Logarithm Properties DAY 4

Two Special Logarithms

A common logarithm is a logarithm with base 10. It is denoted as $\log_{10}$ or just simply log. A natural logarithm is a logarithm with base e. It can be denoted $\log_e$ or simply ln
Your calculator can be used to evaluate $\log_{10}$ or $\log_e$

Properties of Logarithms

Let b, m, and n be positive numbers such that b ≠ 1

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

EXAMPLE TWO→ Expand the logarithmic expression

a) $\ln\left(\frac{(2x)^2}{xz^6}\right)$

b) $\log\left(\frac{6a^2}{(2b)^5}\right)$

Step 1: Simplify (no radicals)

Step 2: Expand.
Numerators are positive.
Denominators are negative.

Step 3: Use the power rule.

c) $\log_a\left(\frac{x}{x+1}\right)$

d) $\ln\left(\frac{x\sqrt{x+1}}{(x-2)^3}\right)$

EXAMPLE THREE→ Condense the logarithmic expression

Step 1: Use the power rule.

Step 2: Condense to one log.
Positive on the numerator.
Negative on the denominator.

Step 3: Simplify (no radicals)

(a) $\log(9) - \log(3) + 3\log(2)$

(b) $\ln(4) + 3\ln(a) - 2\ln(b) - 2\ln(2)$

(c) $\log\left(\frac{x^2}{x^2-1}\right)+\log\left(\frac{x+1}{x}\right)$

(d) $\log(x^2+2x+1)-\log(x^2-1)$

Change of Base

If a, b, and c are positive numbers with b ≠ 1 and c ≠ 1, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

This means we can re-write any logarithm with a base "e" or "10" to evaluate it in our calculators!

EXAMPLE FOUR→ Evaluate the logarithmic expressions and check using your calculator

(a) $\log_4 64 =$ _____

(b) $\log_2 32 =$ _____

(c) $\log_3 \frac{1}{27} =$ _____

(d) $\log_{81} 3 =$ _____