NOTES: SYNTHETIC SUBSTITUTION

DAY 3

Textbook Chapter 5.2

OBJECTIVE: Today you will learn about how to evaluate a function using synthetic substitution and also how to write a model for a polynomial function.

METHOD 1: Evaluate by Direct Substitution

Use ***Direct Substitution*** to evaluate $f(x) = -3x^3 + x^2 - 12x - 5$ when $x = -2$

$$f(-2) = 3(-2)^3 + (-2)^2 - 12(-2) - 5$$
$$= 3(-8) + 4 + 24 - 5$$
$$= 47$$

$(-2, 47)$

$f(-2) = 47$

METHOD 2: Evaluate by Synthetic Substitution

Use ***Synthetic Substitution*** to evaluate $f(x) = -3x^3 + x^2 - 12x - 5$ when $\mathbf{x = -2}$

−2	−3	1	−12	−5
	↓	6	−14	52
	−3	7	−26	47

1. Bring Down
2. Multiply and Add
3. Multiply and Add
4. Multiply and Add…

answer!

$-2, 47$

$f(-2) = 47$

Use ***Synthetic Substitution*** to evaluate $f(x) = -2x^4 - x^3 + 4x - 5$ when $x = -1$

**Write the coefficients of $f(x)$ in order of descending exponents *(there is $0x^2$)*

−1	−2	−1	0	4	−5
	↓	2	−1	1	−5
	−2	1	−1	5	−10

$-1, -10$

$f(-1) = -10$

NOTES: MODELING POLYNOMIAL FUNCTIONS

Textbook Chapter 5.7

The Fundamental Theorem of Algebra

An **n^{th} degree** polynomial function has exactly **n** **solutions.**

1. $x^4 + 8x^2 - 2x + 2 = 0$ Number of zeros: 4 Number of real zeros: 0 (4 imag.)

2. $g(x) = x^3 - x^2 - 3x - 3$ Number of zeros: 3 Number of real zeros: 1 (2 imag.)

Complex Conjugate Theorem

If f is a polynomial function, and a + bi is an imaginary zero, then a – bi must also be a zero.

If f is a polynomial function, and $a + \sqrt{b}$ is an irrational zero, then $a - \sqrt{b}$ must also be a zero.

Given a zero, identify another zero

3. $-\sqrt{5}$ $+\sqrt{5}$ 4. $6-2i$ $6+2i$ 5. $-3-2\sqrt{7}$ $-3+2\sqrt{7}$ 6. $3i$ $-3i$

Write a Polynomial Function given the Zeros

7. Find the polynomial with a leading coefficient of 2 that has the given zeros: 4, 1, –2 ($a=2$)

Write f(x) in factored form: $f(x) = 2(x-4)(x-1)(x+2)$

Change to Standard Form:
$f(x) = 2(x^2-4x-x+4)(x+2)$
$f(x) = 2(x^3+2x^2-5x^2-10x+4x+8)$
$f(x) = 2(x^3-3x^2-6x+8)$
$\boxed{f(x) = 2x^3-6x^2-12x+16}$

8. Find the polynomial with a leading coefficient of 1 that has the given zeros: 5, 3, –2i

Write f(x) in factored form: $f(x) = 1(x-5)(x-3)(x-2i)(x+2i)$

Change to Standard Form:
$f(x) = (x^2-8x+15)(x^2-2xi+2xi-4i^2)$
$f(x) = (x^2-8x+15)(x^2+4)$
$f(x) = x^4+4x^2-8x^3-32x+15x^2+60$
$\boxed{f(x) = x^4-8x^3+19x^2-32x+60}$

PRACTICE: SYNTHETIC SUBSTITUTION

DAY 3

Evaluate with synthetic substitution.

1. $f(x) = 2x^4 - 5x^3 - 4x + 8$ for $x = 3$

$$\begin{array}{c|ccccc} 3 & 2 & -5 & 0 & -4 & 8 \\ & \downarrow & 6 & 3 & 9 & 15 \\ \hline & 2 & 1 & 3 & 5 & \boxed{23} \end{array}$$

$\boxed{f(3) = 23}$

2. $f(x) = x^3 - 2x^2 - 23x + 60$ for $x = 3$

$$\begin{array}{c|cccc} 3 & 1 & -2 & -23 & 60 \\ & \downarrow & 3 & 3 & -60 \\ \hline & 1 & 1 & -20 & 0 \checkmark \end{array}$$

$f(3) = 0$

3 is a zero

$(x-3)$ is a factor

Given Factors, Identify the Zeros

3. $(x - 5)(x - 2)$ 5, 2

4. $(x + 6)^2$ -6, -6

5. $3x(x - 2)$ 0, 2

6. $4x^3(2x - 1)$ 0, 1/2

7. $(3x - 1)(6x + 5)$ 1/3, -5/6

Given a List of Zeros, write as a List of Factors

8. 7 $(x-7)$

9. $x = -2$ $(x+2)$

10. $x = 5, -8$ $(x-5)(x+8)$

11. $x = \pm 6$ $(x+6)(x-6)$

12. $x = \pm 2\sqrt{3}$ (x^2-12)

13. $x = \frac{2}{3}$ $3x-2=0$ $(3x-2)$

14. $x = -\frac{7}{3}$ $(x+7/3)$

15. $x = -\frac{1}{4}, 5$ $(4x-1)(x-5)$

$x - 2\sqrt{3} = 0$

$+2\sqrt{3} \quad +2\sqrt{3}$

$x = 2\sqrt{3}$

$x^2 = (2\sqrt{3})^2$

$x^2 = 4(3)$

$x^2 = 12$

$x^2 - 12 = 0$

$(x + \frac{1}{4} = 0) \cdot 4$

$4x + 1 = 0$

Given one zero, find another other zero.

16. $x = 2 + 5i$ $2-5i$

17. $x = 3-\sqrt{5}$ $3+\sqrt{5}$

18. $x = \sqrt{11}$ $-\sqrt{11}$

19. $x = -8i$ $8i$

20. Write the polynomial function (in factored form) with leading coefficient 1 and the following zeros: –2, 3, 4

$f(x) = 1(x+2)(x-3)(x-4)$

$f(x) = (x^2-x-6)(x-4)$

$f(x) = x^3-4x^2-x^2+4x-6x+24$

$\boxed{f(x) = x^3-5x^2-2x+24}$

21. Write the polynomial function (in standard form) with leading coefficient 2 and the following zeros: 5, 2i

$y = 2(x-5)(x+2i)(x-2i)$

$y = 2(x-5)(x^2+2i-2i-4i^2)$

$y = 2(x-5)(x^2+4)$

$y = 2(x^3-5x^2+4x-20)$

$\boxed{y = 2x^3-10x^2-8x-40}$

22. Write the polynomial function (in standard form) with leading coefficient 2 and the following zeros: 5, $\sqrt{3}$

$y = 2(x-5)(x-\sqrt{3})(x+\sqrt{3})$

$y = 2(x-5)(x^2-x\sqrt{3}+x\sqrt{3}-\sqrt{3}\sqrt{3})$

$y = 2(x-5)(x^2-3)$

$y = 2(x^3-5x^2-3x+15)$

$\boxed{y = 2x^3-10x^2-6x+30}$