

HOMWORK: THE AMBIGUOUS CASE

NAME: _____ DAY 4 DUE: _____

Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle. Solve all triangles.

1. $a = 3, b = 2, \alpha = 50^\circ$.

$$\frac{\sin \beta}{2} = \frac{\sin 50}{3}$$

$$\sin \beta = \frac{2 \sin 50}{3}$$

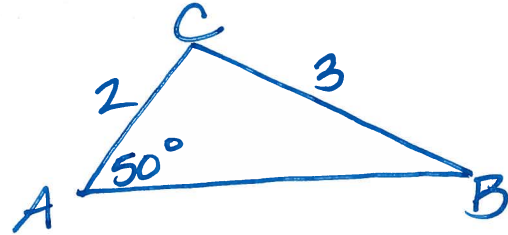
$$\sin \beta \approx 0.511$$

$$\beta \approx \sin^{-1}(0.511)$$

$$\boxed{\beta \approx 30.710^\circ}$$

$$\gamma = 180^\circ - 50^\circ - 30.710^\circ$$

$$\boxed{\gamma \approx 99.29^\circ}$$



Opp side longer
= One Triangle

$$\frac{\sin 99.3^\circ}{c} = \frac{\sin 50^\circ}{3}$$

$$\frac{c \sin 50^\circ}{\sin 50^\circ} = \frac{3 \sin 99.3^\circ}{\sin 50^\circ}$$

$$\boxed{c \approx 3.86}$$

2. $b = 5, c = 3, \beta = 100^\circ$

$$\frac{\sin 100^\circ}{5} = \frac{\sin \gamma}{3}$$

$$\frac{5 \sin \gamma}{5} = \frac{3 \sin 100^\circ}{5}$$

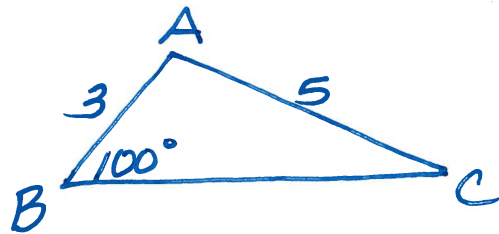
$$\sin \gamma = 0.590$$

$$\gamma = \sin^{-1}(0.590)$$

$$\boxed{\gamma \approx 36.16^\circ}$$

$$\alpha = 180^\circ - 36.16^\circ - 100^\circ$$

$$\boxed{\alpha \approx 43.84^\circ}$$



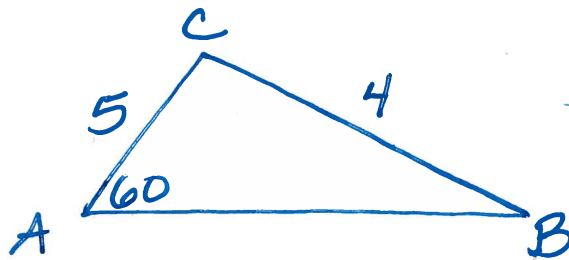
Opp side longer
= One Triangle

$$\frac{\sin 100}{5} = \frac{\sin 43.84}{a}$$

$$\frac{a \sin 100}{\sin 100} = \frac{5 \sin 43.84}{\sin 100}$$

$$\boxed{a \approx 3.51}$$

3. $a=4, b=5, \alpha=60^\circ$.



$$h = 5 \sin 60$$

$$h = 4.33$$

4 is too short!

No Triangle

4. $b=4, c=6, \beta=20^\circ$.

$$\frac{\sin 20^\circ}{4} = \frac{\sin \gamma}{6}$$

$$\frac{4 \sin \gamma}{4} = \frac{6 \sin 20}{4}$$

$$\sin \gamma \approx 0.513$$

$$\gamma \approx \sin^{-1}(0.513)$$

$$\boxed{\gamma \approx 30.86^\circ}$$

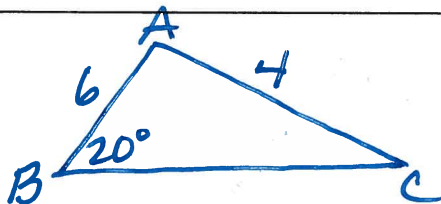
$$\alpha = 180 - 30.86 - 20$$

$$\boxed{\alpha \approx 129.14^\circ}$$

$$\frac{\sin 20^\circ}{4} = \frac{\sin 129.1^\circ}{a}$$

$$\frac{a \sin 20}{\sin 20} = \frac{4 \sin 129.1}{\sin 20}$$

$$\boxed{a \approx 9.08}$$



$$h = 6 \sin 20$$

$$h = 2.05$$

2 TRIANGLES!

$$\gamma_2 \approx 180 - 30.86 \approx \boxed{149.14}$$

$$\alpha_2 = 180 - 149.14 - 20^\circ$$

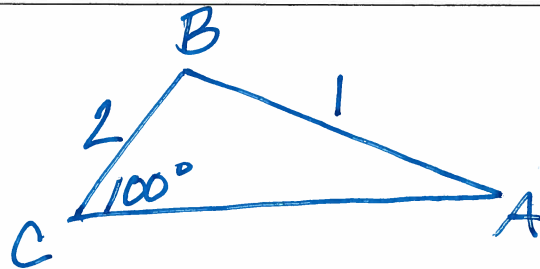
$$\alpha_2 \approx \boxed{10.86^\circ}$$

$$\frac{\sin 20^\circ}{4} = \frac{\sin 10.9^\circ}{a}$$

$$\frac{a \sin 20^\circ}{\sin 20^\circ} = \frac{4 \sin 10.9^\circ}{\sin 20^\circ}$$

$$\boxed{a \approx 2.21}$$

5. $a=2, c=1, \gamma=100^\circ$.



$$h = 2 \sin 100^\circ$$

$$h \approx 1.97$$

1 is too small.

No Triangle

6. $a=2, c=1, \gamma=25^\circ$.

$$\frac{\sin \alpha}{2} = \frac{\sin 25^\circ}{1}$$

$$1 \sin \alpha = 2 \sin 25^\circ$$

$$\sin \alpha \approx 0.845$$

$$\alpha \approx \sin^{-1}(0.845)$$

$$\alpha \approx 57.7^\circ$$

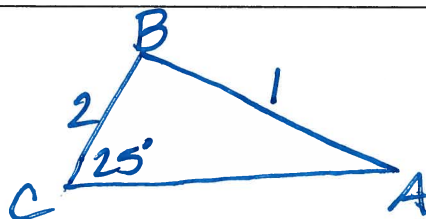
$$B = 180 - 57.7 - 25$$

$$B = 97.3^\circ$$

$$\frac{\sin 97.3^\circ}{b} = \frac{\sin 25^\circ}{1}$$

$$b \sin 25^\circ = \frac{\sin 97.3^\circ}{\sin 25^\circ}$$

$$b \approx 2.35$$



$$h = 2 \sin 25$$

$$h = 0.845$$

2 Triangles!

OR

$$\alpha_2 = 180 - 57.7^\circ$$

$$\alpha_2 \approx 122.3^\circ$$

$$B_2 = 180 - 122.3 - 25^\circ$$

$$B_2 \approx 32.7^\circ$$

$$\frac{\sin 32.7^\circ}{b} = \frac{\sin 25^\circ}{1}$$

$$b \sin 25^\circ = \frac{\sin 32.7^\circ}{\sin 25^\circ}$$

$$b \approx 1.28$$