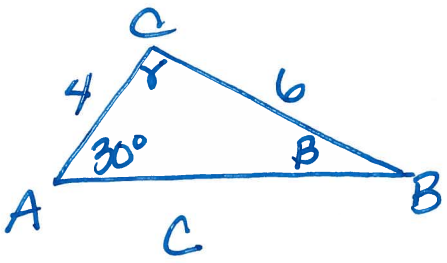
Ch 7.25 – Ambiguous Case

DAY 4

Consider the following for ΔABC, $\alpha = 30°$, $a = 6$, $b = 4$. Try to solve this using the Law of Sines.



$$\frac{6}{\sin 30°} = \frac{4}{\sin \beta}$$

$$\frac{6\sin\beta}{6} = \frac{4\sin 30°}{6}$$

$$\sin\beta = \frac{4\sin 30}{6}$$

$$\sin\beta = \frac{2}{3}\left(\frac{1}{2}\right)$$

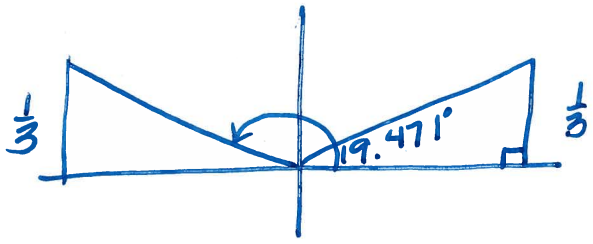
$$\sin\beta = \frac{1}{3}$$

$$\beta = \sin^{-1}\left(\frac{1}{3}\right)$$

$$\boxed{\beta \approx 19.471°}$$

We should notice that our calculator only gives us one possibility for β but is this correct?

Looking at our unit circle, we know that $\sin\beta = \frac{1}{3}$ in two places.



$180° - 19.471°$

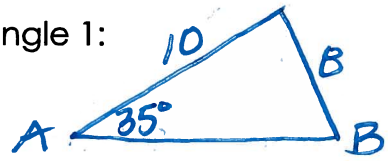
$$\boxed{160.529°}$$

When we are presented a triangle with two sides and the angle opposite one of the sides it is referred to as the Ambiguous Case. The given information may result in one triangle, two triangles, or no triangle.

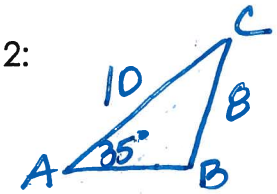
Example 1: Consider triangle ABC with $m\angle A = 35°$, a = 8, b = 10

When this occurs, there are 2 possible triangles, and you must find both solutions!

Triangle 1:



Triangle 2:



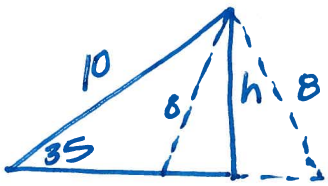
For example, if you are given; triangle ABC with $m\angle A = 35°$, a = 8, b = 10 proceed through the following checklist. *If all 4 answers are yes, there are two triangles are possible! Find BOTH !*

1. SSA ? (yes)
2. given angle is acute ? (yes)
3. opposite side < second side ? (yes)
4. opposite side > height? (yes)

$h = b\sin\alpha = 5.74$

$h < opp < second\ side$

$5.74 < 8 < 10$ ✓



$\frac{\sin B}{10} = \frac{\sin 35}{8}$

$8\sin B = 10\sin 35$

$\sin B = \frac{10\sin 35}{8}$

$B \approx 45.81°$

$B_2 = 180 - 45.81$

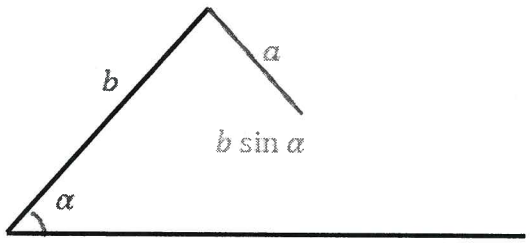
$B_2 = 134.20$

β= 45.81° ; 134.20°

γ= 99.20° ; 10.80°

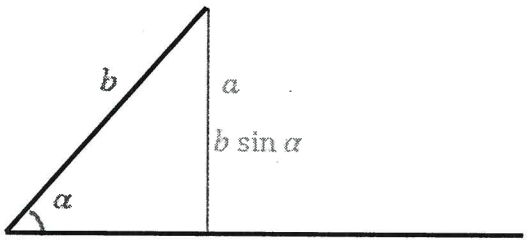
No Triangle

If $a < b\sin\alpha$ then side a isn't long enough to form a triangle.



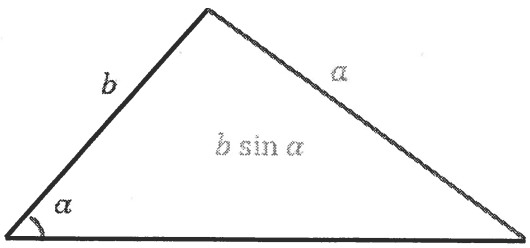
One Triangle – Right

If $a = b\sin\alpha$ then side a is just long enough to form a right triangle.



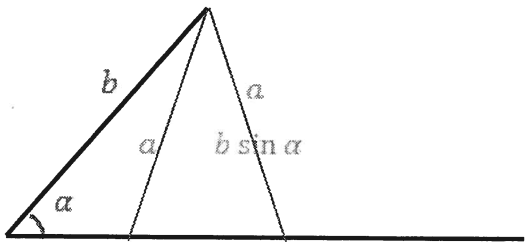
One Triangle - Oblique

If $a \geq b$, then only one triangle can be formed.



Two Triangles

If $a < b$ and $a > b\sin\alpha$, then two distinct triangles can be formed.



Erase

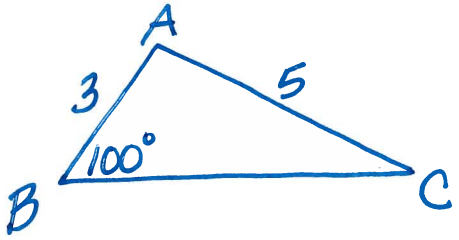
Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all.

7. $a = 3, b = 2, \alpha = 50$

Opposite side is larger than the second side.

One Triangle

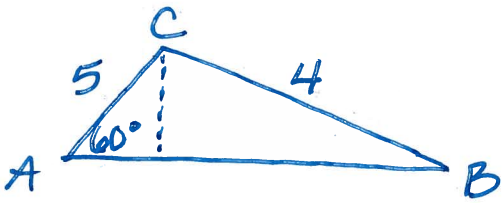
8. $b = 5, c = 3, \beta = 100$



One Triangle

Opposite side is larger than the second side.

9. $a = 4, b = 5, \alpha = 60$



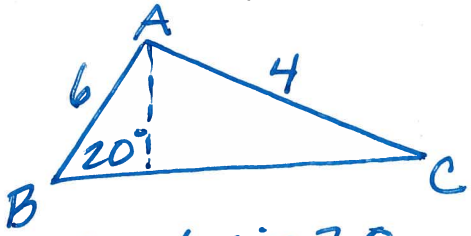
$h = 5 \sin 60$

$h = 4.33$

4 is too short

No Triangle

10. $b = 4, c = 6, \beta = 20$



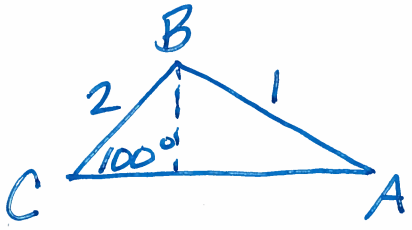
$h = 6 \sin 20$

$h = 2.05$

$2.05 < 4 < 6$

2 Triangles

11. $a = 2, c = 1, \gamma = 100$



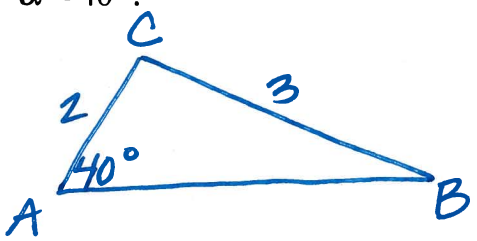
$h = 2 \sin 100$

$h = 1.97$

1 is too short

No Triangle

12. Solve ΔABC if $a=3,\ b=2,\ \alpha=40°$.



longer

β= 25.374
γ= 114.626
c = 4.243

$\frac{\sin B}{2} = \frac{\sin 40}{3}$

$\frac{3\sin B}{3} = \frac{2\sin 40}{3}$

$\sin B \approx 0.429$

$B \approx \sin^{-1}(0.429)$

$B \approx 25.374°$

$\gamma \approx 180 - 40 - 25.374$

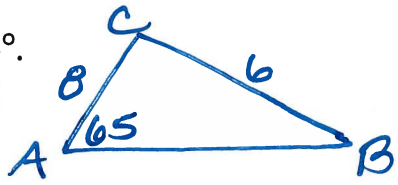
$\gamma \approx 114.626°$

$\frac{\sin 114}{c} = \frac{\sin 40}{3}$

$\frac{c\sin 40}{\sin 40} = \frac{3\sin 114}{\sin 40}$

$c \approx 4.243$

13. Solve ΔABC if $a=6,\ b=8,\ \alpha=35°$.



β= 49.886; 130.114
γ= 95.114; 114.886
c = 10.419; 2.687

$\frac{\sin B}{8} = \frac{\sin 35}{6}$

$\sin B = \frac{8\sin 35}{6}$

$B \approx \sin^{-1}(0.765)$

$B \approx 49.886°$

$\gamma = 95.114$

$\frac{\sin 95.114}{c} = \frac{\sin 35}{6}$

$c = \frac{6\sin 95.114}{\sin 35}$

$c \approx 10.419$

$h = 8\sin 65$

$h = 7.25$

2 Triangles

$B = 130.114°$

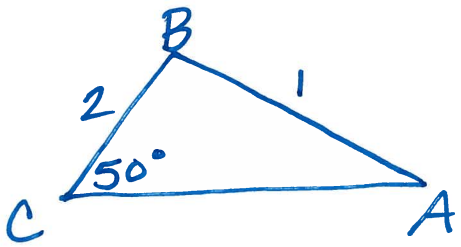
$\gamma = 14.886$

$\frac{\sin 114.886}{c} = \frac{\sin 35}{6}$

$\frac{c\sin 35}{\sin 35} = \frac{6\sin 114.886}{\sin 35}$

$c \approx 2.687$

14. Solve ΔABC if $a=2,\ c=1,\ \gamma=50°$.



β= ; ;
γ= ; ;
c = ; ;

$h = 2\sin 50$

$h = 1.53$

No Triangle