = -8, -2}$$

13.  $\log_6 3x + \log_6(x-1) = 3$

$$\log_6(3x)(x-1) = 3$$

$$\log_6(3x^2 - 3x) = 3$$

$$6^3 = 3x^2 - 3x$$

$$\frac{3x^2 - 3x - 216}{3} = \frac{0}{3}$$

$$x^2 - x - 72 = 0$$

$$(x-9)(x+8) = 0$$

$$x = 9, -8$$

$$\boxed{x = 9}$$

14.  $\log_3(x-9) + \log_3(x-3) = 2$

$$\log_3(x-9)(x-3) = 2$$

$$3^2 = (x-9)(x-3)$$

$$9 = x^2 - 12x + 27$$

$$0 = x^2 - 12x - 18$$

$$x = \frac{+12 \pm \sqrt{(-12)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{12 \pm \sqrt{216}}{2}$$

$$x = \frac{12 \pm 6\sqrt{6}}{2}$$

$$x = 6 \pm 3\sqrt{6}$$

$$\boxed{x = 6 + 3\sqrt{6}}$$

15.  $3^{x+4} = 6^{2x-5}$

$$\begin{aligned} \log 3^{x+4} &= \log 6^{2x-5} \\ (x+4) \log 3 &= (2x-5) \log 6 \\ x \log 3 + 4 \log 3 &= 2x \log 6 - 5 \log 6 \\ x \log 3 - 2x \log 6 &= -5 \log 6 - 4 \log 3 \\ x(\log 3 - 2 \log 6) &= -5 \log 6 - 4 \log 3 \end{aligned}$$

$$x = \frac{-5 \log 6 - 4 \log 3}{\log 3 - 2 \log 6}$$

16.  $10^{3x-8} = 2^{5-x}$

$$\begin{aligned} \log 10^{3x-8} &= \log 2^{5-x} \\ 3x-8 &= (5-x) \log 2 \\ 3x-8 &= 5 \log 2 - x \log 2 \\ 3x + x \log 2 &= 5 \log 2 + 8 \\ x(3 + \log 2) &= 5 \log 2 + 8 \end{aligned}$$

$$x = \frac{5 \log 2 + 8}{3 + \log 2}$$

17.  $\log_2(x+1)^3 = \log_8 3x$

$$\begin{aligned} \log_8(x+1)^3 &= \log_8 3x \\ (x+1)^3 &= 3x \\ (x+1)(x+1)(x+1) &= 3x \\ x^3 + x^2 + 2x^2 + 2x + x + 1 &= 3x \\ x^3 + 3x^2 + 1 &= 0 \end{aligned}$$

$$x \approx -3.1 \quad \boxed{\text{No Solution}}$$

18.  $\log_3 x^2 = \log_9 6x$

$$\begin{aligned} \log_9 x^2 &= \log_9 6x \\ x^2 &= 6x \\ x^2 - 6x &= 0 \\ x(x-6) &= 0 \end{aligned}$$

$$x = \emptyset, 6 \quad \boxed{x = 6}$$

19. One hundred grams of radium are stored in a container. The amount  $R$  (in grams) of radium present after  $t$  years can be modeled by  $R = 100e^{-0.99943t}$ . After how many years will only 5 grams of radium be left?

$$\frac{5}{100} = \frac{100e^{-0.99943t}}{100}$$

$$\ln \frac{1}{20} = \ln e^{-0.99943t}$$

$$\frac{\ln \frac{1}{20}}{-0.99943} = \frac{-0.99943t}{-0.99943}$$

$$t = \frac{\ln \frac{1}{20}}{-0.99943}$$

$$t \approx 2.997 \text{ years}$$

20. You deposit \$800 in an account that pays 2.25% annual interest compounded continuously. About how long will it take for the balance to triple? = 2400

$$A = Pe^{rt}$$

$$\frac{2400}{800} = \frac{800e^{0.0225t}}{800}$$

$$3 = e^{0.0225t}$$

$$\frac{\ln 3}{0.0225} = \frac{0.0225t}{0.0225}$$

$$t \approx 48.83 \text{ years}$$

about 48 years and 10 months

