

Ch 7.25 – Ambiguous Case

DAY 4

Consider the following for $\triangle ABC$, $\alpha = 30^\circ$, $a = 6$, $b = 4$. Try to solve this using the Law of Sines.

We should notice that our calculator only gives us one possibility for β but is this correct?

Looking at our unit circle, we know that $\sin \beta = \frac{1}{3}$ in two places.

When we are presented a triangle with two sides and the angle opposite one of the sides it is referred to as the Ambiguous Case. The given information may result in one triangle, two triangles, or no triangle.

Example 1: Consider triangle ABC with $m\angle A = 35^\circ$, $a = 8$, $b = 10$

When this occurs, there are 2 possible triangles, and you must find both solutions!

Triangle 1:

Triangle 2:

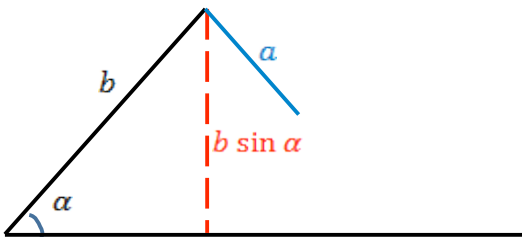
For example, if you are given; triangle ABC with $m\angle A = 35^\circ$, $a = 8$, $b = 10$ proceed through the following checklist. *If all 4 answers are yes, there are two triangles are possible! Find BOTH!*

1. SSA ? (yes)
2. given angle is acute ? (yes)
3. opposite side < second side ? (yes)
4. opposite side > height? (yes)

$\beta =$ _____ ; _____ $\gamma =$ _____ ; _____

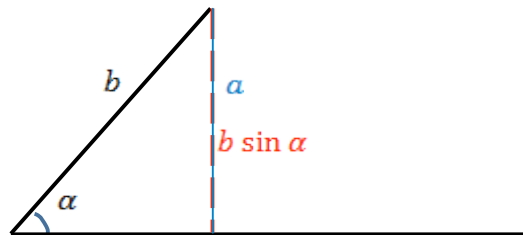
No Triangle

If $a < b \sin \alpha$ then side a isn't long enough to form a triangle.



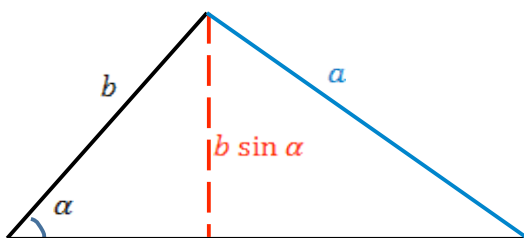
One Triangle – Right

If $a = b \sin \alpha$ then side a is just long enough to form a right triangle.



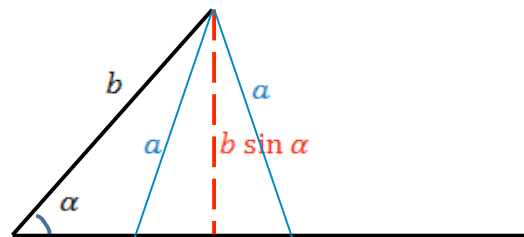
One Triangle - Oblique

If $a \geq b$, then only one triangle can be formed.



Two Triangles

If $a < b$ and $a > b \sin \alpha$, then two distinct triangles can be formed.



Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle that results.

7. $a = 3, b = 2, \alpha = 50$

8. $b = 5, c = 3, \beta = 100$

9. $a = 4, b = 5, \alpha = 60$

10. $b = 4, c = 6, \beta = 20$

11. $a = 2, c = 1, \gamma = 100$

12. Solve $\triangle ABC$ if $a=3$, $b=2$, $\alpha=40^\circ$.

$\beta =$ _____ ; _____
$\gamma =$ _____ ; _____
$c =$ _____ ; _____

13. Solve $\triangle ABC$ if $a=6$, $b=8$, $\alpha=35^\circ$.

$\beta =$ _____ ; _____
$\gamma =$ _____ ; _____
$c =$ _____ ; _____

14. Solve $\triangle ABC$ if $a=2$, $c=1$, $\gamma=50^\circ$.

$\beta =$ _____ ; _____
$\gamma =$ _____ ; _____
$c =$ _____ ; _____