Common Logarithms = Base 10

Sometimes you will see a logarithm with no base, such as those seen below in the table. These are assumed to have a base of 10. The "log" key on your calculator uses base 10, even though it doesn't say so. Check it out!

Knowing that the following logs are all base 10, evaluate each of them:

$\log 100{,}000{,}000$	=	8
$\log 10{,}000{,}000$	=	7
$\log 1{,}000{,}000$	=	6
$\log 100{,}000$	=	5
$\log 10{,}000$	=	4
$\log 1{,}000$	=	3
$\log 100$	=	2
$\log 10$	=	1
$\log 1$	=	0
$\log 0.1$	=	-1
$\log 0.01$	=	-2
$\log 0.001$	=	-3
$\log 0.0001$	=	-4
$\log 0.00001$	=	-5
$\log 0.000001$	=	-6
$\log 0.0000001$	=	-7
$\log 0.00000001$	=	-8

Try the following on your calculator. Round decimals to the nearest hundredth.

1. $\log 50$ 1.70
2. $\log 400$ 2.60
3. $\log 7{,}000$ 3.85
4. $\log 20{,}000$ 4.30

Logarithms allow us to work with very large and very small numbers easily!

Now let's look at some properties of logarithms!

1. Does log10 + log 100 = log (10 + 100)?

$1 + 2 \neq \log(110)$ No!

$1 + 2 = 3$ 1000

2. Does log1,000 + log10,000 = log (1,000 + 10,000)?

$3 + 4 \neq \log(11,000)$

10,000,000

Let's try some with base 2.

3. $\log_2 4 + \log_2 8 = \log_2$ $\boxed{32}$

$2 + 3 = 5$

4. $\log_2 16 - \log_2 2 = \log_2$ $\boxed{8}$

$4 - 1 = 3$

What did we learn from this?

$\log_b M + \log_b N = \log_b$ $\boxed{MN}$

$\log_b M - \log_b N = \log_b$ $\boxed{M/N}$

Log Properties

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M)^n = n \cdot \log_b M$$

Using Properties of Logs: Use $\log_5 3 \approx 0.683$ and $\log_5 7 \approx 1.209$ to approximate:

1. $\log_5 21 =$ $\log_5 7 + \log_5 3$
   $0.683 + 1.209$
   1.892

2. $\log_5\left(\frac{3}{7}\right) =$ $\log_5 3 - \log_5 7$
   $0.683 - 1.209$
   -0.526

3. $\log_5 49 =$ $\log_5 7 + \log_5 7$ $\qquad \log_5 7^2 = 2(\log_5 7)$
   $1.209 + 1.209$
   2.418

4. $\log_5 27 =$ $\log_5 3 + \log_5 3 + \log_5 3$ $\qquad \log_5 27$
   $0.683 + 0.683 + 0.683$ $\qquad \log_5 3^3$
   2.049 $\qquad 3 \cdot (\log_5 3)$

5. $\log_5 63 =$ $\log_5 9 + \log_5 7$
   $\log_5 3^2 + \log_5 7$
   $2\log_5 3 + \log_5 7$
   $2(0.683) + 1.209 = 2.575$

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Using Properties of Logs: Expand the following logarithmic expressions:

1. $\log_5 (xy)$

$\log_5 x + \log_5 y$

2. $\log_2 6x^5$

$\log_2 (6 \cdot x^5)$

$\log_2 6 + \log_2 x^5$

$\log_2 6 + 5\log_2 x$

3. $\log_3 \frac{4}{5}$

$\log_3 4 - \log_3 5$

4. $\log_4 3ab^2$

$\log_4 (3 \cdot a \cdot b^2)$

$\log_4 3 + \log_4 a + \log_4 b^2$

$\log_4 3 + \log_4 a + 2\log_4 b$

5. $\log_5 \frac{3x}{y^2}$

$\log_5 3x - \log_5 y^2$

$\log_5 3 + \log_5 x - 2\log_5 y$

6. $\log \sqrt{x}$

$\log x^{1/2}$

$\frac{1}{2} \log x$

Using Properties of Logs: Condense the following logarithmic expressions:

1. $\log_3 8 + \log_3 5$

$\log_3 (8 \cdot 5)$

$\log_3 (40)$

2. $\log_2 3 + \log_2 y$

$\log_2 (3y)$

3. $\log_4 100 - \log_4 5$

$\log_4 \left(\frac{100}{5}\right)$

$\log_4 20$

4. $3\log_5 a + \log_5 b - 2\log_5 c$

$\log_5 a^3 + \log_5 b - \log_5 c^2$

$\log_5 \frac{a^3 b}{c^2}$

5. $\log_2 8 + \log_2 10 - \log_2 5$

$\log_2 \left(\frac{8 \cdot 10}{5}\right)$

$\log_2 \left(\frac{80}{5}\right)$

$\log_2 (16)$

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6. $\frac{1}{2}\log_2 16 + \log_2 1$

$\log_2 16^{1/2} + \log_2 1$

$\log_2 4 + \log_2 1$

$\log_2 (4 \cdot 1)$

$\log_2 4$

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Logarithm Properties

DAY 4

Two Special Logarithms

A common logarithm is a logarithm with base 10. It is denoted as $\log_{10}$ or just simply log. A natural logarithm is a logarithm with base e. It can be denoted $\log_e$ or simply ln
Your calculator can be used to evaluate $\log_{10}$ or $\log_e$

Properties of Logarithms

Let b, m, and n be positive numbers such that $b \neq 1$

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \log_b m$

EXAMPLE TWO→ Expand the logarithmic expression

Step 1: Simplify (no radicals)

Step 2: Expand.
Numerators are positive.
Denominators are negative.

Step 3: Use the power rule.

a) $\ln\left(\frac{(2x)^2}{xz^6}\right)$

$\ln\left(\frac{4x^2}{xz^6}\right)$

$\ln\left(\frac{4x}{z^6}\right)$

$\boxed{\ln 4 + \ln x - 6\ln z}$

b) $\log\left(\frac{6a^2}{(2b)^5}\right)$

$\log\left(\frac{6a^2}{2^5b^5}\right)$

$\log\left(\frac{3a^2}{16b^5}\right)$

$\boxed{\log 3 + 2\log a - \log 16 - 5\log b}$

c) $\log_a\left(\frac{x}{x+1}\right)$

$\boxed{\log_a x - \log_a (x+1)}$

d) $\ln\left(\frac{x\sqrt{x+1}}{(x-2)^3}\right)$

$\ln\left(\frac{x(x+1)^{1/2}}{(x-2)^3}\right)$

$\ln x + \ln(x+1)^{1/2} - \ln(x-2)^3$

$\boxed{\ln x + \frac{1}{2}\ln(x+1) - 3\ln(x-2)}$

OR $\boxed{\ln x + \frac{\ln(x+1)}{2} - 3\ln(x-2)}$

EXAMPLE THREE→ Condense the logarithmic expression

Step 1: Use the power rule.

Step 2: Condense to one log.
Positive on the numerator.
Negative on the denominator.

Step 3: Simplify (no radicals)

(a) $\log(9) - \log(3) + 3\log(2)$

$\log 9 - \log 3 + \log 2^3$

$\log\left(\frac{9 \cdot 8}{3}\right) = \boxed{\log(24)}$

(b) $\ln(4) + 3\ln(a) - 2\ln(b) - 2\ln(2)$

$\ln 4 + \ln a^3 - \ln b^2 - \ln 2^2$

$\ln\left(\frac{4a^3}{4b^2}\right) = \boxed{\ln\left(\frac{a^3}{b^2}\right)}$

(c) $\log\left(\frac{x^2}{x^2-1}\right) + \log\left(\frac{x+1}{x}\right)$

$\log\left(\left(\frac{x^2}{x^2-1}\right)\left(\frac{x+1}{x}\right)\right)$

$\log\left(\frac{x^2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)(x)}\right)$

$\boxed{\log \frac{x}{(x-1)}}$

(d) $\log(x^2+2x+1) - \log(x^2-1)$

$\log(x+1)^2 - \log(x^2-1)$

$\log \frac{(x+1)^2}{(x^2-1)}$

$\boxed{\log\left(\frac{x+1}{x-1}\right)}$

<u>Change of Base</u>

If a, b, and c are positive numbers with b ≠ 1 and c ≠ 1, then:

$$\log_c a = \frac{\log_b a}{\log_b c}$$

This means we can re-write any logarithm with a base "e" or "10" to evaluate it in our calculators!

EXAMPLE FOUR→ Evaluate the logarithmic expressions and check using your calculator

(a) $\log_4 64 =$ 3

(b) $\log_2 32 =$ 5

(c) $\log_3 \frac{1}{27} =$ −3

(d) $\log_{81} 3 =$ 1/4 $\quad 81^{1/4} = 3$