Exponential and Logarithmic Equations

Name: ______________________ Date: ____________

Exponential Equations (x is in the exponent)	Instructions
1. $2^x = 10$ $\log 2^x = \log 10$ $\frac{x \log 2}{\log 2} = \frac{\log 10}{\log 2}$ $x = \frac{\log 10}{\log 2}$	1. Isolate the base/exponent (b^x) 2. **Log** both sides. 3. Use the power rule to get x out of the exponent. 4. Solve for x.
2. $e^x = 10$ $\ln e^x = \ln 10$ $\frac{x \ln e}{\ln e} = \frac{\ln 10}{\ln e}$ $x = \frac{\ln 10}{\ln e}$	1. 1. Isolate the base/exponent (b^x) 2. **Natural Log** both sides. 3. Use the power rule to get x out of the exponent. 4. Solve for x.
3. $2^{3x+2} = 2^{5x}$ $3x+2 = 5x$ $-3x \quad -3x$ $2 = 2x$ $1 = x$	Bases are the same: 1. Set the exponents equal to each other 2. Solve the equation.
4. $2^{3x} = 8^{4x-1}$ $2^{3x} = (2^3)^{4x-1}$ $2^{3x} = 2^{12x-3}$ $3x = 12x - 3$ $-12x \quad -12x$ $-9x = -3$ $x = 1/3$	Bases are not the same: 1. Rewrite the bases so that they are the same! 2. Simplify the powers 3. Set the exponents equal to each other 4. Solve the equation

Log Definition: $\log_b x = y$ ←→ $b^y = x$

Log Properties

1. $\log_b x^n =$ $n \log_b x$
2. $\log_b x + \log_b y =$ $\log_b (xy)$
3. $\log_b x - \log_b y =$ $\log_b \left(\frac{x}{y}\right)$
4. $\log_a x = \frac{\log_b x}{\log_b a}$

Special Cases

1. $\log_b b =$ 1
2. $\log_b 1 =$ 0
3. $\ln_e e^x =$ x
4. $\ln_e e =$ 1
5. $\log_b b^x =$ x
6. $b^{\log_b x} =$ x
7. $e^{\ln x} =$ x
8. $\ln e^x =$ x

Logarithmic Equations (logs or natural logs)	**Instructions**
1. $\log_4 x = 3$ $4^3 = x$ $64 = x$ $x = 64$	1. Isolate the $\log_b x$ 2. Convert to exponential form using the log definition 3. Solve for x.
2. $\ln(2x - 4) = 7$ $e^7 = 2x - \cancel{4}$ $+4 \quad \cancel{+4}$ $\frac{e^7+4}{2} = \frac{\cancel{2}x}{\cancel{2}}$ $x = \frac{e^7+4}{2}$	1. Isolate the ln x 2. Exponentiate both sides using ***e*** or definition 3. ***e*** and ln "undo" eachother 4. Solve.
3. $\log_3(x - 4) = \log_3(2x)$ $\cancel{x} - 4 = 2x$ $-\cancel{x} \quad -x$ $-4 = x$ $x = -4$	Logs or Natural Logs on both sides: 1. They must have the same bases 2. Set the values equal to each other 3. Solve.
4. $\log(x + 2) + \log(x - 3) = \log(x + 29)$ $\log(x+2)(x-3) = \log(x+29)$ $x^2 - x - 6 = \cancel{x} + \cancel{29}$ $-x - 29 \quad -\cancel{x} - \cancel{29}$ $x^2 - 2x - 35 = 0$ $(x-7)(x+5) = 0$ $x = 7, -5$	Multiple logs on one side or both sides: 1. They must have the same bases 2. Condense the logs together using log properties. 3. Either a. Convert to exponential form using the definition b. Set the values equal to each other 4. Solve (sometimes it will be a quadratic!)

Special Hints:

- Sometimes you will need to solve a quadratic equation!
- Recall the relationship between radicals and fractional exponents: $\sqrt[n]{x^m}$ = $x^{m/n}$
- Recall exponent properties: $(x^n)(x^m)$ = x^{n+m}, $(x^n)^m$ = x^{nm}, x^0 = 1
- x to an even power will have **two solutions** (don't forget the "±" sign)

Exponential and Logarithmic Equations **DAY 5**

EXAMPLE ONE→ Equating exponents and the "inside" of a logarithm

(a) $4^{x+3} = 4^{2x-1}$

$x+3 = 2x-1$
$-x \qquad -x$
$3 = x-1$
$+1 \qquad +1$
$\boxed{4 = x}$

(b) $\log_4(x^2) = \log_4(x+2)$

$x^2 = x+2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$\boxed{x = 2, -1}$

EXAMPLE TWO→ Changing the bases

a) $9^{2x} = 27^{x-1}$

$(3^2)^{2x} = (3^3)^{x-1}$
$3^{4x} = 3^{3x-3}$
$4x = 3x-3$
$-3x \qquad -3x$
$\boxed{x = -3}$

b) $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

$(3^4)^{3-x} = (3^{-1})^{5x-6}$
$3^{12-4x} = 3^{-5x+6}$
$12 - 4x = -5x + 6$
$-12 \; +5x \qquad +5x \; -12$
$\boxed{x = -6}$

c) $\log_2 x = \log_4(4x-4)$

$\dfrac{\log_4 x}{\log_4 2} = \log_4(4x-4)$
$\dfrac{\log_4 x}{1/2} = \log_4(4x-4)$
$2\log_4 x = \log_4(4x-4)$
$\log_4 x^2 = \log_4(4x-4)$
$x^2 = 4x-4$
$x^2 - 4x + 4 = 0$
$(x-2)^2 = 0$
$x - 2 = 0$
$\boxed{x = 2}$

d) $\log_4 (x)^2 = \log_{16}(2x+15)$

$\log_{16} x^2 = \log_{16}(2x+15)$
$x^2 = 2x+15$
$x^2 - 2x - 15 = 0$
$(x-5)(x+3) = 0$
$\boxed{x = 5, -3}$

EXAMPLE THREE→ Re-Writing the function

(a) $4^x = 11$

$\log_4 11 = x$

$\boxed{x = \log_4 11}$

(b) $2^x = 5$

$\log_2 5 = x$

$\boxed{x = \log_2 5}$

(c) $\log_4(5x - 1) = 3$

$4^3 = 5x - 1$

$64 = 5x - 1$

$+1 \quad\quad +1$

$65 = 5x$

$\boxed{x = 13}$

(d) $\log_7(3x - 2) = 2$

$7^2 = 3x - 2$

$49 = 3x - 2$

$+2 \quad\quad +2$

$51 = 3x$

$\boxed{x = 17}$

EXAMPLE FOUR→ Using Properties and re-writing

(a) $\log(2x) + \log(x - 5) = 2$

$\log(2x)(x-5) = 2$

$10^2 = (2x)(x-5)$

$100 = 2x^2 - 10x$

$2x^2 - 10x - 100 = 0$

$x^2 - 5x - 50 = 0$

$(x-10)(x+5) = 0$

$x = 10, \cancel{-5}$ $\boxed{x = 10}$

(b) $\log(5x) + \log(x - 1) = 2$

$\log(5x)(x-1) = 2$

$10^2 = (5x)(x-1)$

$100 = 5x^2 - 5x$

$5x^2 - 5x - 100 = 0$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5, \cancel{-4}$

$\boxed{x = 5}$

EXAMPLE FIVE→ Difficult Solving!

$6^{3x-1} = 10^{x+7}$

$\log 6^{3x-1} = \log 10^{x+7}$

$(3x-1)\log 6 = x + 7$

$3x\log 6 - \log 6 = x + 7$

$+\log 6 \quad\quad +\log 6$

$3x\log 6 = x + 7 + \log 6$

$-x \quad\quad -x$

$3x\log 6 - x = 7 + \log 6$

$\dfrac{x(3\log 6 - 1)}{(3\log 6 - 1)} = \dfrac{7 + \log 6}{(3\log 6 - 1)}$

$\boxed{x = \dfrac{7 + \log 6}{3\log 6 - 1}}$