PRACTICE: SERIES

DAY 16

Write the series in **SIGMA NOTATION.**

1 (Same)	Series: $3 + 6 + 9 + 12 + 15 + 18$ Sequence: $3, 6, 9, 12, 15, 18$ Formula: $a_n = 3+3(n-1) = 3n$	$n =$ 6 $i =$ 1	$\sum_{i=1}^{6} 3i$
2 (Same)	Series: $6 + 8 + 10 + 12 + 14$ Sequence: $6, 8, 10, 12, 14$ Formula: $a_n = 6+2(n-1) = 2n+4$	$n =$ 5 $i =$ 1	$\sum_{i=1}^{5} 2i+4$
3	Series: $20 + 30 + 40 + 80$ Sequence: $20, 30, 40, 80$ Formula: $a_n = 20+10(n-1) = 10n+10$	$n =$ 4 $i =$ 1	$\sum_{i=1}^{4} 10i+10$

Determine if the series is arithmetic or geometric, or neither

4. $\sum_{i=1}^{10} 4 + 2i$ Arithmetic	5. $\sum_{i=1}^{7} 3i$ Arithmetic	6. $\sum_{i=1}^{4} 5(-3)^i$ Geometric
7. $\sum_{i=1}^{20} i$ Arithmetic	8. $\sum_{i=1}^{6} 8\left(\frac{3}{4}\right)^{i-1}$ Geometric	9. $\sum_{i=1}^{5} i^2 - 4$ Neither

Does each infinite series have a sum? Why or why not?

10. $\sum_{i=1}^{\infty}\left(-\frac{1}{2}\right)^{i-1}$ yes, $r = -\frac{1}{2}$ $\lvert r\rvert = \lvert -\frac{1}{2}\rvert = \frac{1}{2}$	11. $\sum_{i=1}^{\infty} 3\left(\frac{5}{4}\right)^{i-1}$ No. $\frac{5}{4} > 1$	12. $\sum_{i=1}^{\infty} 3\left(\frac{1}{4}\right)^{i-1}$ yes. $\frac{1}{4} > 1$

$\lvert r\rvert < 1$

Find the sum of each series!

A **13.** $\sum_{i=1}^{9} 12-7i$

$a_n = 12-7n$ $\quad n=9$, $a_1 = 5$, $a_9 = -51$

$S_9 = \frac{9}{2}(5-51)$

$S_9 = \frac{9}{2}(-46)$

$S_9 = -207$

14. $\sum_{i=1}^{6} 2(3)^{i-1}$ $\quad n=6$, $a_1=2$, $r=3$

G

$a_n = 2(3)^{n-1}$

$S_6 = 2\left(\frac{1-3^6}{1-3}\right)$

$S_6 = 2\left(\frac{-728}{-2}\right)$

$S_6 = 728$

15. $\sum_{i=1}^{\infty} 3\left(-\frac{1}{2}\right)^{i-1}$ $\quad n=\infty$, $a_1=3$, $r=-\frac{1}{2}$ $\quad \infty$

$S_\infty = \frac{3}{1+\frac{1}{2}}$

$S_\infty = \frac{3}{1.5}$

$S_\infty = 2$

∞ **16.** $\sum_{i=1}^{\infty} 3\left(-\frac{1}{2}\right)^{i}$ $\quad n=\infty$, $a_1 = -\frac{3}{2}$, $r=-\frac{1}{2}$

$S_\infty = \frac{-\frac{3}{2}}{1+\frac{1}{2}}$

$S_\infty = \frac{-1.5}{1.5}$

$S_\infty = -1$

17. $\sum_{i=1}^{4} 64\left(-\frac{1}{4}\right)^{i-1}$ $\quad n=4$, $a_1=64$, $r=\frac{-1}{4}$

G

$a_n = 64\left(-\frac{1}{4}\right)^{n-1}$

$S_4 = 64\left(\frac{1+\frac{1}{4}^4}{1+\frac{1}{4}}\right)$

$S_4 = 64\left(\frac{1.003\ldots}{1.25}\right)$

$S_4 = 51.4$

18. $\sum_{i=1}^{4} -3+2i$ $\quad n=4$, $a_1=-1$, $a_4=5$ $\quad$ A

$a_n = -3+2n$

$S_4 = \frac{4}{2}(-1+5)$

$S_4 = 2(4)$

$S_4 = 8$

A 19. $\sum_{i=1}^{7} 3i$ $\quad n=7$, $a_1=3$, $a_7=21$

$a_n = 3n$

$S_7 = \frac{7}{2}(3+21)$

$S_7 = \frac{7}{2}(24)$

$S_7 = 84$

20. $\sum_{i=1}^{20} -12+4i$ $\quad n=20$, $a_1=-8$, $a_{20}=68$

A $\quad a_n = -12+4n$

$S_{20} = \frac{20}{2}(-8+68)$

$S_{20} = 10(60)$

$S_{20} = 600$

21. $\sum_{i=0}^{4} 5(-3)^{i}$ $\quad n=5$, $a_1=-15$, $r=-3$ $\quad$ G

$a_n = 5(-3)^n$

$S_5 = 5\left(\frac{1+3^5}{1+3}\right)$

$S_5 = 5\left(\frac{1+243}{4}\right)$

$S_5 = 5\left(\frac{244}{4}\right)$

$S_5 = 305$

Find the sum of the following series.

A 1. $5 + 8 + 11 + \ldots + 41$

$n = \frac{41-5}{3} + 1 = 13$

$S_{13} = \frac{13}{2}(5+41)$

$S_{13} = 299$

A 2. $2 + 4 + 8 + \ldots + 64$

$n = \frac{64-2}{2} + 1 = 32$

$S_{32} = \frac{32}{2}(2+64)$

$S_{32} = 1056$

3. $\sum_{i=1}^{\infty} 9\left(\frac{2}{3}\right)^{i-1}$ ∞

$n = \infty$, $a_1 = 9$, $r = \frac{3}{2}$

$a_n = 9\left(\frac{3}{2}\right)^{n-1}$

$\text{Sum} = \infty$

A 4. $\sum_{i=1}^{9} 2+i$

$n = 9$, $a_1 = 3$, $a_9 = 11$

$a_n = 2+n$

$S_9 = \frac{9}{2}(3+11)$

$S_9 = \frac{9}{2}(14)$

$S_9 = 63$

G 5. $\sum_{i=1}^{5} 3\left(\frac{1}{4}\right)^{i-1}$

$n = 5$, $a_1 = 3$, $r = 1/4$

$a_n = 3\left(\frac{1}{4}\right)^{n-1}$

$S_5 = 3\left(\frac{1-\frac{1}{4}^5}{1-\frac{1}{4}}\right)$

$S_5 = 3\left(\frac{0.999\ldots}{0.75}\right)$

$S_5 \approx 3(1.332\ldots) \approx 3.996$

6. $\sum_{i=1}^{10} 8\left(\frac{3}{2}\right)^{i-1}$ G

$n = 10$, $a_1 = 8$, $r = \frac{3}{2}$

$a_n = 8\left(\frac{3}{2}\right)^{n-1}$

$S_{10} = 8\left(\frac{1-\frac{3}{2}^{10}}{1-\frac{3}{2}}\right)$

$S_{10} = 8(113.330\ldots)$

$S_{10} \approx 906.641$

A 10. $\sum_{i=1}^{100} 2+i$

$n = 100$, $a_1 = 3$, $a_n = 102$

$a_n = 2+n$

$S_{100} = \frac{100}{2}(3+102)$

$S_{100} = 50(105)$

$S_{100} = 5250$

A 11. $\sum_{i=1}^{20} 50-2i$

$n = 20$, $a_1 = 48$, $a_{20} = 10$

$a_n = 50-2n$

$S_{20} = \frac{20}{2}(48+10)$

$S_{20} = 10(58)$

$S_{20} = 580$

12. $\sum_{i=1}^{\infty} 9\left(\frac{2}{3}\right)^{i-1}$

$n = \infty$, $a_1 = 9$, $r = 2/3$

$a_n = 9\left(\frac{2}{3}\right)^{n-1}$

$S_\infty = \frac{9}{1-2/3}$

$S_\infty = 27$

∞ 13. $\sum_{i=1}^{\infty} 64\left(\frac{1}{4}\right)^{i-1}$

$n = \infty$, $a_1 = 64$, $r = \frac{1}{4}$

$a_n = 64\left(\frac{1}{4}\right)^{n-1}$

$S_\infty = \frac{64}{1-1/4}$

$S_\infty \approx 85.333$

14. $\sum_{i=4}^{8} \frac{1}{9}(3)^i$

$n = 8$, $a_1 = \frac{1}{3}$, $r = 3$

G $a_n = \frac{1}{9}(3)^n$

$S_8 = \frac{1}{3}\left(\frac{1-3^8}{1-3}\right)$

$S_8 = \frac{1}{3}\left(\frac{-6560}{-2}\right)$

$S_8 \approx 1093.333$

15. $\sum_{i=0}^{4} 6(3)^i$

$n = 5$, $a_0 = 6$, $r = 3$

G $a_n = 6(3)^n$

$S_5 = 6\left(\frac{1-3^5}{1-3}\right)$

$S_5 = 6\left(\frac{-242}{-2}\right)$

$S_5 = 726$

$60 + 40 + 20 = 120$

16. A ball is dropped off of a 40 foot cliff. Each time it hits the ground, the ball bounces up half of the previous height. Assuming the ball will bounce forever, what it the total vertical distance the ball will travel? (hint: you need to count the distance the ball travels both up and down. If you get 40 feet, you are only accounting for about half the distance!)

17. You are pushing your neighbor on a swing. You push the swing one time 12 feet on the first swing. On the next swing, your friend travels 9 feet, on the next swing your friend travels 6.75 feet, and the pattern continues. What is the total distance your friend travels after 5 swing? After 10 swings?

12

9

6.75

$a_n = 12(0.75)^{n-1}$

$2\left(\sum_{i=1}^{5} 12(0.75)^{n-1}\right)$ $n=5$, $a_1 = 12$, $r = 0.75$

$2\left(12\left(\frac{1-0.75^5}{1-0.75}\right)\right)$

$2(12(3.050\ldots)) \approx 73.219$

$2\left(\sum_{i=1}^{10} 12(0.75)^{n-1}\right)$ $n=10$, $a_1 = 12$, $r = 0.75$

$2\left(12\left(\frac{1-0.75^{10}}{1-0.75}\right)\right)$

$2(12(3.774\ldots))$

≈ 90.594

18. You buy a car for $12,000. Each year, the car depreciates in value by 20%. How much will the car be worth in 10 years?

$a_n = 12{,}000(1-0.2)^n$, $n = \#$years

$a_{10} = 12{,}000(0.8)^{10}$

$a_{10} = 12{,}000(0.107\ldots)$

$a_{10} \approx \$1288.49$

19. You buy an antique wardrobe for $300. Each year, the wardrobe appreciates by 7%. How much will the wardrobe be worth in 10 years?

$a_n = 300(1+0.07)^n$, $n = \#$ years

$a_{10} = 300(1.07)^{10}$

$a_{10} = 300(1.967\ldots)$

$a_{10} \approx \$590.145$

Sequences and Series Summary

Explicit Formula: A formula that expresses the value of each term using term numbers

Compare: term value and term number.

Recursive Formula: A formula that expresses the value of each term using the previous term

Compare: term value and previous term value

	Sequence (a_n)	Series S_n (Σ)
Arithmetic	$a_n = a_1 + d(n-1)$ $a_1 = \square$ $a_n = (a_{n-1}) + d$	$S_n = \frac{n}{2}(a_1 + a_n)$
Geometric	$a_n = a_1(r)^{n-1}$ $a_1 = \square$ $a_n = r(a_{n-1})$	$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$
Finite	To find n: $\frac{a_n - a_1}{d} + 1$	$\sum_{i=1}^{10} 2(3)^{i-1}$ $n=10$, $a_1 = 2$ $\sum_{i=1}^{10} 2(3)^{i}$ $n=10$, $a_1 = 6$ $\sum_{i=0}^{10} 2(3)^{i}$ $n=11$, $a_0 = 2$
Infinite		$S_\infty = \frac{a_1}{1-r}$ $\lvert r \rvert < 1$

$n = \#$ terms
$a_1 = 1^{st}$
$a_n = $ last

if $|r| > 1$, $S_\infty = \infty$