$y = a\sqrt{x-h} + K$ vertex : (h, K)
a : stretch/shrink up/down

NOTES: RADICAL AND INVERSE FUNCTIONS DAY 11

Textbook Chapter 6.4

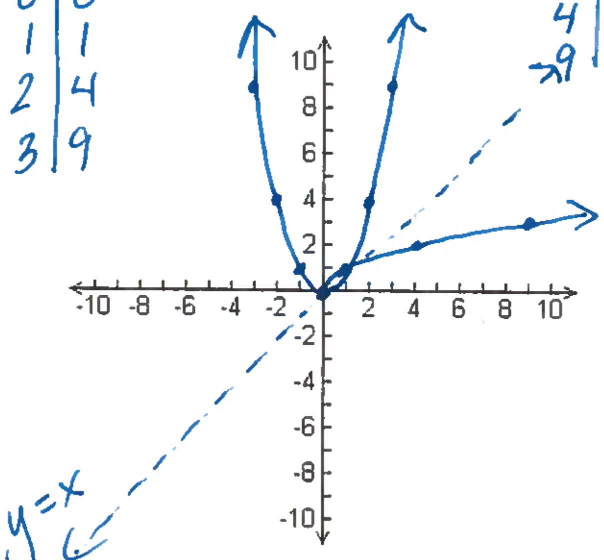
OBJECTIVE: Today you will learn about inverse functions!

Graph both functions. What is their relationship?

1. $f(x) = x^2$, $g(x) = \sqrt{x}$

x	y
0	0
1	1
2	4
3	9

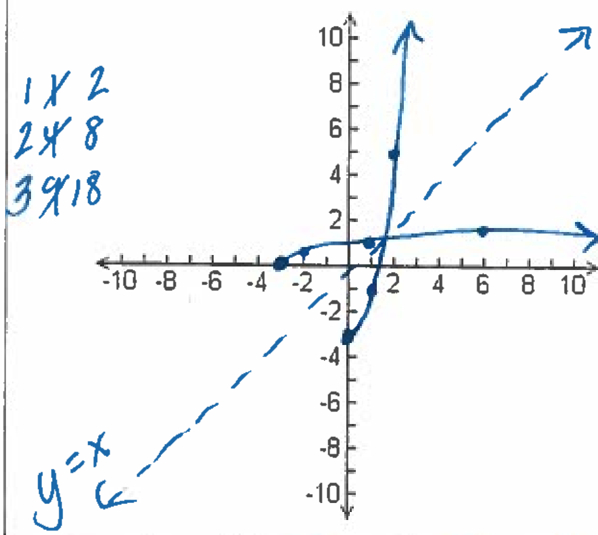
x	y
0	0
1	1
4	2
9	3



2. $f(x) = 2(x)^2 - 3$ $(0, -3)$ $g(x) = \frac{1}{2}\sqrt{x+3} + 0$ $(-3, 0)$

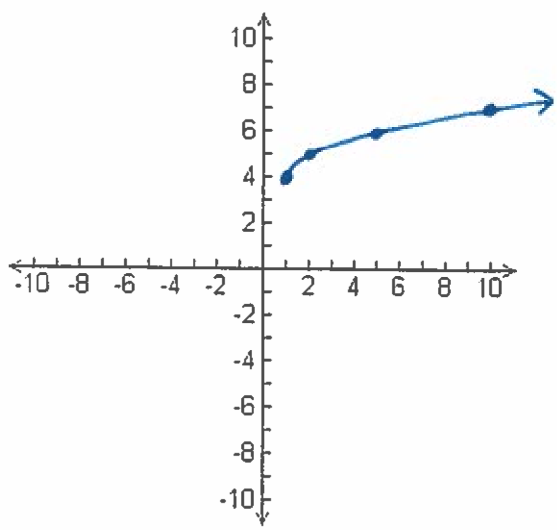
1	~~1~~	2
2	~~4~~	8
3	~~9~~	18

1	~~1~~	1/2
4	~~2~~	1
9	~~3~~	3/2



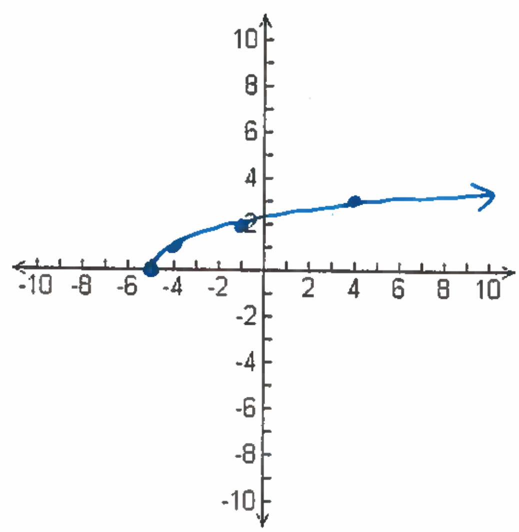
3. $y = \sqrt{x-1} + 4$ $(1, 4)$

1	1
4	2
9	3



4. $y = 2\sqrt{x+5} + 0$ $(-5, 0)$

1	1
4	2
9	3



$x \to \boxed{y = x^2} \to y$ $x \to \boxed{y = \sqrt{x}} \to y$

Inverses!

* Tables are switched
* Reflect over line $y = x$

$f^{-1}(x)$ = inverse of $f(x)$

Find the Vertex Practice: (all types)

Draw the Shape. $y=x^3$

5. $y=\sqrt{x-4}+8$ Vertex: (4 , 8)
6. $y=-\sqrt{x}$ Vertex: (0 , 0)
7. $y=-|x-1|$ Vertex: (1 , 0)
8. $y=\sqrt{x}-7$ Vertex: (0 , -7)
9. $y=\frac{1}{3}(x+10)^2$ Vertex: (-10 , 0)
10. $y=3\sqrt{x+1}$ Vertex: (-1 , 0)

1. Switch x and y.
2. Solve for y.

Find the inverse function.

1. $y = 3x - 3$ $x = 3\boxed{y} - 3$ $+3 \quad +3$ $\frac{1x+3}{3} = \frac{3y}{3}$ $\frac{1}{3}x + \frac{3}{3} = y$ $y = \frac{1}{3}x + 1$	2. $f(x) = x^2$ $y = x^2$ $\sqrt{x} = \sqrt{y^2}$ $\sqrt{x} = y$ $f^{-1}(x) = \sqrt{x}$ Change $f(x) = y$ 1. Switch x, y. 2. Solve for y. Rewrite $y = f^{-1}(x)$
3. $y = -\frac{4}{5}x + 11$ $x = -\frac{4}{5}\boxed{y} + 11$ $-11 \quad -11$ $\left(-\frac{5}{4}\right)(x-11) = \left(-\frac{4}{5}y\right)\left(-\frac{5}{4}\right)$ $-\frac{5}{4}x + \frac{55}{4} = y$ $y = -\frac{5}{4}x + \frac{55}{4}$	4. $f(x) = \frac{1}{2}x^3 - 2$ $y = \frac{1}{2}x^3 - 2$ $x = \frac{1}{2}\boxed{y^3} - 2$ $+2 \quad +2$ $2(x+2) = \left(\frac{1}{2}y^3\right)2$ $\sqrt[3]{2x+4} = \sqrt[3]{y^3}$ $y = \sqrt[3]{2x+4}$ Change $f(x) = y$ 1. Switch x, y 2. Solve for y. Rewrite $y = f^{-1}(x)$ $f^{-1}(x) = \sqrt[3]{2x+4}$

Verify that the functions are inverses: $f(x) = \cancel{4}\,16x^2 + 8$ $\quad g(x) = \frac{1}{4}\sqrt{x-8}$

$f(g(x)) = 16\left(\frac{1}{4}\sqrt{x-8}\right)^2 + 8$

$= 16\left(\frac{1}{4}\right)^2\left(\sqrt{x-8}^{\,2}\right) + 8$

$= 16\left(\frac{1}{16}\right)(x-8) + 8$

$= (x-8) + 8$

$= x$ ✓

$x \to \boxed{g} \to y \qquad y \to \boxed{f} \to x$

$g(f(x)) = \frac{1}{4}\sqrt{16x^2+8-8}$

$= \frac{1}{4}\sqrt{16x^2}$

$= \frac{1}{4}(4x)$

$= x$ ✓

Inverses!!

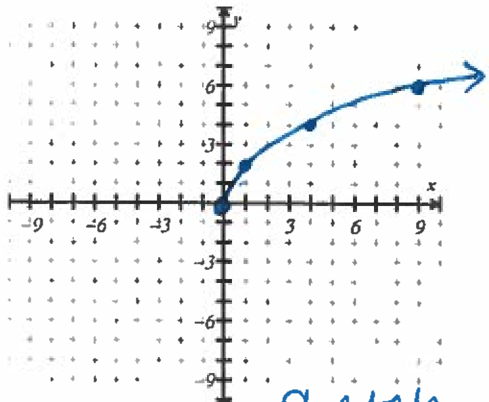
Graphing the Radical Functions

DAY 11

1) $y = 2\sqrt{x}$ Initial Point: (0 , 0)

Chart

1 ~~1~~ 2
4 ~~2~~ 4
9 ~~3~~ 6
16 ~~4~~ 8

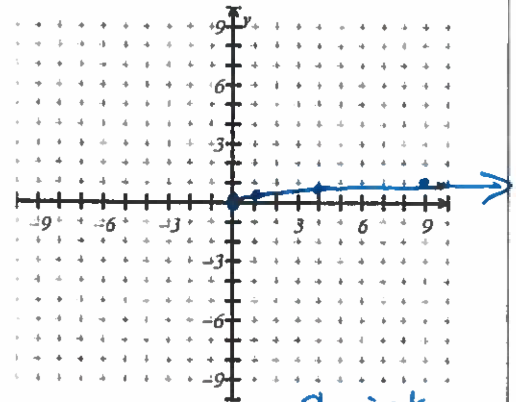


Stretch

2) $y = \frac{1}{3}\sqrt{x}$ Initial Point: (0 , 0)

Chart

1 ~~1~~ 1/3
4 ~~2~~ 2/3
9 ~~3~~ 1
16 ~~4~~ 4/3

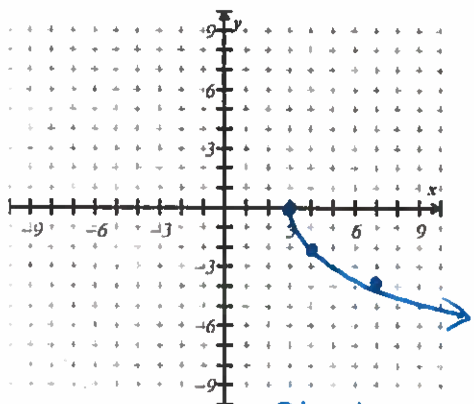


Shrink

3) $y = -2\sqrt{x-3}$ Initial Point: (3 , 0)

Chart

1 ~~1~~ -2
4 ~~2~~ -4
9 ~~3~~ -6
16 ~~4~~ -8

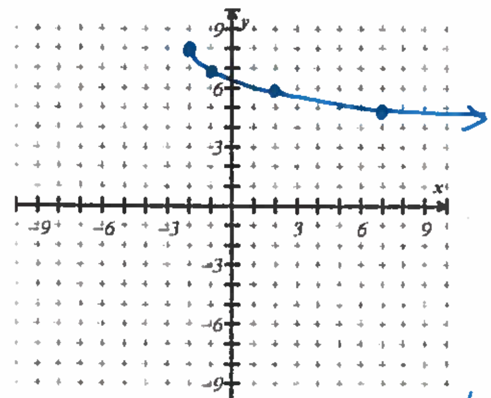


Stretch

4) $y = -\sqrt{x+2} + 8$ Initial Point: (-2 , 8)

Chart

1 ~~1~~ -1
4 ~~2~~ -2
9 ~~3~~ -3
16 ~~4~~ -4

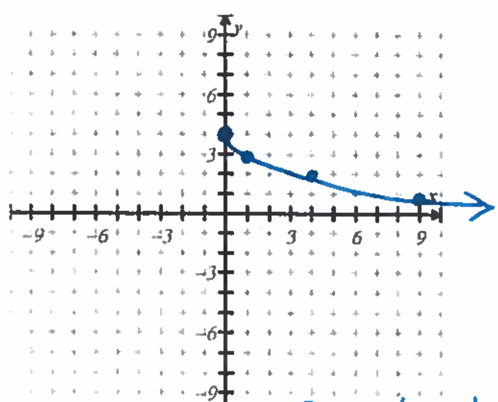


Standard

5) $r(x) = -\sqrt{x} + 4$ Initial Pt: (0 , 4)

Chart

1 ~~1~~ -1
4 ~~2~~ -2
9 ~~3~~ -3
16 ~~4~~ -4

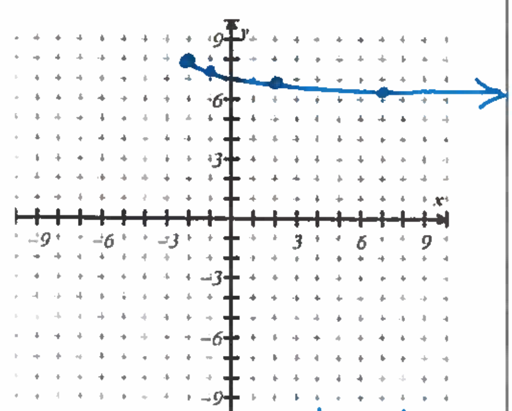


Standard

6) $y = -\frac{1}{2}\sqrt{x+2} + 8$ Initial Pt: (-2 , 8)

Chart

1 ~~1~~ -1/2
4 ~~2~~ -1
9 ~~3~~ -3/2
16 ~~4~~ -2



Shrink

Find the inverses.

7. $y = 3x + 5$

$$x = 3\boxed{y} + 5$$
$$-5 \qquad -5$$
$$\frac{x-5}{3} = \frac{3y}{3}$$
$$\frac{1x-5}{3} = y$$
$$\frac{1}{3}x - \frac{5}{3} = y$$
$$y = \frac{1}{3}x - \frac{5}{3}$$

8. $y = 2x^2 - 10$

$$x = 2\boxed{y^2} - 10$$
$$+10 \qquad +10$$
$$\frac{x+10}{2} = \frac{2y^2}{2}$$
$$\sqrt{\frac{1}{2}x + 5} = \sqrt{y^2}$$
$$y = \sqrt{\frac{1}{2}x + 5}$$

☆ 9. $y = 2\sqrt{3x-1} + 4$

$$x = 2\boxed{\sqrt{3y-1}} + 4$$
$$-4 \qquad -4$$
$$\frac{x-4}{2} = \frac{2\sqrt{3y-1}}{2}$$
$$\left(\frac{1}{2}x - 2\right)^2 = \sqrt{3y-1}^{\,2}$$
$$\left(\frac{1}{2}x - 2\right)^2 = 3y - 1$$
$$+1 \qquad +1$$
$$\left(\frac{1}{2}x - 2\right)^2 + 1 = 3y$$
$$y = \frac{1}{3}\left(\frac{1}{2}x - 2\right)^2 + 1$$

10. $y = 4x^3 - 10$

$$x = 4\boxed{y^3} - 10$$
$$+10 \qquad +10$$
$$\frac{x+10}{4} = \frac{4y^3}{4}$$
$$\sqrt[3]{\frac{1}{4}x + \frac{10}{4}} = \sqrt[3]{y^3}$$
$$\sqrt[3]{\frac{1}{4}x + \frac{5}{2}} = y$$
$$y = \sqrt[3]{\frac{1}{4}x + \frac{5}{2}}$$

Verify that the functions are inverses.

11. $f(x) = \frac{5x-3}{2}$ $\qquad g(x) = \frac{2x+3}{5}$

$$f(g(x)) = \frac{\cancel{5}\left(\frac{2x+3}{\cancel{5}}\right) - 3}{2}$$
$$= \frac{2x + \cancel{3} - \cancel{3}}{2}$$
$$= \frac{2x}{2}$$
$$= x \checkmark$$

$$g(f(x)) = \frac{\cancel{2}\left(\frac{5x - \cancel{3}}{\cancel{2}}\right) + \cancel{3}}{5}$$
$$= \frac{5x}{5}$$
$$= x \checkmark$$

12. $f(x) = 2x^2 - 3$ $\qquad g(x) = \sqrt{\frac{x+3}{2}}$

$$f(g(x)) = 2\left(\sqrt{\frac{x+3}{2}}\right)^2 - 3$$
$$= \cancel{2}\left(\frac{x+3}{\cancel{2}}\right) - 3$$
$$= x + 3 - 3$$
$$= x \checkmark$$

$$g(f(x)) = \sqrt{\frac{(2x^2-3)+3}{2}}$$
$$= \sqrt{\frac{2x^2}{2}}$$
$$= \sqrt{x^2}$$
$$= x \checkmark$$